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# STATIC, FREE VIBRATION, AND STABILITY ANALYSIS OF THIN, ELASTIC SHELLS OF REVOLUTION

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# STATIC, FREE VIBRATION, AND STABILITY ANALYSIS OF THIN, ELASTIC SHELLS OF REVOLUTION

*ARTURS KALNINS*

## FOREWORD

This final technical report was prepared by the Office of Research of Lehigh University, Bethlehem, Pennsylvania, under Air Force Contract No. AF 33(615)-3870. It was administered under the Structures Division and the Vehicle Dynamics Division, Air Force Flight Dynamics Laboratory, with Messrs T.N. Bernstein (FDTR) and R.F. Taylor (FDDS), acting as Project Engineers.

This report was completed in September 1968 and covers the work performed from April 1966 to July 1968. The supervision of the project, the development of the mathematical analyses, and the programming of the Static and Axisymmetric Eigenvalue Programs were carried out by Dr. Arturs Kalnins, Professor of Mechanics at Lehigh University. Dr. A. B. Perlman, Assistant Professor of Mechanical Engineering at Tufts University, Medford, Massachusetts, suggested and developed the method of solution and wrote the computer program for the Nonsymmetric Eigenvalue Program.

This technical report has been reviewed and is approved.

FRANCIS J. JANIK, JR.
Chief, Theoretical Mechanics Branch
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## ABSTRACT

This project was undertaken to present workable methods of analyses for thin, elastic shells of revolution, and to provide computer programs for performing such analyses. By means of these methods, the following problems for a thin, elastic shell of revolution can be solved: (1) stresses and deflections can be determined when the shell is subjected to arbitrary mechanical and/or thermal loads; (2) natural frequencies and mode shapes can be found for free vibration when the shell is subjected to or is free of prestress; (3) buckling loads, according to the classical stability theory, can be found when the shell is subjected to axisymmetric or sinusoidal nonsymmetric prestress. The results of the static and free-vibration analyses have been verified and compared to experiments on many occasions and should be regarded as acceptable. The buckling load, however, may or may not correspond to the actual collapse load of the shell.


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## SYMBOLS

| | |
|---|---|
| $v$, $u$, $\beta$ | displacement and rotation vectors |
| $T_1$, $T_2$, $T_3$ | unit tangent vectors |
| $u_1$, $u_2$, $u_3$ | components of displacement |
| $\beta_1$, $\beta_2$ | components of rotation |
| $A_1$, $A_2$, $a_1$, $a_2$ | components of metric tensor |
| $N_i$, $M_i$ | resultant force and couple vectors |
| $N_{11}$, $N_{12}$, $N_{22}$ | membrane stress resultants |
| $Q_1$, $Q_2$ | transverse shear resultants |
| $M_{11}$, $M_{12}$, $M_{22}$ | moment resultants |
| $R_{11}$, $R_{12}$, $R_{22}$, $r_{11}$, $r_{12}$, $r_{22}$ | curature parameters |
| $p_1$, $p_2$, $p_3$ | surface loads |
| $E_1$, $E_2$ | Young's moduli |
| $\nu_{12}$, $\nu_{21}$ | Poisson's ratios |
| $T$ | temperature |
| $T_A$, $T_M$ | average temperature and its moment |
| $\alpha_1$, $\alpha_2$ | coefficients of thermal expansion |
| $z_i$ | coordinates of layers |
| $\theta$, $s$, $r$, $\phi$ | coordinates for a shell of revolution |
| $\xi_1$, $\xi_2$, $\xi_3$ | coordinates for a general shell |

## PART I. THEORY AND ANALYSIS OF THIN, ELASTIC SHELLS

### INTRODUCTION

The formulation of the basic theory which governs the deformation of a thin, elastic shell dates back to the end of the nineteenth century. At that time, it represented a natural outgrowth of the earlier developments of the three-dimensional, linear theory of elasticity and a complete understanding of the theories for the deflection of slender beams and thin plates. Using exactly the same fundamental assumptions which had been made for beams and plates, only the concept of a curved surface, through differential geometry, had to be added in order to produce a theory for the deflection of a thin shell. The motivation for the derivation of a shell theory at that time probably did not come from expected immediate applications, although the aim of the initial papers on this topic was said to be directed at the development of a mathematical theory for the analysis of vibration of bells.

The greatest difficulty in the application of shell theory to actual analysis was found in the fact that the governing equations were so complicated that either further simplifying assumptions had to be made or only a few very simple shell shapes could be analyzed. While the simplifying assumptions, such as the assumption of purely extensional (membrane) or inextensional states, did extend somewhat the class of shells which could be analyzed, their use introduced inaccuracies which sometimes could but at other times could not be recognized.

The advent of a high-speed digital computer, as in many other areas, has opened up great possibilities in shell analysis. One class of shells whose static, stability, and free-vibration analyses can now be regarded as feasible comprises all thin shells which are rotationally symmetric about one axis. The symmetry must include the geometry as well as the physical properties of the shell, so that no distinction can be made between any two points of the shell which are equidistant from the axis of symmetry. All shell properties, geometrical as well as physical, can vary arbitrarily along the meridian of the shell, and the shell can be represented by an arbitrary reference surface, as long as it is continuous and rotationally symmetric. The shell wall can consist of any number of layers which can be made of different orthotropic materials.

It can be now safely said that at least the static and free-vibration analyses of such shells of revolution have reached a state of art where the stresses and deflections or the natural frequencies and mode shapes can be determined in a routine manner.

The purpose of this report is to give to the designer workable methods for the analysis of a class of shells which includes all shells revolution. The methods are backed up by computer programs, also included in this report, by means of which solutions can be obtained. While the results for the static and free-vibration analyses for a properly described shell should be acceptable without any reservation, the results of the stability programs should be used with caution, because no simple estimate is now available which could relate the critical load, as predicted by the programs, with the actual collapse load of the shell. Further research in this direction is necessary.

## II. GOVERNING EQUATIONS FOR SHELL ANALYSIS

### 1. Introduction

The analysis of thin, elastic shells of revolution considered in this report can be classified into two cases:

1. Stress analysis of a shell subjected to mechanical and thermal surface loads and edge loads.

2. Free vibration and stability analysis of a prestressed shell.

The first case is reduced to a boundary value problem governed by a system of nonhomogeneous, linear, partial differential equations. The equations are separable with respect to the meridional and circumferential coordinates of the shell. The solution for each separable component of the loads is obtained by solving a typical two-point boundary value problem governed by eight first-order, linear, ordinary differential equations. The method of solution is given in [1]*.

The second case leads to an eigenvalue problem which is governed by a homogeneous system of linear, differential equations and homogeneous boundary conditions. The second case can be further divided into two groups of problems:

*Numbers in brackets refer to References at the end of this Part.

1. Free vibration and stability with axisymmetric prestress (including zero prestress).

2. Free vibration and stability with nonsymmetric prestress.

The partial differential equations for the eigenvalue problem with axisymmetric prestress are again separable, and the solution is obtained by solving a typical eigenvalue problem governed by a system of eight first-order, homogeneous, linear, ordinary differential equations. The method of solution is given in [2].

In the case with nonsymmetric prestress, the partial differential equations are not separable, and the problem cannot be solved exactly. The solution is approximated by some selected components of a Fourier series. By means of the method of weighted residuals, an approximate solution for the cases with nonsymmetric prestress is obtained by solving an eigenvalue problem which is governed by 8×k first-order, homogeneous, linear, ordinary differential equations, where k is the number of Fourier components used in the solution. Again, the method of solution is that given in [2].

In order to arrive at a governing system of equations, which is applicable for all the analyses considered in this report, we shall first write down the governing equations for a linear theory of shells when referred to a general orthogonal coordinate system. Such a system of equations can be found in a paper by Knowles and Reissner [3]. These equations will be complemented by the inclusion of the inertia terms, orthotropic layers, and an arbitrary reference surface as given in the theory derived by Kalnins [4].

For the stress analysis problem of a shell, these linear equations will constitute the governing system of equations. For the free-vibration and stability analysis of a prestressed shell, it will be regarded that the equilibrium equations are those for the <u>deformed</u> shell element. Then the governing equations for the prestressed shell will be developed by assuming that the solution consists of a prestressed state and an infinitesimal superimposed state. After subtracting out the equilibrium equations for the prestressed shell and omitting all square terms in the variables of the superimposed state, a linear, homogeneous system of equations for the free-vibration and stability problem of a prestressed shell will be obtained.

## 2. Governing Equations For Orthogonal Coordinates

The theory of shells derived in [4] is based on three assumptions:

1. Points on a normal of a reference surface before deformation remain on a straight line after deformation.

2. Distances between the points on a normal do not change during deformation.

3. Stresses are replaced by stress resultants.

The analysis of shells considered in this report will be applicable to a thin shell for which the following additional assumptions will be made:

4. Points on a normal of the reference surface before deformation remain on the same normal after deformation.

5. The ratio of the thickness to the minimum radius of curvature is negligible with respect to one.

Assumption #1 means that the displacement vector is assumed in the form

$$\underset{\sim}{v}(\xi_1,\xi_2,\xi_3) = \underset{\sim}{u}(\xi_1,\xi_2) + \xi_3\ \underset{\sim}{\beta}(\xi_1,\xi_2) \tag{2.1}$$

where $\xi_1$, $\xi_2$ denote the coordinates of an orthogonal coordinate system lying on the reference surface, and $\xi_3$ is the coordinate along the normal of the reference surface. The origin of $\xi_3$ is on the reference surface. The vectors $\underset{\sim}{u}$ and $\underset{\sim}{\beta}$ can be resolved into components defined by

$$\begin{aligned} \underset{\sim}{u} &= u_1\ \underset{\sim}{I}_1 + u_2\ \underset{\sim}{I}_2 + u_3\ \underset{\sim}{I}_3 \\ \underset{\sim}{\beta} &= \beta_1\ \underset{\sim}{I}_1 + \beta_2\ \underset{\sim}{I}_2 + \beta_3\ \underset{\sim}{I}_3 \end{aligned} \tag{2.2}$$

where $\underset{\sim}{T}_1$ and $\underset{\sim}{T}_2$ are the unit tangent vectors of the $\xi_1$, $\xi_2$ coordinate curves, and the unit normal of the reference surface is defined by

$$\underset{\sim}{T}_3 = \underset{\sim}{T}_1 \times \underset{\sim}{T}_2 \tag{2.3}$$

Assumption #2, which will be made throughout this analysis, requires that $\beta_3 = 0$.

The governing equations of a shell theory which is based on these five assumptions can be displayed in the following way. The equations of equilibrium are given in vector form by

$$(A_2 \underset{\sim}{N}_1)_{,1} + (A_1 \underset{\sim}{N}_2)_{,2} + A_1 A_2 \underset{\sim}{P} = 0$$

$$(A_2 \underset{\sim}{M}_1)_{,1} + (A_1 \underset{\sim}{M}_2)_{,2} + A_1 A_2 (\underset{\sim}{T}_1 \times \underset{\sim}{N}_1 + \underset{\sim}{T}_2 \times \underset{\sim}{N}_2) \tag{2.4}$$

$$+ A_1 A_2 \underset{\sim}{M} = 0$$

where $\underset{\sim}{N}_1$, $\underset{\sim}{N}_2$, and $\underset{\sim}{M}_1$, $\underset{\sim}{M}_2$, denote the resultant stress vector and stress couple on the edges $\xi_1$ = const. and $\xi_2$ = const., respectively; $A_1$, $A_2$ are the nonzero components of the metric tensor of the $(\xi_1,\xi_2)$ coordinate system; and

$$\underset{\sim}{P} = \underset{\sim}{p} - b_1 \ddot{\underset{\sim}{u}} - b_2 \ddot{\underset{\sim}{\beta}}$$

$$\underset{\sim}{M} = \underset{\sim}{m} - b_2 \ddot{\underset{\sim}{U}} - b_3 \ddot{\underset{\sim}{B}} \tag{2.5}$$

$$\underset{\sim}{U} = \underset{\sim}{I}_3 \times \underset{\sim}{u}$$

$$\underset{\sim}{B} = \underset{\sim}{I}_3 \times \underset{\sim}{\beta} \tag{2.6}$$

Commas designate differentiation with respect to the $\xi_1$ or $\xi_2$ coordinates, and dots denote derivatives with respect to time.

If the ratio of the thickness to the minimum radius of curvature is negligible with respect to one (Assumption #5), then the parameters $b_1$, $b_2$, $b_3$ are given in [4] by

$$b_n = \sum_{i=1}^{m} \rho^i Z_n^i \tag{2.7}$$

where n = 1,2,3, and

$$Z_n^i = (z_{i+1}^n - z_i^n)/n \tag{2.8}$$

In equation (2.7), m denotes the total number of layers, $\rho^i$ is the mass density of the i-th layer, and $\xi_3 = z_i$, $\xi_3 = z_{i+1}$ are the coordinates of the bounding surfaces of the i-th layer, as shown in Figure 1.

The surface loads applied to the two bounding surfaces of the shell are represented by a surface load vector, $\underset{\sim}{p}$, measured per unit area of the reference surface, and defined by

$$\underset{\sim}{p} = \underset{\sim}{p}_1 + \underset{\sim}{p}_2 \tag{2.9a}$$

where $\underset{\sim}{p}_1$ and $\underset{\sim}{p}_2$ denote the surface loads, measured per unit area of the reference surface, which are applied on the bounding surfaces $\xi_3 = z_1$ and $\xi_3 = z_{m+1}$, respectively. Similarly, the surface moment vector is defined by

$$\underset{\sim}{m} = z_1 \underset{\sim}{I}_3 \times \underset{\sim}{p}_1 + z_{m+1} \underset{\sim}{I}_3 \times \underset{\sim}{p}_2 \tag{2.9b}$$

Details of all these definitions can be found in [4].

The resultant stress vector and couple on an edge $\xi_1$ = constant can be resolved into components defined by

$$\begin{aligned} \underset{\sim}{N}_1 &= N_{11} \underset{\sim}{I}_1 + N_{12} \underset{\sim}{I}_2 + Q_1 \underset{\sim}{I}_3 \\ \underset{\sim}{M}_1 &= - M_{12} \underset{\sim}{I}_1 + M_{11} \underset{\sim}{I}_2 \end{aligned} \tag{2.10a}$$

and on $\xi_2$ = constant by

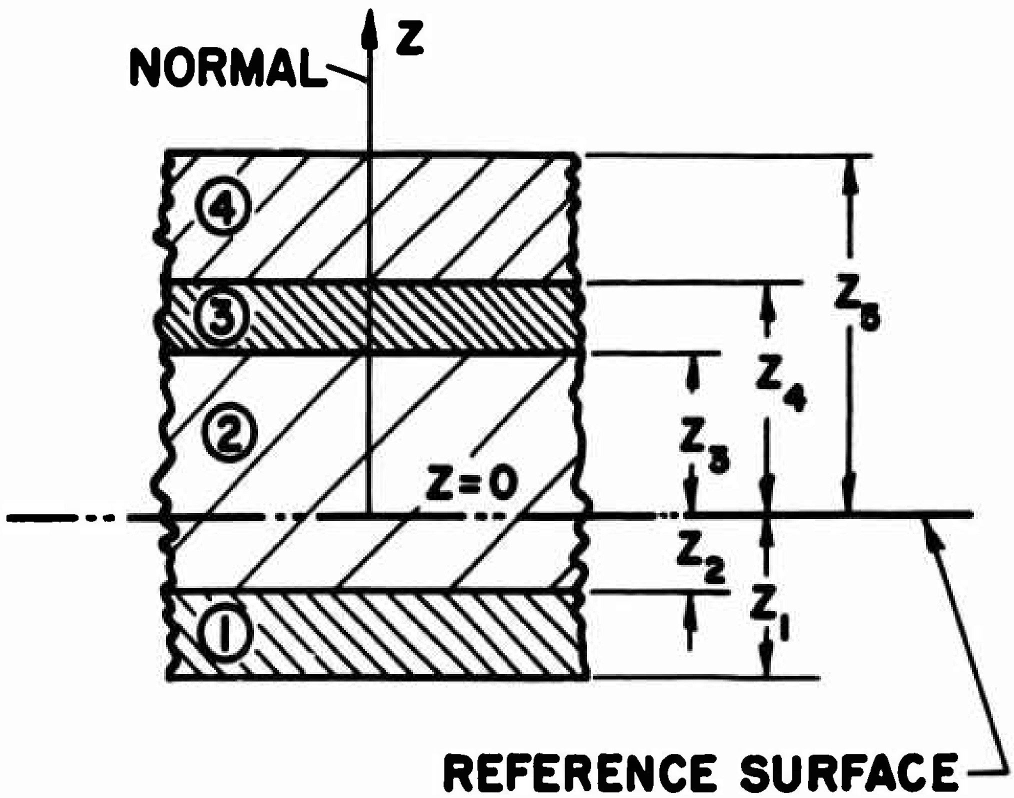


Figure 1. Layered Element of Shell.

$$\underset{\sim}{M}_2 = N_{21}\,\underset{\sim}{T}_1 + N_{22}\,\underset{\sim}{T}_2 + Q_2\,\underset{\sim}{T}_3$$

$$\underset{\sim}{M}_2 = -\,M_{22}\,\underset{\sim}{T}_1 + M_{21}\,\underset{\sim}{T}_2 \tag{2.10b}$$

The Gauss formulas found in [3] for the derivatives of the unit vectors for an orthogonal coordinate system on a surface are given by

$$\begin{aligned}
\underset{\sim}{T}_{1,1} &= -\,A_{1,2}\,\underset{\sim}{T}_2/A_2 - A_1\,\underset{\sim}{T}_3/R_{11} \\
\underset{\sim}{T}_{2,1} &= \phantom{-\,}A_{1,2}\,\underset{\sim}{T}_1/A_2 - A_1\,\underset{\sim}{T}_3/R_{21} \\
\underset{\sim}{T}_{3,1} &= \phantom{-\,}A_1\,\underset{\sim}{T}_1/R_{11} + A_1\,\underset{\sim}{T}_2/R_{12}
\end{aligned} \tag{2.11a}$$

and

$$\begin{aligned}
\underset{\sim}{T}_{2,2} &= -\,A_{2,1}\,\underset{\sim}{T}_1/A_1 - A_2\,\underset{\sim}{T}_3/R_{22} \\
\underset{\sim}{T}_{1,2} &= \phantom{-\,}A_{2,1}\,\underset{\sim}{T}_2/A_1 - A_2\,\underset{\sim}{T}_3/R_{12} \\
\underset{\sim}{T}_{3,2} &= \phantom{-\,}A_2\,\underset{\sim}{T}_2/R_{22} + A_2\,\underset{\sim}{T}_1/R_{21}
\end{aligned} \tag{2.11b}$$

where the curvature components are defined by

$$\begin{aligned}
A_1/R_{11} &= \underset{\sim}{T}_{3,1} \cdot \underset{\sim}{T}_1 \\
A_1/R_{12} &= \underset{\sim}{T}_{3,1} \cdot \underset{\sim}{T}_2 \\
A_2/R_{21} &= \underset{\sim}{T}_{3,2} \cdot \underset{\sim}{T}_1 \\
A_2/R_{22} &= \underset{\sim}{T}_{3,2} \cdot \underset{\sim}{T}_2
\end{aligned} \tag{2.12}$$

Substituting the resultant stress and stress couple vectors, as given by equations (2.10), into equation (2.1), and making use of the Gauss formulas, leads to the following equations of equilibrium:

$$(A_2 N_{11})_{,1} + (A_1 N_{21})_{,2} + A_{1,2} N_{12} - A_{2,1} N_{22}$$

$$+ A_1 A_2 (Q_1/R_{11} + Q_2/R_{12} + P_1) = 0 \qquad (2.13a)$$

$$(A_2 N_{12})_{,1} + (A_1 N_{22})_{,2} + A_{2,1} N_{21} - A_{1,2} N_{11}$$

$$+ A_1 A_2 (Q_1/R_{21} + Q_2/R_{22} + P_2) = 0 \qquad (2.13b)$$

$$(A_2 Q_1)_{,1} + (A_1 Q_2)_{,2} - A_1 A_2 (N_{11}/R_{11} + N_{12}/R_{12}$$

$$+ N_{21}/R_{21} + N_{22}/R_{22} - P_3) = 0 \qquad (2.13c)$$

$$(A_2 M_{11})_{,1} + (A_1 M_{21})_{,2} + A_{1,2} M_{12} - A_{2,1} M_{22}$$

$$- A_1 A_2 (Q_1 - M_1) = 0 \qquad (2.13d)$$

$$(A_2 M_{12})_{,1} + (A_1 M_{22})_{,2} + A_{2,1} M_{21} - A_{1,2} M_{11}$$

$$- A_1 A_2 (Q_2 - M_2) = 0 \qquad (2.13e)$$

The positive directions of the resultants are shown in Figure 2, and $P_i, M_i$ denote the components of $\underset{\sim}{P}$ and $\underset{\sim}{M}$, respectively.

The relations between the stress resultants and shell strains, given in [4], for a layered, orthotropic, thin shell, can be written as

$$
\begin{aligned}
N_{11} &= C_{11}\,\epsilon_{11} + C_{12}\,\epsilon_{22} + E_{11}\,k_{11} + E_{12}\,k_{22} + H_1 \\
N_{22} &= C_{12}\,\epsilon_{11} + C_{22}\,\epsilon_{22} + E_{12}\,k_{11} + E_{22}\,k_{22} + H_2 \\
N_{12} &= N_{21} = F(\gamma_1 + \gamma_2) + J(\delta_1 + \delta_2) \\
M_{11} &= E_{11}\,\epsilon_{11} + E_{12}\,\epsilon_{22} + D_{11}\,k_{11} + D_{12}\,k_{22} + H_3 \\
M_{22} &= E_{12}\,\epsilon_{11} + E_{22}\,\epsilon_{22} + D_{12}\,k_{11} + D_{22}\,k_{22} + H_4 \\
M_{12} &= M_{21} = J(\gamma_1 + \gamma_2) + K(\delta_1 + \delta_2)
\end{aligned}
\tag{2.14}
$$

where, after using Assumption #5, the material parameters are given by

$$
\begin{aligned}
C_{\alpha\beta} &= B_{\alpha\beta 1} \\
E_{\alpha\beta} &= B_{\alpha\beta 2} \\
D_{\alpha\beta} &= B_{\alpha\beta 3}
\end{aligned}
\tag{2.15}
$$

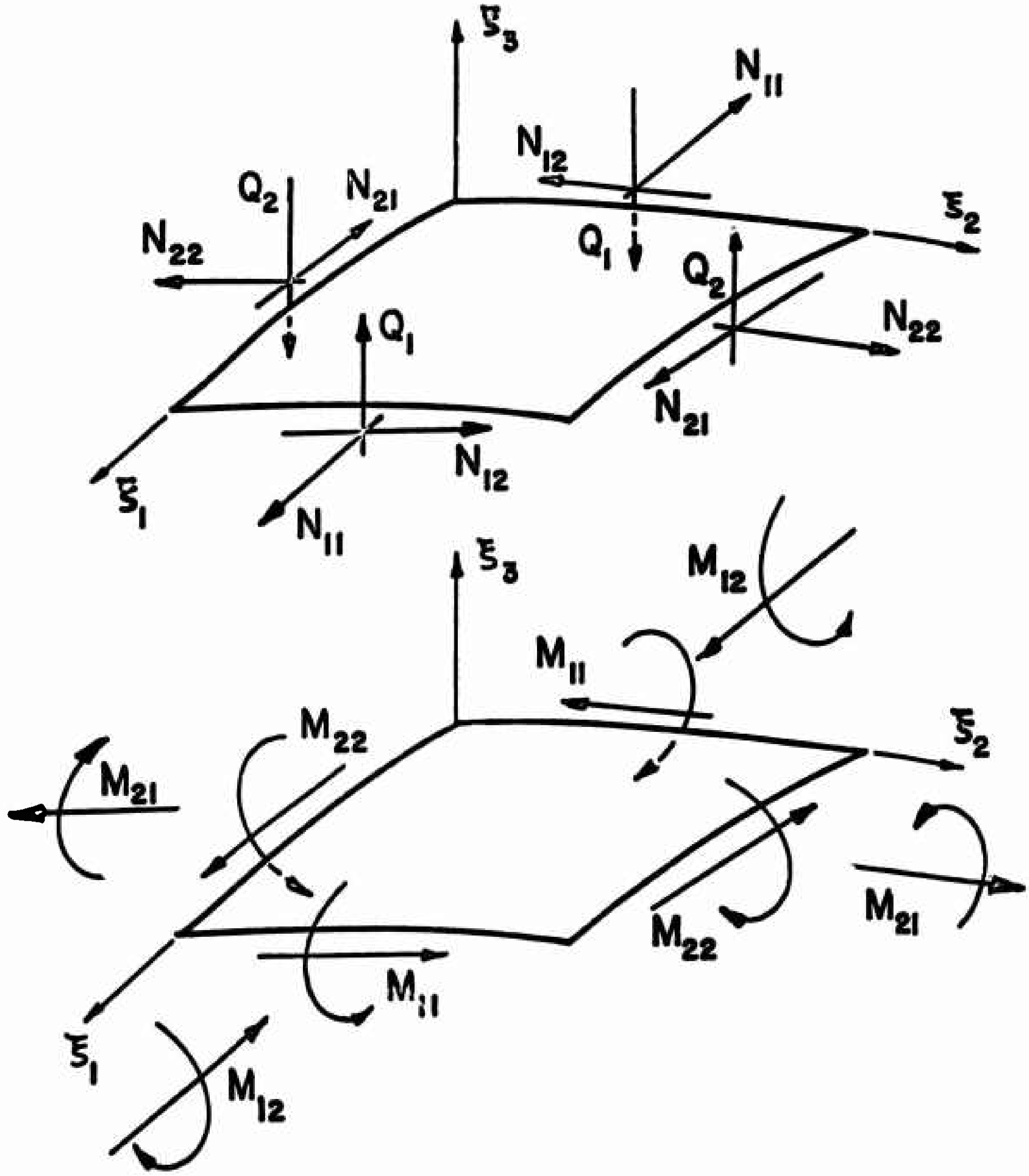


Figure 2. Stress-resultants on an Element.

for $\alpha$, $\beta$ = 1,2 and

$$B_{\alpha\beta n} = \sum_{i=1}^{m} B^{i}_{\alpha\beta} \, Z^{i}_{n} \tag{2.16}$$

Similarly,

$$\begin{aligned} F &= G_{121} \\ J &= G_{122} \\ K &= G_{123} \end{aligned} \tag{2.17}$$

where

$$G_{12n} = \sum_{i=1}^{m} G^{i}_{12} \, Z^{i}_{n} \tag{2.18}$$

For an orthotropic shell

$$\begin{aligned} B_{11} &= E_1/(1 - \nu_{12}\,\nu_{21}) \\ B_{12} &= \nu_{12}\,E_1/(1 - \nu_{12}\,\nu_{21}) = \nu_{21}\,E_2/(1 - \nu_{12}\,\nu_{21}) \\ B_{22} &= E_2/(1 - \nu_{12}\,\nu_{21}) \end{aligned} \tag{2.19}$$

where $E_1$ and $E_2$ are Young's moduli in the $\xi_1$ and $\xi_2$ directions, respectively; $G_{12}$ is the shear modulus in the tangent plane of the $(\xi_1,\xi_2)$ coordinate surface; $\nu_{12}$ denotes the contraction (Poisson's ratio) in the $\xi_1$ direction, caused by a positive normal stress in the $\xi_2$ direction; and the superscript i refers to the properties of the i-th layer.

The temperature terms in equations (2.14) are represented by $H_k$, where k = 1,2,3,4, and are based on the assumption of a linear temperature field with respect to the $\xi_3$ coordinate. Thus, the temperature distribution within the shell is assumed to be given by

$$T(\xi_1,\xi_2,\xi_3) = T_A(\xi_1,\xi_2) + \xi_3\, T_M(\xi_1,\xi_2) \tag{2.20}$$

Then, for a thin shell, the $H_k$ terms have the form

$$\begin{aligned} H_1 &= H_{11}\, T_A + H_{12}\, T_M \\ H_2 &= H_{21}\, T_A + H_{22}\, T_M \\ H_3 &= H_{12}\, T_A + H_{13}\, T_M \\ H_4 &= H_{22}\, T_A + H_{23}\, T_M \end{aligned} \tag{2.21}$$

where, for $\alpha$ = 1,2 and n = 1,2,3,

$$H_{\alpha n} = \sum_{i=1}^{m} A_\alpha^i Z_n^i \tag{2.22}$$

and

$$\begin{aligned} A_1 &= \alpha_1 + \nu_{12}\alpha_2 \\ A_2 &= \alpha_2 + \nu_{21}\alpha_1 \end{aligned} \tag{2.23}$$

where $\alpha_1$ and $\alpha_2$ denote the coefficients of thermal expansion in the $\xi_1$ and $\xi_2$ directions, respectively.

If the linear distribution of the temperature throughout the shell is assumed in the form of equation (2.20), then the prescription of the temperature distribution on the two bounding surfaces of the shell, given by $\xi_3 = z_1$ and $\xi_3 = z_{m+1}$, define uniquely $T_A$ and $T_M$. Denoting the prescribed temperature on $\xi_3 = z_1$ by $T_L$ and that on $\xi_3 = z_{m+1}$ by $T_U$, we have that

$$\begin{aligned} T_A &= (T_L z_{m+1} - T_U z_1)/(z_{m+1} - z_1) \\ T_M &= (T_U - T_L)/(z_{m+1} - z_1) \end{aligned} \tag{2.24}$$

The relations between the shell strains, appearing in equations (2.14), and the displacement components, as defined by equations (2.2), can be found from [3] and written as

$$
\begin{aligned}
\varepsilon_{11} &= u_{1,1}/A_1 + A_{1,2}\, u_2/A_1 A_2 + u_3/R_{11} \\
\gamma_1 &= u_{2,1}/A_1 - A_{1,2}\, u_1/A_1 A_2 + u_3/R_{12} \\
k_{11} &= \beta_{1,1}/A_1 + A_{1,2}\, \beta_2/A_1 A_2 \\
\delta_1 &= \beta_{2,1}/A_1 - A_{1,2}\, \beta_1/A_1 A_2
\end{aligned}
\tag{2.25a}
$$

and

$$
\begin{aligned}
\varepsilon_{22} &= u_{2,2}/A_2 + A_{2,1}\, u_1/A_1 A_2 + u_3/R_{22} \\
\gamma_2 &= u_{1,2}/A_2 - A_{2,1}\, u_2/A_1 A_2 + u_3/R_{21} \\
k_{22} &= \beta_{2,2}/A_2 + A_{2,1}\, \beta_1/A_1 A_2 \\
\delta_2 &= \beta_{1,2}/A_2 - A_{2,1}\, \beta_2/A_1 A_2
\end{aligned}
\tag{2.25b}
$$

On account of Assumption #4, we have the relations

$$
\begin{aligned}
\beta_1 &= u_1/R_{11} + u_2/R_{12} - u_{3,1}/A_1 \\
\beta_2 &= u_2/R_{22} + u_1/R_{21} - u_{3,2}/A_2
\end{aligned}
\tag{2.26}
$$

This completes the list of the governing equations for a thin, elastic shell which are referred to an arbitrary reference surface and an orthogonal coordinate system on this surface. There are twenty-one equations, represented by equations (2.13), (2.14), (2.25), and (2.26), and twenty-one unknowns. Together with the boundary conditions, they constitute a properly posed boundary value problem for the analysis of a thin elastic shell.

The appropriate boundary conditions, if Assumption #4 is employed, on an edge $\xi_1$ = const. are the following:

1. Either $N_{11}^*$ or $u_1$ prescribed,
2. Either $N_{12}^*$ or $u_2$ prescribed,
3. Either $Q_1^*$ or $u_3$ prescribed,
4. Either $M_{11}$ or $\beta_1$ prescribed,

where the <u>effective</u> stress resultants are defined by

$$N_{11}^* = N_{11} + M_{12}/R_{12}$$

$$N_{12}^* = N_{12} + M_{12}/R_{22} \tag{2.27}$$

$$Q_1^* = Q_1 + M_{12,2}/A_2$$

Similar boundary conditions are obtained on the edge $\xi_2$ = const. by exchanging the indices 1 and 2.

The governing equations presented here can be applied to the linear, infinitesimal-deflection analysis of a thin shell, for which the geometric shell parameters, given by $A_1$, $A_2$, $\underset{\sim}{I}_1$, $\underset{\sim}{I}_2$, $\underset{\sim}{I}_3$, $R_{11}$, $R_{12}$, $R_{21}$, $R_{22}$, are taken as those of the reference surface lying in the undeformed shell. However, for the free-vibration and stability analysis of a prestressed shell, we must at least consider the equilibrium of the <u>deformed</u> shell element.

It should be noted that the equations of equilibrium, equations (2.13), are equally applicable to an undeformed as to a deformed shell element, because a deformed shell is just another shell. In order to apply equations (2.13) to a finitely deformed shell, we must simply regard that the geometric shell parameters are the components of the metric, unit tangent vectors, and curvatures corresponding to a convected coordinate system of the deformed shell. These geometric shell parameters are, of course, not known until the problem is solved, but they can be expressed in terms of the known geometric shell parameters of the undeformed reference surface and the components of the displacement vector. Such expressions will be derived in the following section.

We should also note at this time that since our equations assume an orthogonal coordinate system on the reference surface, we have already made the assumption that the $(\xi_1, \xi_2)$ convected coordinate system is orthogonal in the undeformed shell and remains orthogonal throughout deformation. However, the coordinate system need not coincide with the lines of curvature neither on the reference surface in the undeformed nor in the deformed state.

## 3. Geometric Shell Parameters For A Deformed Shell

The object of this section is to derive the relations between the geometric shell parameters of two different deformation states (States I and II) of the shell, when the displacement vector between these two states is given. States I and II can be the undeformed and deformed states of the shell, or they can be any other two states.

Let us denote the components of the metric and curvature and the unit tangent vectors of the coordinate curves of State I by $a_1$, $a_2$, $r_{11}$, $r_{12}$, $r_{21}$, $r_{22}$, $\underset{\sim}{t}_1$, $\underset{\sim}{t}_2$, $\underset{\sim}{t}_3$, and those of State II by $A_1$, $A_2$, $R_{11}$, $R_{12}$, $R_{21}$, $R_{22}$, $\underset{\sim}{T}_1$, $\underset{\sim}{T}_2$, $\underset{\sim}{T}_3$. The $(\xi_1,\xi_2)$ coordinate system, which describes the points of the reference surface of the shell, will be assumed to be an orthogonal convected coordinate system, so that a given material point of the reference surface retains the same values of $\xi_1$ and $\xi_2$ for any deformation state of the shell. However, the normal coordinate of a point in State I will be denoted by $\xi_3^{I}$ and the normal coordinate of the same point in State II will be denoted by $\xi_3^{II}$. Then the position vector of a specific material point of the shell in State I is given by

$$\underset{\sim}{r}(\xi_1,\xi_2,\xi_3) = \underset{\sim}{s}(\xi_1,\xi_2) + \xi_3^{I}\,\underset{\sim}{t}_3\,(\xi_1,\xi_2) \tag{2.28a}$$

and in State II by

$$\underset{\sim}{R}(\xi_1,\xi_2,\xi_3) = \underset{\sim}{S}(\xi_1,\xi_2) + \xi_3^{II}\,\underset{\sim}{T}_3\,(\xi_1,\xi_2) \tag{2.28b}$$

and the displacement vector is defined by

$$\underset{\sim}{v} = \underset{\sim}{R} - \underset{\sim}{r} \tag{2.29}$$

For the purpose of deriving the equations for a stability analysis of a shell, it will be convenient to set the normal component of $\underset{\sim}{\beta}$ equal to zero, which was one of the assumptions introduced in the preceding section. However, since we can resolve the displacement vector either along the tangent vectors of State I or along those of State II, our choice of the resolution will affect the direction of the normal component of $\underset{\sim}{\beta}$.

For example, if we use Assumption #1 in the form

$$\underset{\sim}{v} = \underset{\sim}{u} + \xi_3^{I}\,\underset{\sim}{\beta} \tag{2.30}$$

and the resolution

$$\underset{\sim}{u} = u_1 \underset{\sim}{t}_1 + u_2 \underset{\sim}{t}_2 + u_3 \underset{\sim}{t}_3 \tag{2.31a}$$

$$\underset{\sim}{\beta} = \beta_1 \underset{\sim}{t}_1 + \beta_2 \underset{\sim}{t}_2 \tag{2.31b}$$

then a point, which was in State I at $\xi_3^I = 1$, will be at

$$\xi_3^{II} = (1 + \beta_1^2 + \beta_2^2)^{1/2} \tag{2.32}$$

in State II. Similarly, if we use Assumption #1 in the form

$$\underset{\sim}{v} = \underset{\sim}{u} + \xi_3^{II} \underset{\sim}{\beta} \tag{2.33}$$

and the resolution

$$\underset{\sim}{u} = U_1 \underset{\sim}{T}_1 + U_2 \underset{\sim}{T}_2 + U_3 \underset{\sim}{T}_3 \tag{2.34a}$$

$$\underset{\sim}{\beta} = B_1 \underset{\sim}{T}_1 + B_2 \underset{\sim}{T}_2 \tag{2.34b}$$

then a point, which was in State I at $\xi_3^I = 1$, will be in State II at

$$\xi_3^{II} = (1 - B_1^2 - B_2^2)^{1/2} \tag{2.35}$$

This means that if either equation (2.31b) or (2.34b) is employed, the distances between the points on a normal will change, and that Assumption [illegible] is not compatible with setting the normal component of $\underset{\sim}{\beta}$ equal to zero for a large rotation of the normal. For our purposes, it will be appropriate to make the assumption that the States I and II are such that the squares of the components of $\underset{\sim}{\beta}$ are negligible with respect to one. Then, according to equation (2.32) and (2.35), the normal coordinate of a point in State I equals the normal coordinate of the same point in State II, and the coordinate system is also convected in the $\xi_3$ direction.

With this assumption, we have that

$$\xi_3^{I} = \xi_3^{II} = \xi_3 \tag{2.36}$$

and the displacement vector is given by

$$\underset{\sim}{v} = \underset{\sim}{u} + \xi_3\,\underset{\sim}{\beta} \tag{2.37}$$

where

$$\underset{\sim}{u} = \underset{\sim}{S} - \underset{\sim}{s} \tag{2.38a}$$

$$\underset{\sim}{\beta} = \underset{\sim}{T}_3 - \underset{\sim}{t}_3 \tag{2.38b}$$

The nonzero components of the metric and the unit tangent vectors of State I are given by

$$a_1^2 = \underset{\sim}{S}_{,1} \cdot \underset{\sim}{S}_{,1}$$

$$a_2^2 = \underset{\sim}{S}_{,2} \cdot \underset{\sim}{S}_{,2} \tag{2.39}$$

$$\underset{\sim}{t}_1 = \underset{\sim}{S}_{,1}/a_1$$

$$\underset{\sim}{t}_2 = \underset{\sim}{S}_{,2}/a_2 \tag{2.40}$$

and those of State II by

$$A_1^2 = \underset{\sim}{S}_{,1} \cdot \underset{\sim}{S}_{,1}$$

$$A_2^2 = \underset{\sim}{S}_{,2} \cdot \underset{\sim}{S}_{,2} \tag{2.41}$$

$$\underset{\sim}{T}_1 = \underset{\sim}{S}_{,1}/A_1$$

$$\underset{\sim}{T}_2 = \underset{\sim}{S}_{,2}/A_2 \tag{2.42}$$

Using equations (2.38), the relations between the unit tangent vectors in the two states are given by

$$
\begin{aligned}
\underset{\sim}{T}_1 &= (a_1\underset{\sim}{t}_1 + \underset{\sim}{u}_{,1})/A_1 \\
\underset{\sim}{T}_2 &= (a_2\underset{\sim}{t}_2 + \underset{\sim}{u}_{,2})/A_2 \qquad (2.43) \\
\underset{\sim}{T}_3 &= \underset{\sim}{t}_3 + \underset{\sim}{\beta}
\end{aligned}
$$

The connection between the components of the metric in the two states is found from equations (2.39) and (2.41), which, with the use of equation (2.38a), can be written as

$$
\begin{aligned}
A_1^2 - a_1^2 &= (\underset{\sim}{u}_{,1} + \underset{\sim}{s}_{,1}) \cdot (\underset{\sim}{u}_{,1} + \underset{\sim}{s}_{,1}) - \underset{\sim}{s}_{,1} \cdot \underset{\sim}{s}_{,1} \\
&= 2a_1\underset{\sim}{t}_1 \cdot \underset{\sim}{u}_{,1} + \underset{\sim}{u}_{,1} \cdot \underset{\sim}{u}_{,1} \qquad (2.44)
\end{aligned}
$$

or

$$
A_1 = a_1(1 + 2\underset{\sim}{t}_1 \cdot \underset{\sim}{u}_{,1}/a_1 + \underset{\sim}{u}_{,1} \cdot \underset{\sim}{u}_{,1}/a_1^2)^{1/2} \qquad (2.45)
$$

Resolving $\underset{\sim}{u}$ along the tangent vectors of State I, as given by equation (2.31a), differentiating with respect to $\xi_1$, and using the Gauss formulas, leads to

$$\underset{\sim}{u}_{,1}/a_1 = \varepsilon_{11}\underset{\sim}{t}_1 + \gamma_1\underset{\sim}{t}_2 - \beta_1\underset{\sim}{t}_3 \tag{2.46}$$

where the shell strains are given by equations (2.25) and $\beta_1$ by equations (2.26). Substituting equation (2.46) into equation (2.45) we find that

$$A_1 = a_1(1 + 2\varepsilon_{11} + \varepsilon_{11}^2 + \gamma_1^2 + \beta_1^2)^{1/2} \tag{2.47}$$

Let us recall that we already have assumed that $\beta_1^2$ is negligible with respect to one. Let us now assume further that the States I and II are such that the squares of any of the shell strains, defined by equations (2.25), are negligible with respect to one. Then it follows from equation (2.47) that

$$A_1 = a_1(1 + \varepsilon_{11}) \tag{2.48a}$$

and, by a similar procedure, that

$$A_2 = a_2(1 + \varepsilon_{22}) \tag{2.48b}$$

The connection between the components of curvature of the two states is found from the definitions given by equations

(2.12). Using equations (2.43), we can write that

$$A_1^2/R_{11} = (\underset{\sim}{t}_{3,1} + \underset{\sim}{\beta}_{,1}) \cdot (a_1\underset{\sim}{t}_1 + \underset{\sim}{u}_{,1}) \tag{2.49}$$

or

$$A_1^2/R_{11} = a_1\underset{\sim}{t}_{3,1} \cdot \underset{\sim}{t}_1 + a_1\underset{\sim}{\beta}_{,1} \cdot \underset{\sim}{t}_1 + \underset{\sim}{t}_{3,1} \cdot \underset{\sim}{u}_{,1}$$

$$+ \underset{\sim}{\beta}_{,1} \cdot \underset{\sim}{u}_{,1} \tag{2.50}$$

Resolving also $\underset{\sim}{\beta}$ along the tangent vectors of State I, as given by equation (2.31b), we find that

$$\underset{\sim}{\beta}_{,1}/a_1 = k_{11}\underset{\sim}{t}_1 + \delta_1\underset{\sim}{t}_2 - \mu_1\underset{\sim}{t}_3 \tag{2.51}$$

where the shell strains are given by equations (2.25) and

$$\mu_1 = \beta_1/r_{11} + \beta_2/r_{21} \tag{2.52}$$

Substituting equation (2.46) and equation (2.51) into equation (2.50) and using the definition of the components of curvature for State I, we find that

$$(A_1^2/a_1^2)/R_{11} = 1/r_{11} + k_{11} + \gamma_1/r_{12} + \varepsilon_{11}/r_{11}$$

$$+ k_{11}\varepsilon_{11} + \gamma_1\delta_1 + \beta_1\mu_1 \qquad (2.53)$$

As before, we again omit any terms which have squares of shell strains in comparison to those which have none, and get

$$(A_1^2/a_1^2)/R_{11} = 1/r_{11} + k_{11} + \gamma_1/r_{12} + \varepsilon_{11}/r_{11} \qquad (2.54)$$

or using equation (2.48a), we find that

$$1/R_{11} = 1/r_{11} + k_{11} + \gamma_1/r_{12} + \varepsilon_{11}(1/r_{11} - 2/R_{11}) \qquad (2.55)$$

Since the difference in the curvature components contains shell strains of power one, and those having power two are negligible, then equation (2.55) can finally be written as

$$1/R_{11} = 1/r_{11} + k_{11} + \gamma_1/r_{12} - \varepsilon_{11}/r_{11} \qquad (2.56a)$$

Similarly, from equations (2.12) we get that

$$A_1A_2/R_{12} = (\underset{\sim}{t}_{3,1} + \underset{\sim}{\beta}_{,1}) \cdot (a_2\underset{\sim}{t}_2 + \underset{\sim}{u}_{,2})$$

which, after omitting the squares in shell strains and using equations (2.48), leads to

$$1/R_{12} = 1/r_{12} + \delta_1 + \gamma_2/r_{11} - \varepsilon_{11}/r_{12} \tag{2.56b}$$

By exchanging the indices 1 and 2, we can obtain the remaining relations of the curvature components in the form

$$1/R_{21} = 1/r_{21} + \delta_2 + \gamma_1/r_{22} - \varepsilon_{22}/r_{21} \tag{2.56c}$$

$$1/R_{22} = 1/r_{22} + k_{22} + \gamma_2/r_{21} - \varepsilon_{22}/r_{22} \tag{2.56d}$$

Equations (2.48) and (2.56) give the desired expressions between the geometric shell parameters of States I and II when the shell strains between these states, as defined by equations (2.25) in terms of the components of the displacement vector, are given. It should be recalled that in deriving these expressions, terms containing squares of shell strains have been neglected in comparison with terms which contain none.

4. <u>Governing Equations of Equilibrium for Infinitesimal Perturbations</u>.

The stability and the free-vibration problem of a shell with initial prestress are both concerned with a <u>prestress load system</u> which produces in the shell a <u>prestressed</u> state. The stresses and displacements of the prestressed state are supposed to be previously calculated by some, linear or non-linear, shell theory.

The stability problem asks whether or not there exists a static <u>perturbed</u> state, infinitesimally close to the pre-stressed state, which is still in equilibrium with the pre-stress load system. The free-vibration problem requires the existence of such a perturbed state in the presence of the inertia terms. The object of this section is, therefore, the derivation of the governing equations of equilibrium for the case when infinitesimal perturbations are superimposed on the prestressed state. Such perturbations must necessarily involve not only the perturbations in the stress resultants of the prestressed state, but also in geometry of the shell, which is described by the components of the metric and curvature.

After much of the theoretical work of this report was completed, papers by Koiter [8], Cohen [9], and Budiansky [10] appeared in the literature, which addressed themselves exactly at the same problem, i.e., the appropriate equations for the stability analysis of an arbitrary shell. While undoubtedly all the desired information regarding such equations could have been obtained from any one of these references, the point of view which had been adopted for the purposes of this report is, in our opinion, sufficiently different to warrant the presentation of our derivation of the stability equations. The author must hasten to acknowledge, however, the benefit of having read the three references, which no doubt has helped him in the preparation of the final version of the derivation of the stability equations given in this report.

In our derivation, we shall require at the outset that the coordinate system used to describe the shell in the prestressed state is restricted to be orthogonal, although not necessarily directed along the lines of curvature. Such a requirement makes our equations less general than those given in [8, 10], where they are written in tensor form and therefore applicable to arbitrary coordinates. We feel that the orthogonality restriction will not introduce a serious inconvenience to the shell analyst, because, whenever possible, orthogonal coordinate systems should be preferred to nonorthogonal ones.

The infinitesimal perturbations in the prestressed state can be regarded as infinitesimal increments in each of the quantities appearing in the equations of equilibrium, as given by equations (2.13). An appealing interpretation of the perturbation state is given in [10] in terms of the rates of change of all the quantities of the prestressed state with respect to some monotonically increasing parameter, which could be imagined as time. Then the stability and free-vibration problems are concerned with the existence of the rates of the prestress quantities corresponding to prescribed rates of the load terms. Of course, the rate concept can be translated into infinitesimal increments (differentials) by simply multiplying the rates by the differential in time. Thus, if a solution in rates exists, then clearly there also exists a perturbed state, at a time dt away from the prestressed state, given for each quantity by

$$( \ ) = ( \ )^p + (\dot{\ })dt \tag{2.57}$$

where the superscript p denotes the prestressed state, and a dot designates the rate of change evaluated at the prestressed state.

Using such a rate concept, let us now turn our attention to equations (2.13) which represent the exact conditions of equilibrium of the prestressed state, if the metric and curvature components are those of the prestressed state. From here on, we shall distinguish all quantities belonging to the prestressed state by the superscript p.

In order to derive the appropriate equations for the rates of all quantities, we must simply differentiate all equations (2.13) with respect to time. In doing so, we should note that it follows from equations (2.48) and (2.56) that the rates of change of the metric and curvature components can be expressed as

$$
\begin{aligned}
\dot{A}^p_1 &= \dot{\varepsilon}_{11} A^p_1 \\
\dot{A}^p_2 &= \dot{\varepsilon}_{22} A^p_2
\end{aligned}
\tag{2.58}
$$

and

$$
\begin{aligned}
(1/\dot{R}^p_{11}) &= \dot{k}_{11} + \dot{\gamma}_1/R^p_{12} - \dot{\varepsilon}_{11}/R^p_{11} \\
(1/\dot{R}^p_{12}) &= \dot{\delta}_1 + \dot{\gamma}_2/R^p_{11} - \dot{\varepsilon}_{11}/R^p_{12} \\
(1/\dot{R}_{21}) &= \dot{\delta}_2 + \dot{\gamma}_1/R^p_{22} - \dot{\varepsilon}_{22}/R^p_{21} \\
(1/\dot{R}_{22}) &= \dot{k}_{22} + \dot{\gamma}_2/R^p_{21} - \dot{\varepsilon}_{22}/R^p_{22}
\end{aligned}
\tag{2.59}
$$

After performing the differentiations of equations (2.13a), (2.13c), (2.13e), and using (2.59), they can be written in the form

$$(A_2^p\dot{N}_{11})_{,1} + (A_1^p\dot{N}_{21})_{,2} + A^p_{1,2}\dot{N}_{12} - A^p_{2,1}\dot{N}_{22}$$

$$+ A_1^pA_2^p(\dot{Q}_1/R^p_{11} + \dot{Q}_2/R^p_{12}) + (\dot{\varepsilon}_{22}A_2^pN^p_{11})_{,1}$$

$$+ (\dot{\varepsilon}_{11}A_1^pN^p_{21})_{,2} + (\dot{\varepsilon}_{11}A_1^p)_{,2}N^p_{12}$$

$$- (\dot{\varepsilon}_{22}A_2^p)_{,1}N^p_{22} + A_1^pA_2^pQ_1^p(\dot{k}_{11} + \dot{\gamma}_1/R^p_{12} + \dot{\varepsilon}_{22}/R^p_{11})$$

$$+ A_1^pA_2^pQ_2^p(\dot{\delta}_1 + \dot{\gamma}_2/R^p_{11} + \dot{\varepsilon}_{22}/R^p_{12})$$

$$+ \partial(\hat{A}_1^pA_2^pP_1^p)/\partial t = 0 \tag{2.63}$$

$$(A_2^p\dot{Q}_1)_{,1} + (A_1^p\dot{Q}_2)_{,2} - A_1^pA_2^p(\dot{N}_{11}/R^p_{11} + \dot{N}_{12}/R^p_{12}$$

$$+ \dot{N}_{21}/R^p_{21} + \dot{N}_{22}/R^p_{22}) + (\dot{\varepsilon}_{22}A_2^pQ_1^p)_{,1}$$

$$+ (\dot{\varepsilon}_{11}A_1^pQ_2^p)_{,2} - A_1^pA_2^p[N^p_{11}(\dot{k}_{11} + \dot{\gamma}_1/R^p_{12} + \dot{\varepsilon}_{22}/R^p_{11})$$

$$+ N^p_{12}(\dot{\delta}_1 + \dot{\gamma}_2/R^p_{11} + \dot{\varepsilon}_{22}R^p_{12})$$

$$+ N^p_{21}(\dot{\delta}_2 + \dot{\gamma}_1/R^p_{22} + \dot{\varepsilon}_{11}R^p_{21})$$

$$+ N^p_{22}(\dot{k}_{22} + \dot{\gamma}_2/R^p_{21} + \dot{\epsilon}_{11}/R^p_{22})]$$

$$+ \partial(A^p_1 A^p_2 P^p_3)/\partial t = 0$$

$$(A^p_2 \dot{M}_{11})_{,1} + (A^p_1 \dot{M}_{21})_{,2} + A^p_{1,2}\dot{M}_{12} - A^p_{2,1}\dot{M}_{22} - A^p_1 A^p_2 \dot{Q}_1$$

$$+ (\dot{\epsilon}_{22} A^p_2 M^p_{11})_{,1} + (\dot{\epsilon}_{11} A^p_1 M^p_{21})_{,2} + (\dot{\epsilon}_{11} A^p_1)_{,2} M^p_{12}$$

$$- (\dot{\epsilon}_{22} A^p_2)_{,1} M^p_{22} - (\dot{\epsilon}_{11} + \dot{\epsilon}_{22}) A^p_1 A^p_2 Q^p_1$$

$$+ \partial(A^p_1 A^p_2 M^p_1)/\partial t = 0$$

Equations (2.13b) and (2.13c) give similar expressions where only the indices 1 and 2 must be exchanged.

Our equations of equilibrium were obtained in one step from the well-known equations of equilibrium of a shell element. Because we have restricted the coordinate system to remain orthogonal also in the perturbed state, our equations contain the assumption that the rate of the membrane shear strain is zero.

To complete the system of equations, the stress-strain and strain-displacement equations, (2.14), (2.25), and (2.26), must be added, where all stress resultants, shell strains, and displacement components are regarded as their rates, while the metric and curvature components are those of the prestressed state. Such equations would not, however, be exact, because the rates of change of the metric and curvature components are neglected. Therefore, if equations (2.14), (2.25), and (2.26) were used, then any terms containing the products of the displacement components of the prestressed state and rate variables of the perturbation would have been neglected. Such an assumption will be made throughout this report.

## 5. Simplified Equations for Stability Analysis

Having derived the governing equations for infinitesimal perturbations of a given prestressed state, let us now turn our attention to the stability problem of a thin, elastic shell.

The concept of stability involves a *sequence* of prestress load systems which are applied to the shell in a certain prescribed step-wise fashion in such a way that at each subsequent step some loads of the prestress load system have increased in magnitude. We shall assume that the initial step is taken as one where the prestress loads are absent, and that at this step the shell is in an *unstressed* state and *stable*.

After the loading of each step of the prestress load system is completed and the shell has reached a static prestressed state, we shall perturb the prestressed state by applying some superimposed load system which produces an *infinitesimal* perturbation. By definition, the shell will be declared *unstable*, or we shall say that it *buckles*, when a prestress load system is found at which a perturbed state is possible without the application of any superimposed loads. The first such prestress load system, encountered in the step-wise process of increasing the prestress loads, will be designated the *critical* load system, or simply, the *buckling* load.

Mathematically, the buckling load of the shell is reached when the rate equations, given in the preceding section, together with the appropriate boundary conditions, are satisfied. The rates of the load terms, which occur as the last term in each of the equilibrium equations (2.63), however, must be assigned values to correspond to the kinds of prestress loads applied to the shell.

Since each surface or edge load is distinguished by its intensity (per unit area or length) and direction, there can be general prescribed relations between the intensity and direction and the deformation of the shell. Each such prescribed relation would determine the precise form of the load rate terms included in equations (2.63). Because of the approximations planned in this section, all the terms involving the load rates will be neglected, and therefore the form of the possible expressions for the load rate terms will not be pursued any further.

The definition of instability which we have employed means that when the prestress load system has reached the buckling load, then both the prestressed and perturbed states are in equilibrium with the same critical load system. Therefore, at the same loads, two solutions are possible which indicates a point of bifurcation in the solution.

It may happen that the shell does not fail at a point of bifurcation, so that the buckling load does not coincide with the collapse load. However, for the purpose of presenting an analysis for arbitrary shells of revolution, it will be regarded that a bifurcation point represents a state of the shell at which the collapse of the shell may start. No attempt will be made in this report to pursue the state of the shell beyond such a bifurcation point.

There is no fundamental difficulty in retaining in the stability analysis all the terms which appear in equations (2.63). However, the question may be raised, whether or not all the terms are equally significant and under what conditions can they be simplified. It is the object of this section to examine arguments on the basis of which some of the terms may be neglected. The reason for such an objective is the desire to obtain the simplest possible system of equations for stability analysis, and at the same time to understand the precise limitations of such equations. After this examination will be completed, we shall be able to judge whether or not, for a given case, our stability equations are valid. If the prestressed or perturbed states do not meet the imposed limitations, we shall also know how to modify the stability equations by retaining more terms from the exact equations (2.63) to make them valid.

It is proposed now to develop the equations of equilibrium for the stability analysis with the following limitations:

1. As far as equilibrium is concerned, the element of the shell does not stretch when going from the prestressed to the disturbed state.

2. The bending stresses of the prestressed state are negligible with respect to the membrane stresses.

The first assumption means that we may set in equations (2.63) $\epsilon_{11} = \epsilon_{22} = 0$. If this is accepted, then the shell is assumed to buckle inextensionally, as in the infinitesimal bending of a ring or a plate. This assumption may be justified on physical grounds for a case of a smooth shell of revolution, subjected to an axisymmetric prestress. For such a case, the superimposed state contains a number of circumferential waves, which means that each initially circular strip will change to a wavy strip when buckling begins. Such a deformation need not involve any stretching of the strip, so that $\epsilon_{11}$ and $\epsilon_{22}$ may indeed be close to zero.

It should be emphasized that the first assumption is only used for the consideration of equilibrium, but will not be used in the stress-strain and strain-displacement relations. Therefore, after the superimposed state for a critical load is found, it will be possible to check the relative magnitudes of the membrane and bending strains. Since, as given in [4], the three-dimensional strain can be written in the form

$$\epsilon_{11} = \epsilon_{11} + \zeta_3 k_{11} \qquad (2.64)$$

the validity of this assumption may be estimated by comparing for the superimposed state the membrane strain with the maximum value of the bending strain. If the first is much smaller than the second, then the inextensibility assumption will be justified.

The second assumption means that we can set in equations (2.63)

$$M_{11}^{p} = M_{12}^{p} = M_{21}^{p} = M_{22}^{p} = Q_{1}^{p} = Q_{2}^{p} = 0$$

The validity of this assumption can be easily checked by examining the magnitudes of the stress resultants of the prestressed state. For the case when the reference surface is the middle surface of the shell, the membrane stress is given by

$$\sigma_{\alpha\beta}^{m} = N_{\alpha\beta}/h \tag{2.65a}$$

and the maximum bending stress by

$$\sigma_{\alpha\beta}^{b} = 6M_{\alpha\beta}/h^2 \tag{2.65b}$$

where $\alpha, \beta = 1,2$ and $h$ denotes the thickness of the shell. If in the prestressed state

$$\sigma^{m}_{\alpha\beta} >> \sigma^{b}_{\alpha\beta} \tag{2.66}$$

then the second assumption is justified. If not, it is not justified.

If the elastic limit of the material is not to be exceeded, then the loss of stability occurs predominantly in thin shells for which the inequality (2.66) will be satisfied. This does not mean, however, that the basic state should be obtained by means of the membrane theory. If this were done, then not all boundary conditions of the prestressed state could be satisfied. For a general approach, the prestressed state should be obtained by means of the bending theory of shells. After this is done, the inequality (2.66) should be examined, and, if it is reasonably satisfied, only then the stability equations based on the second assumption should be employed.

After making use of these two assumptions in equations (2.63), the equations of equilibrium for the stability analysis have the following form

$$(A^{p}_{2}N_{11})_{,1} + (A^{p}_{1}N_{21})_{,2} + A^{p}_{1,2}N_{12} - A^{p}_{2,1}N_{22}$$
$$+ A^{p}_{1}A^{p}_{2}(Q_{1}/R^{p}_{11} + Q_{2}/R^{p}_{12}) = 0$$

$$(A_2^p N_{12})_{,1} + (A_1^p N_{22})_{,2} + A_{2,1}^p N_{21} - A_{1,2}^p N_{11}$$

$$+ A_1^p A_2^p (Q_1/R_{21}^p + Q_2/R_{22}^p) = 0$$

$$(A_2^p Q_1)_{,1} + (A_1^p Q_2)_{,2} - A_1^p A_2^p (N_{11}/R_{11}^p + N_{12}/R_{12}^p$$

$$+ N_{21}/R_{21}^p + N_{22}/R_{22}^p) = A_1^p A_2^p [N_{11}^p (k_{11} + \gamma_1/R_{12}^p)$$

$$+ N_{12}^p (\delta_1 + \gamma_2/R_{11}^p) + N_{21}^p (\delta_2 + \gamma_1/R_{22}^p) \tag{2.67}$$

$$+ N_{22}^p (k_{22} + \gamma_2/R_{21}^p)]$$

$$(A_2^p M_{11})_{,1} + (A_1^p M_{21})_{,2} + A_{1,2}^p M_{12} - A_{2,1}^p M_{22} - Q_1 = 0$$

$$(A_2^p M_{12})_{,1} + (A_1^p M_{22})_{,2} + A_{2,1}^p M_{21} - A_{1,2}^p M_{11} - Q_2 = 0$$

At this point, the decision must be made whether the stability analysis should be valid for a prestressed state which has large deflections and/or rotations, in which case the shapes of the shell in the unstressed and prestressed states may differ considerably. If we wish to admit such cases, then the prestressed state must be calculated for each prestress load system using an appropriate nonlinear theory. Then it also becomes necessary to consider the equilibrium equations in the prestressed state by using the components of the metric and curvature of the prestressed state, as shown in equations (2.67). Of course, these components are not given with the problem, but must be calculated from the metric and curvature components of the unstressed state and the shell strains of the prestressed state. The shell strain-displacement relations are then nonlinear for the prestressed state, and, consequently, additional stability terms will appear in the shell strain-displacement relations for the superimposed state. These terms will contain products of displacement quantities of the prestressed and superimposed states.

While there is no doubt that such a procedure would lead to a more accurate prediction of the bifurcation points of the solution, in many cases it may be unnecessary. For example, the deformation of the prestressed state in an axially loaded column produces a slightly shorter column at the time of buckling and is negligible, as far as the calculation of the critical load is concerned. Similarly, the prestressed deformations in a spherical shell, subjected to an external pressure, need not be calculated by a nonlinear theory, because the shell just prior to buckling is another spherical shell with a slightly shorter radius. As a matter of fact, it is difficult to think of a buckling problem in which the prestressed state is not predominantly a compressive membrane state involving very little bending. The bending occurs at the critical load, when going from the prestressed to the disturbed state.

If this argument is correct, then not only are the first two assumptions justified, but we would be also justified to assume that:

3. The deflections and rotations of the prestressed state are infinitesimal.

This will mean that in equations (2.67) and (2.25), we can set

$$
\begin{aligned}
A_1^p &= a_1 \\
A_2^p &= a_2 \\
R_{11}^p &= r_{11} \\
R_{12}^p &= r_{12} \\
R_{21}^p &= r_{21} \\
R_{22}^p &= r_{22}
\end{aligned}
\tag{2.68}
$$

where the terms on the right-hand side refer to the unstressed state. Moreover, we can also neglect any stability terms which otherwise would occur in the shell strain-displacement equations, if a large-deflection prestressed state were admitted.

Although it imposes a definite limitation on the prestressed state, such a procedure would save a considerable amount of computation. First, the prestressed state has to be calculated by means of a linear shell theory only once, because other prestressed states can be obtained by superposition, and second, the only quantities which must be saved from the prestressed state are the membrane stress resultants $N^p_{11}$, $N^p_{12}$, $N^p_{21}$, and $N^p_{22}$. This procedure will be adopted in the stability analysis described in this report.

As far as the free-vibration problem of a prestressed shell is concerned, the arguments used to simplify the stability terms may no longer be as sound as for the stability problem. However, for the analysis described in this report, it will be simply assumed that:

1. The vibration of the prestressed shell is predominantly inextensional.

2. The prestressed state is a membrane state.

3. The deformations of the prestressed state are infinitesimal.

If such limitations are acceptable, then the system of equations used for both the free vibration and stability analyses will consist of equations (2.67), (2.68), (2.14), and (2.25). Inertia terms, appearing in equations (2.13) must be added to equations (2.67) for the free-vibration problem. Together with some homogeneous boundary conditions, these equations constitute a linear eigenvalue problem.

# III. FUNDAMENTAL EQUATIONS FOR SHELL ANALYSIS

## 1. Introduction

The governing equations derived in Section II consist of a system of twenty-one equations and contain twenty-one unknowns. The boundary-value problem can be formulated in terms of eight differential equations, henceforth called the fundamental equations, involving only eight unknowns, called the fundamental variables. The other variables are related to the fundamental variables by algebraic relations. Such a formulation has been first used for a general shell in [5], and will form the foundation for all the shell analyses described in this report.

According to the method of analysis proposed in [5], one of the two coordinates on the reference surface of the shell must be selected as a preferred coordinate, say $\xi_1$, and then the fundamental equations will form a system of eight first-order partial differential equations with respect to $\xi_1$, and the fundamental variables will be those quantities which appear in the boundary conditions on the edge $\xi_1$ = constant. Thus, if $\xi_1$ is selected as the preferred coordinate, the fundamental variables used in the formulation of the boundary value problem of a shell are the elements of the following matrix

$$
y = \begin{bmatrix} u_1 \\ u_2 \\ u_3 \\ \beta_1 \\ N_{11}^* \\ N_{12}^* \\ Q_1^* \\ M_{11} \end{bmatrix}
$$

where the asterisk designates the effective stress resultants defined by equations (2.27).

The method of analysis used in [5] requires the solutions of initial-value problems of the fundamental variables within the interval of the $\xi_1$ coordinate. For this purpose, it is necessary to calculate the derivatives of the fundamental variables with respect to $\xi_1$ at a given value of $\xi_1$, when the physical and geometrical parameters of the shell, the fundamental variables themselves, and their derivatives with respect to $\xi_2$ are known at that value of $\xi_1$.

There is a convenient way of arranging the governing equations for the purpose of such a calculation. It has been given for a shell of revolution in [6] and for a general shell in [5]. It is convenient, because if all equations are calculated consecutively, the end product is the required derivatives of the fundamental variables with respect to $\xi_1$.

The fundamental equations for an arbitrary shell and for a shell of revolution will now be listed separately.

## 2. Arbitrary Shell

According to the scheme given in [5], the calculation of the derivatives of the fundamental variables with respect to $\xi_1$ for a classical theory of shells can be arranged in the following order:

$$\varepsilon_{22} = u_{2,2}/a_2 + a_{2,1}u_1/a_1a_2 + u_3/r_{22} \tag{3.1}$$

$$\gamma_2 = u_{1,2}/a_2 - a_{2,1}u_2/a_1a_2 + u_3/r_{12} \tag{3.2}$$

$$\beta_2 = u_2/r_{22} + u_1/r_{12} - u_{3,2}/a_2 \tag{3.3}$$

$$k_{22} = \beta_{2,2}/a_2 + a_{2,1}\beta_1/a_1a_2 \tag{3.4}$$

$$\delta_2 = \beta_{1,2}/a_2 - a_{2,1}\beta_2/a_1a_2 \tag{3.5}$$

$$u_{3,1} = a_1(u_1/r_{11} + u_2/r_{21} - \beta_1) \tag{3.6}$$

$$\lambda_1 = a_{1,2}u_2/a_1a_2 + u_3/r_{11} \tag{3.7}$$

$$\lambda_2 = - a_{1,2}u_1/a_1a_2 + u_3/r_{12} \tag{3.8}$$

$$\lambda_3 = u_2(1/r_{22})_{,1} + u_1(1/r_{12})_{,1} - u_{3,12}/a_2$$

$$+ u_{3,2}a_{2,1}/a_2^2 \tag{3.9}$$

$$\lambda_4 = \lambda_3/a_1 - a_{1,2}\beta_1/a_1a_2 - \lambda_1/r_{12} \tag{3.10}$$

$$d_1 = J + K/r_{22} \tag{3.11}$$

$$N_3 = N_{11}^* - C_{12}\varepsilon_{22} - E_{12}k_{22} - H_1 - J(\lambda_2 + \gamma_2)/r_{12}$$

$$- K(\lambda_4 + \delta_2)/r_{12} \tag{3.12}$$

$$N_4 = N_{12}^* - (F + J/r_{22})(\lambda_2 + \gamma_2) - d_1(\lambda_4 + \delta_2) \tag{3.13}$$

$$M_3 = M_{11} - E_{12}\varepsilon_{22} - D_{12}k_{22} - H_3 \tag{3.14}$$

$$d_2 = F + 2J/r_{22} + K/r_{22}^2 \tag{3.15}$$

$$d_3 = C_{11} + K/r_{12}^2 - E_{11}^2/D_{11} - d_1^2/d_2 r_{12}^2 \tag{3.16}$$

$$\varepsilon_{11} = [N_3 - E_{11}M_3/D_{11} - d_1 N_4/d_2 r_{12}]/d_3 \tag{3.17}$$

$$k_{11} = (M_3 - E_{11}\varepsilon_{11})/D_{11} \tag{3.18}$$

$$u_{1,1} = a_1(\varepsilon_{11} - a_{1,2}u_2/a_1 a_2 - u_3/r_{11}) \tag{3.19}$$

$$\beta_{1,1} = a_1(k_{11} - a_{1,2}\beta_2/a_1 a_2) \tag{3.20}$$

$$u_{2,1} = a_1(N_4 - d_1\varepsilon_{11}/r_{12})/d_2 \tag{3.21}$$

$$\gamma_1 = u_{2,1}/a_1 + \lambda_2 \tag{3.22}$$

$$\delta_1 = u_{2,1}/a_1 r_{22} + \varepsilon_{11}/r_{12} + \lambda_4 \tag{3.23}$$

$$M_{12} = J(\gamma_1 + \gamma_2) + K(\delta_1 + \delta_2) \tag{3.24}$$

$$N_{22} = C_{12}\varepsilon_{11} + C_{22}\varepsilon_{22} + E_{12}k_{11} + E_{22}k_{22} + H_2 \tag{3.25}$$

$$M_{22} = E_{12}\varepsilon_{11} + E_{22}\varepsilon_{22} + D_{12}k_{11} + D_{22}k_{22} + H_4 \tag{3.26}$$

$$N_{11} = N_{11}^{*} - M_{12}/r_{12} \tag{3.27}$$

$$N_{12} = N_{12}^{*} - M_{12}/r_{22} \tag{3.28}$$

$$Q_1 = Q_1^{*} - M_{12,2}/a_2 \tag{3.29}$$

$$\begin{aligned} a_2B = {} & (a_1M_{22})_{,2} + a_{2,1}M_{21} - a_{1,2}M_{11} \\ & - a_1a_2(b_2\ddot{u}_2 + b_3\ddot{\beta}_2 - m_2) \end{aligned} \tag{3.30}$$

$$\begin{aligned} a_2N_{11,1}^{*} = {} & a_2M_{12}(1/r_{12})_{,1} - a_{2,1}N_{11}^{*} - a_2B/r_{12} \\ & - (a_1N_{21})_{,2} - a_{1,2}N_{12} + a_{2,1}N_{22} \\ & - a_1a_2(Q_1/r_{11} + p_1 - b_1\ddot{u}_1 - b_2\ddot{\beta}_1) \end{aligned} \tag{3.31}$$

$$a_2N^*_{12,1} = a_2M_{12}(1/r_{22})_{,1} - a_{2,1}N^*_{12} - a_2B/r_{22}$$

$$- (a_1N_{22})_{,2} - a_{2,1}N_{21} + a_{1,2}N_{11}$$

$$- a_1a_2(Q_1/r_{12} + p_2 - b_1\ddot{u}_2 - b_2\ddot{\beta}_2) \tag{3.32}$$

$$a_2Q^*_{1,1} = - a_{2,1}Q^*_1 - (a_{2,1}M_{12}/a_2)_{,2} - B_{,2}$$

$$+ a_1a_2(N_{11}/r_{11} + N_{12}/r_{12} + N_{21}/r_{21}$$

$$+ N_{22}/r_{22} - p_3 + b_1\ddot{u}_3) + a_1a_2[N^p_{11}(k_{11} + \gamma_1/r_{12})$$

$$+ N^p_{12}(\delta_1 + \gamma_2/r_{11}) + N^p_{21}(\delta_2 + \gamma_1/r_{22})$$

$$+ N^p_{22}(k_{22} + \gamma_2/r_{21})] \tag{3.33}$$

$$a_2M_{11,1} = - a_{2,1}M_{11} - (a_1M_{21})_{,2} - a_{1,2}M_{12} + a_{2,1}M_{22}$$

$$+ a_1a_2(Q_1 - m_1 + b_2\ddot{u}_1 + b_3\ddot{\beta}_1) \tag{3.34}$$

As shown by equations (2.14), it is permissible to assume in these equations that $N_{12} = N_{21}$ and $M_{12} = M_{21}$.

If it is desired to eliminate the derivatives of the curvature terms with respect to $\xi_1$ from equation (3.9), this can be achieved by means of the Codazzi formulas in the form

$$a_2(1/r_{22})_{,1} = a_1(1/r_{21})_{,2} + a_{1,2}(1/r_{12} + 1/r_{21}) + a_{2,1}(1/r_{11} - 1/r_{22}) \tag{3.35a}$$

$$a_2(1/r_{12})_{,1} = a_1(1/r_{11})_{,2} - a_{2,1}(1/r_{12} + 1/r_{21}) + a_{1,2}(1/r_{11} - 1/r_{22}) \tag{3.35b}$$

Then the only derivative of the shell properties with respect to $\xi_1$ appearing in these equations will be that of the metric component $a_2$.

## 3. Shell Of Revolution

The geometric shell parameters for a shell whose reference surface in the unstressed state is a surface of revolution (see Figure 3) is given by

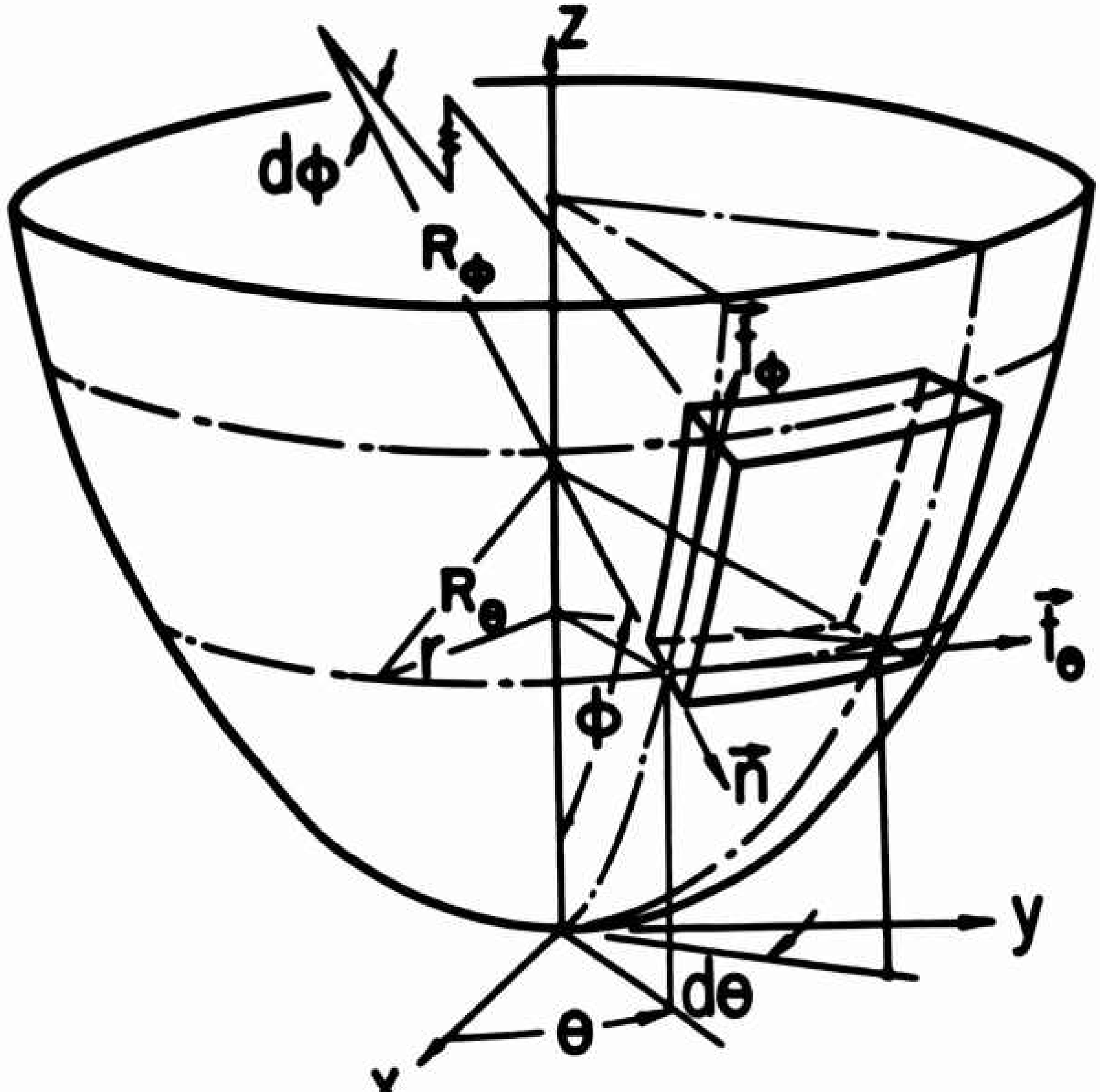


Figure 3. Element of a Shell of Revolution.

$$\xi_1 = \phi$$

$$a_1 = R_\phi$$

$$r_{11} = R_\phi$$

$$\xi_2 = \theta$$

$$a_2 = r$$

$$r_2 = r/\sin\phi$$

$$r_{12} = r_{21} = 0$$

where $R_\phi$ is the radius of curvature of the meridian, $\phi$ is the angle between the normal and the axis of symmetry, and r is the distance from the axis of symmetry. The fundamental equations for a shell of revolution can be written in the following form:

$$\varepsilon_\theta = u_{\theta,\theta}/r + u_\phi \cos\phi/r + w \sin\phi/r \tag{3.36}$$

$$\gamma_\theta = u_{\phi,\theta}/r - u_\theta \cos\phi/r \tag{3.37}$$

$$\beta_\theta = u_\theta \sin\phi/r - w_{,\theta}/r \tag{3.38}$$

$$k_\theta = \beta_{\theta,\theta}/r + \beta_\phi \cos\phi/r \tag{3.39}$$

$$\delta_\theta = \beta_{\phi,\theta}/r - \beta_\theta \cos\phi/r \tag{3.40}$$

$$w_{,s} = u_\phi/R_\phi - \beta_\phi \tag{3.41}$$

$$\lambda_4 = u_\theta(1/R_\phi - \sin\phi/r) \cos\phi/r - w_{,s\theta}/r$$

$$+ w_{,\theta}\cos\phi/r^2 \tag{3.42}$$

$$N_3 = N_\phi - C_{12}\varepsilon_\theta - E_{12}k_\theta - H_1 \tag{3.43}$$

$$N_4 = N^*_{\phi\theta} - (F + J\sin\phi/r)\gamma_\theta - (J + K\sin\phi/r)(\lambda_4 + \delta_\theta) \tag{3.44}$$

$$M_3 = M_\phi - E_{12}\varepsilon_\theta - D_{12}k_\theta - H_3 \tag{3.45}$$

$$d_2 = F + 2J\sin\phi/r + K(\sin\phi/r)^2 \tag{3.46}$$

$$d_3 = C_{11} - E_{11}^2/D_{11} \tag{3.47}$$

$$\varepsilon_\phi = (N_3 - E_{11}M_3/D_{11})/d_3 \tag{3.48}$$

$$k_\phi = (M_3 - E_{11}\varepsilon_\phi)/D_{11} \tag{3.49}$$

$$u_{\phi,s} = \varepsilon_\phi - w/R_\phi \tag{3.50}$$

$$\beta_{\phi,s} = k_\phi \tag{3.51}$$

$$u_{\theta,s} = N_4/d_2 \tag{3.52}$$

$$\gamma_\phi = u_{\theta,s} \tag{3.53}$$

$$\delta_\phi = u_{\theta,s}\sin\phi/r + \lambda_4 \tag{3.54}$$

$$M_{\phi\theta} = J(\gamma_\phi + \gamma_\theta) + K(\delta_\phi + \delta_\theta) \tag{3.55}$$

$$N_\theta = C_{12}\varepsilon_\phi + C_{22}\varepsilon_\theta + E_{12}k_\phi + E_{22}k_\theta + H_2 \tag{3.56}$$

$$M_\theta = E_{12}\varepsilon_\phi + E_{22}\varepsilon_\theta + D_{12}k_\phi + D_{22}k_\theta + H_4 \tag{3.57}$$

$$N_{\phi\theta} = N^*_{\phi\theta} - M_{\phi\theta}\sin\phi/r \tag{3.58}$$

$$Q_\phi = Q_\phi^* - M_{\phi\theta,\theta}/r \tag{3.59}$$

$$B = M_{\theta,\theta}/r + M_{\phi\theta}\cos\phi/r + b_2\ddot{u}_\theta + b_3\ddot{\beta}_\theta - m_\theta \tag{3.60}$$

$$N_{\phi,s} = - N_{\phi\theta,\theta}/r + (N_\theta - N_\phi)\cos\phi/r - Q_\phi/R_\phi$$

$$+ b_1\ddot{u}_\phi + b_2\ddot{\beta}_\phi - p_\phi \tag{3.61}$$

$$N^*_{\phi\theta,s} = M_{\phi\theta}(1/R_\phi - \sin\phi/r)\cos\phi/r - N^*_{\phi\theta}\cos\phi/r$$

$$- B\sin\phi/r - N_{\theta,\theta}/r - N_{\phi\theta}\cos\phi/r + b_1\ddot{u}_\theta$$

$$+ b_2\ddot{\beta}_\theta - p_\theta \tag{3.62}$$

$$M_{\phi,s} = - M_\phi\cos\phi/r - M_{\phi\theta,\theta}/r + M_\theta\cos\phi/r + Q_\phi$$

$$+ b_2\ddot{u}\phi + b_3\ddot{\beta}_\phi - m_\phi \tag{3.63}$$

$$Q^*_{\phi,s} = - Q^*_\phi\cos\phi/r - M_{\phi\theta,\theta}\cos\phi/r^2 - B_{,\theta}/r$$

$$+ N_\phi/R_\phi + N_\theta\sin\phi/r + b_1\ddot{w} - p$$

$$+ N^p_\phi k_\phi + N^p_\theta k_\theta + 2N^p_{\phi\theta}\tau \tag{3.64}$$

where $\tau$ has been defined as

$$\tau = \delta_\phi + \gamma_\theta/R_\phi = \delta_\theta + \gamma_\phi \sin\phi/r \tag{3.65}$$

In these equations, the derivatives with respect to the arc length, s, along the meridian have been given. The conversion to a derivative with respect to $\phi$ is achieved by the formula

$$\partial/\partial s = (1/R_\phi)\, \partial/\partial\phi \tag{3.66}$$

Also, the only relevant Codazzi formula, given by equation (3.35a), in the form

$$\partial r/\partial\phi = R_\phi \cos\phi \tag{3.67}$$

has been used throughout the derivation. The normal displacement has been denoted by w and the normal surface load by p.

The fundamental equations listed in this section define the behavior of a thin shell for all the boundary value problems considered in this report. Now we shall consider a method by means of which all of these problems can be solved.

# IV. REDUCTION TO ORDINARY DIFFERENTIAL EQUATIONS

## 1. Statement Of Problem

The method of solution used in the analysis is applicable to boundary value problems which are governed in a two-dimensional region S, defined by $a_1 \leq \xi_1 \leq b_1$ and $a_2 \leq \xi_2 \leq b_2$, by a system of linear differential equations stated in the form

$$\partial y/\partial\xi_1 = F(\xi_1,\xi_2,y,\partial y/\partial\xi_2,\partial^2 y/\partial\xi_2^2,\ldots) \tag{4.1}$$

The symbol $y = y(\xi_1,\xi_2)$ denotes an $(m,1)$ column matrix whose elements are m unknown dependent variables, and F denotes m linear functions in the elements of y and their derivatives with respect to $\xi_2$, arranged as the elements of a column matrix. In this formulation, $\xi_1$ is a preferred coordinate.

The method of solution admits general boundary conditions on the edges of the region S. For the present purpose, it will be assumed that the boundary conditions are stated in the form

$$T_a(\xi_2)y(a_1,\xi_2) = u_a(\xi_2) \tag{4.2a}$$

$$T_b(\xi_2)y(b_1,\xi_2) = u_b(\xi_2) \tag{4.2b}$$

$$y(\xi_1,a_2) = y(\xi_1,b_2) \tag{4.3}$$

The elements of the (m,m) matrices, $T_a$ and $T_b$, are specified by the statement of the boundary conditions on the coordinate curves $\xi_1 = a_1$ and $\xi_1 = b_1$, respectively, and $u_a$, $u_b$ are (m,1) column matrices which contain m/2 prescribed elements. As stated by equations (4.2), the boundary conditions can be specified on either the elements of y or on their linear combinations, but not on their derivatives. The last condition, equation (4.3), is a continuity condition of the elements of y on the coordinate curves $\xi_2 = a_2$ and $\xi_2 = b_2$. The reason for such a continuity condition is that for the cases considered in this report, the $\xi_2$ coordinate curve is a closed curve, so that the curves $\xi_2 = a_2$ and $\xi_2 = b_2$ coincide.

Before presenting the actual procedure of the method of analysis, the system of partial differential equations must be turned into a system of ordinary differential equations, depending only on the coordinate $\xi_1$. Two different possibilities are discussed below.

## 2. Separable Equations

For axisymmetric shells, which have a straight axis of symmetry for its geometric and physical properties, the homogeneous system of equations obtained from equation (4.1) is separable with respect to the $\xi_1$ and $\xi_2$ coordinates. Choosing $\xi_2$ as the circumferential coordinate along a closed latitude circle of the shell, the elements of y can be expressed in a separable form as follows

$$y(\xi_1,\xi_2) = y_n(\xi_1)T_n(\xi_2) \tag{4.4a}$$

where

$$T_n(\xi_2) = \begin{Bmatrix} \cos n\xi_2 \\ \sin n\xi_2 \end{Bmatrix} \tag{4.4b}$$

with the meaning that, depending on the particular element of $y$, the top or the bottom trigonometric function in equation (4.4b) is applicable. If the nonhomogeneous load terms, contained in equation (4.1), are chosen in a similarly separable form, i.e.,

$$b(\xi_1,\xi_2) = b_n(\xi_1)T_n(\xi_2) \tag{4.5}$$

then the $\xi_1$-dependent part of $y$ is governed by a system of $m$ linear ordinary differential equations which can be written as

$$dy_n(\xi_1)/d\xi_1 = F(\xi_1,y_n) + b_n(\xi_1) \tag{4.6}$$

where $F$ denotes $m$ linear functions in the elements of $y_n(\xi_1)$, and the $(m,1)$ column matrix, $b_n$, contains the $\xi_1$-dependent parts of the load terms. Similarly, the prescribed boundary conditions are chosen in a separable form as

$$u_a(\xi_2) = u_{an}T_n(\xi_2)$$

$$u_b(\xi_2) = u_{bn}T_n(\xi_2) \tag{4.7}$$

The condition given by equation (4.3) is automatically satisfied, because the $\xi_2$-coordinate curve is a closed curve and the trigonometric functions, $T_n(\xi_2)$, are periodic.

The case when the homogeneous equations are separable is the simplest one, and, for each value of n, it leads to the solution of boundary value problems governed by a system of m first-order, ordinary differential equations, as given by equation (4.6). If the loads and the boundary conditions are expanded in a Fourier series of the form

$$f(\xi_1,\xi_2) = \sum_{n=o}^{\infty} f_n(\xi_1)T_n(\xi_2) \tag{4.8}$$

then the problem is solved for each set of Fourier coefficients, $f_n(\xi_1)$, separately, and the solution is constructed in a similar series, given by

$$y(\xi_1,\xi_2) = \sum_{n=o}^{\infty} y_n(\xi_1)T_n(\xi_2) \qquad (4.9)$$

The Fourier coefficients, $f_n(\xi_1)$, can be either $b_n(\xi_1)$, $u_{an}$, or $u_{bn}$, and they produce the solution $y_n(\xi_1)$.

With regard to the boundary value problem of a thin, elastic shell of revolution, the governing equations are separable for the linear stress analysis problem of a shell, subjected to arbitrary loads, and for the free-vibration and stability problems with axisymmetric prestress. For the first case, the surface loads on the shell must be expanded in a Fourier series of the form

$$p(\phi,\theta) = \sum_{n=o}^{N} [p_n'(\phi) \cos n\theta + p_n''(\phi) \sin n\theta]$$

$$p_\phi(\phi,\theta) = \sum_{n=o}^{N} [p_{\phi n}'(\phi) \cos n\theta + p_{\phi n}''(\phi) \sin n\theta] \qquad (4.10)$$

$$p_\theta(\phi,\theta) = \sum_{n=o}^{N} [p_{\theta n}'(\phi) \sin n\theta + p_{\phi n}''(\phi) \cos n\theta]$$

where N is selected in such a way that the series gives an acceptable representation of the actual loads. The solution for the fundamental variables is then given in the form

$$w(\phi,\theta) = \sum_{n=o}^{N} [w_n'(\phi) \cos n\theta + w_n''(\phi) \sin n\theta]$$

$$Q_\phi^*(\phi,\theta) = \sum_{n=o}^{N} [Q_{\phi n}'(\phi) \cos n\theta + Q_{\phi n}''(\phi) \sin n\theta]$$

$$u_\phi(\phi,\theta) = \sum_{n=o}^{N} [u_{\phi n}'(\phi) \cos n\theta + u_{\phi n}''(\phi) \sin n\theta]$$

$$N_\phi(\phi,\theta) = \sum_{n=o}^{N} [N_{\phi n}'(\phi) \cos n\theta + N_{\phi n}''(\phi) \sin n\theta]$$

$$\beta_\phi(\phi,\theta) = \sum_{n=o}^{N} [\beta_{\phi n}'(\phi) \cos n\theta + \beta_{\phi n}''(\phi) \sin n\theta] \tag{4.11}$$

$$M_\phi(\phi,\theta) = \sum_{n=o}^{N} [M_{\phi n}'(\phi) \cos n\theta + M_{\phi n}''(\phi) \sin n\theta]$$

$$u_\theta(\phi,\theta) = \sum_{n=o}^{N} [u_{\theta n}'(\phi) \sin n\theta + u_{\theta n}''(\phi) \cos n\theta]$$

$$N_{\phi\theta}^*(\phi,\theta) = \sum_{n=o}^{N} [N_{\phi\theta n}'(\phi) \sin n\theta + N_{\phi\theta n}''(\phi) \cos n\theta]$$

The boundary value problem for the shell of revolution is solved for each value of n separately, and each prescribed primed or double-primed component of the load produces the primed or double-primed components of the solution. The governing equations are the eight first-order, ordinary differential equations, obtained from the fundamental equations of Section III, after making use of the solution in the separable form of equations (4.11) and setting the prestress terms equal to zero.

For the free-vibration and stability problems with axisymmetric prestress, the surface and edge loads are zero, but the solution is again given in the form of equations (4.11). Thus, the governing equations are given by the same system of eight first-order, differential equations of Section III, except that now the prestress terms, appearing in equation (3.64), must be retained and the surface loads set equal to zero. For the stability problem, the inertia terms are set equal to zero, and by varying the prestress terms, the buckling loads are found for each given value of n. For a free-vibration problem, the prestress terms are kept constant, and by varying the frequency, the natural frequencies are found for each given value of n.

3. <u>Nonseparable Equations</u>

For shells of revolution, which have some nonsymmetric shell parameters, such as thickness, prestress, material properties, imperfections, etc., the system of equations (4.1) is not separable. These parameters appear as two-dimensional coefficients in the differential equations, and a separable solution in the form of equations (4.4) is no longer possible. In order to reduce equation (4.1) to a system of first-order, ordinary differential equations, some procedure must be used for the elimination of the derivatives with respect to $\xi_2$ on the right hand side of equation (4.1). This can be achieved in various ways. A Fourier expansion method will be used in this report.

Since it is assumed that the $\xi_2$ coordinate curve is a closed circle, the nonsymmetric shell parameters must be periodic, and they can be expanded in Fourier series in the form

$$P(\xi_1,\xi_2) = \sum_{m=o}^{N} P_m(\xi_1)T_m(\xi_2) \tag{4.12}$$

The $\xi_1$-dependent coefficients, $P_m(\xi_1)$, are given, and the corresponding coefficients of the solution, $y_n(\xi_1)$, must be found. Unlike the case when all the parameters are axisymmetric, it is no longer true that one Fourier coefficient in the load parameters will produce one Fourier coefficient in the solution, $y_n(\xi_1)$, with the same value of n. Instead, an infinite series for an exact solution is in general needed for any choice of the load parameters. Therefore, to solve the problem exactly, an infinite number of differential equations containing the infinite number of unknowns, $y_n(\xi_1)$, would have to be solved. Since that is impossible, one way to solve such a problem is to satisfy equation (4.1) by assuming an approximate solution of the form

$$y(\xi_1,\xi_2) = \sum_{n=n_1,\ldots,n_k} y_n(\xi_1)T_n(\xi_2) \tag{4.13}$$

The indices, $n_i$, where $i = 1,2,\ldots,k$, represent a selected list of wave numbers, and in general they need be neither consecutive nor start with $n = 0$. They must be selected by the user of the method from previous experience.

Substitution of such an approximate solution, given by equation (4.13), into equation (4.1), leads to

$$G_1 + G_2 = 0 \tag{4.14}$$

where

$$G_1 = \sum_{n=n_1,\ldots,n_k} R_n(\xi_1)T_n(\xi_2) \tag{4.15a}$$

$$R_n(\xi_1) = dy_n(\xi_1)/d\xi_1 - F[\xi_1,y_{n_1}(\xi_1),\ldots,y_{n_k}(\xi_1)] \tag{4.15b}$$

and

$$G_2 = \sum_{n\neq n_1,\ldots,n_k} S_n[\xi_1,y_{n_1}(\xi_1),\ldots,y_{n_k}(\xi_1)]T_n(\xi_2) \tag{4.15c}$$

The group of terms denoted by $G_1$ contain the trigonometric functions, $T_n(\xi_2)$, with those values of n which are on the list of the selected wave numbers, while $G_2$ contains those which are not on the list.

If the solution given by equation (4.13) were exact, it would satisfy equation (4.14) exactly. Since this must be so for any value of $\xi_2$, it follows that for an exact solution we must require that

$$R_n = 0 \text{ for } n=n_1,\ldots,n_k \tag{4.16a}$$

$$S_n = 0 \text{ for } n\neq n_1,\ldots,n_k \tag{4.16b}$$

Noting that the unknowns in equation (4.14) are the $\xi_1$-dependent coefficients of the solution, which are k in number, it is concluded that equations (4.16) require the satisfaction of more equations than the number of unknowns. In general, this is not possible, and the solution as given by equation (4.13) can satisfy equation (4.1) only approximately.

The method for arriving at a reasonable approximation can be borrowed from the method of weighted residuals [7], which requires that instead of the actual equation (4.1), its integral with respect to $\xi_2$, multiplied by a suitable weighting function, be satisfied; i.e., that

$$\int_{a_2}^{b_2} [G_1 + G_2]W_i(\xi_2)d\xi_2 = 0 \tag{4.17}$$

where $W_i(\xi_2)$ are the weighting functions.

The easiest choice of the weighting functions for our case is to use the same trigonometric functions, $T_n(\xi_2)$, which are selected to participate in the solution as given by equation (4.13); i.e., the trigonometric functions with indices $n = n_1, n_2, \ldots, n_k$. Because the integrals of products of trigonometric functions with unequal indices are zero, such a selection of $W_i(\xi_2)$ means that the integral of equation (4.17) containing $G_2$ is zero, and that then equations (4.15) reduce to

$$\int_{a_2}^{b_2} [\sum_{n=n_1,\ldots,n_k} R_n(\xi_1)T_n(\xi_2)]T_i(\xi_2)d\xi_2 = 0 \tag{4.18}$$

where $i = n_1, \ldots, n_k$. Equation (4.18) represents k equations containing k unknowns, $y_n(\xi_1)$, and it can be solved.

When the integration with respect to $\xi_2$ in equation (4.18) is carried out, only the terms with $n = i$ remain, and, after omitting factors which arise from the integration, equation (4.18) reduces to

$$R_n(\xi_1) = 0 \tag{4.19}$$

where $n = n_1, \ldots, n_k$. Thus, the method of weighted residuals has shown that the solution can be represented by equation (4.13), when the $\xi_1$-dependent coefficients, $y_n(\xi_1)$, are found from equation (4.19). The error in the solution comes from the fact that equation (4.16b) is not satisfied.

This discussion has given a general approach for the analysis of shells of revolution with some nonsymmetric parameters. For the shell analysis considered in this report, the nonseparable case arises for the free vibration and stability problems with a nonsymmetric prestress. Then the prestress terms, $N^p_\phi$, $N^p_{\phi\theta}$, and $N^p_\theta$, occurring in equation (3.64), are dependent on $\theta$ and the solution is not separable.

Regarding the stability of a shell of revolution, it is important to investigate the character of the instability that nonsymmetric prestress loads can produce. If the prestress loads are assumed expanded in a Fourier series in the form given by equation (4.12), let us consider the character of the instability produced by the separate terms of the series. A special case arises for the Fourier harmonic with $m = 1$. In this case, there is a resultant couple produced by the stress resultants on a latitude circle of the shell, and therefore this problem can be called the "bending" problem of a shell of revolution. The stability analyses for the bending of cylindrical and conical shells have been successfully carried out in the past.

The stability problem of a shell of revolution subjected to prestress loads which are represented by a Fourier harmonic with $m \geq 2$ seems to be in a different category. Not a single published theoretical or experimental investigation is known to the author which deals with such prestress loads. Moreover, as the following arguments will show, the prestressed state with $m \geq 2$ leads to a state of the shell which is quite different from that with $m = 0$ or $m = 1$, as far as the buckling of a shell is concerned.

Consider, for example, a shell of revolution with some prestress loads given by one Fourier harmonic with $m \geq 2$. Then the membrane stress resultants of the prestressed state obtained by a linear theory will have the form

$$N_{\phi}^{p} = N_{\phi m}^{p}(\phi) \cos m\theta \tag{4.20}$$

With such a circumferential variation, positive and negative signs of the stress field will alternate along meridional strips (Figure 5), having the width of the circumference divided by 2m. Thus, any buckling that could occur would be confined to those strips which are in compression, while the ones in tension would be stretched. Moreover, for $m \geq 2$, the resultant force and couple of all the stress resultants on a latitude circle of a shell of revolution are zero, so that no force or couple is directly transmitted along the meridian in the way that it is transmitted for prestress loads with $m = 0$ or $m = 1$.

To illustrate the "transmission" of the resultant force and couple along the meridian, consider a cylindrical shell subjected to an edge load in the form of equation (4.20). For the wave numbers $m = 0$ or $m = 1$, the Fourier coefficient of the membrane stress, $N^p_{\phi m}$, does not decay when going along the generator away from the edge, but stays approximately constant. The resultant force and couple on every latitude circle remains constant, regardless of the length of the shell. For $m \geq 2$, however, since the resultant force and couple of the applied edge load are zero, it follows from St. Venant's principle that $N^p_{\phi m}$ must decay when going away from the loaded edge, and that the characteristic decay length equals the diameter of the latitude circle. Therefore, for $m \geq 2$, the effect of the applied load is felt only near the edge and does not affect the whole shell.

The preceding arguments are being advanced for the purpose of justifying the admission of only one Fourier component of the prestressed state at one time. Our hypothesis is that even though a nonsymmetric prestress load may have to be expanded in a Fourier series with many terms, the components which will affect the stability of the whole will be those with wave numbers $m = 0$ and $m = 1$. Therefore, we see no point in complicating the analysis at this time by the inclusion of more nonsymmetric Fourier components but one. We are, however, going to admit arbitrary values of m, so that the character of the instability produced by prestress loads with $m \geq 2$ can also be explored.

If the prestress loads are represented by one Fourier component with any m, then the membrane stress resultants of the prestressed state, occurring in equations (3.64), are given by

$$
\begin{aligned}
N^p_\phi(\phi,\theta) &= N^p_{\phi m}(\phi) \cos m\theta \\
N^p_\theta(\phi,\theta) &= N^p_{\theta m}(\phi) \cos m\theta \qquad (4.20) \\
N^p_{\phi\theta}(\phi,\theta) &= N^p_{\phi\theta m}(\phi) \sin m\theta
\end{aligned}
$$

The superimposed state can then have either the same plane of symmetry, in which case the superimposed variables in the stability terms of equations (3.64) are given by

$$
\begin{aligned}
k_\phi(\phi,\theta) &= \sum_n k_{\phi n}(\phi) \cos n\theta \\
k_\theta(\phi,\theta) &= \sum_n k_{\theta n}(\phi) \cos n\theta \qquad (4.21) \\
\tau(\phi,\theta) &= \sum_n \tau_n(\phi) \sin n\theta
\end{aligned}
$$

or anti-symmetry, in which case they will be given by

$$k_\phi(\phi,\theta) = \sum_n k_{\phi n}(\phi) \sin n\theta$$

$$k_\theta(\phi,\theta) = \sum_n k_{\theta n}(\phi) \sin n\theta \tag{4.22}$$

$$\tau(\phi,\theta) = \sum_n \tau_n(\phi) \cos n\theta$$

The series in equations (4.21) and (4.22) are to be summed over the same list of wave numbers that is used in equation (4.13).

For the symmetric superimposed state, the stability terms of equations (3.64) will have the form

$$F_3 = \sum_n [(N^p_{\phi m} k_{\phi n} + N^p_{\theta m} k_{\theta n}) \cos m\theta \cos n\theta$$

$$+ 2N^p_{\phi\theta m} \tau_n \sin m\theta \sin n\theta] \tag{4.23}$$

Using the identities

$$2 \cos m\theta \cos n\theta = \cos(n-m)\theta + \cos(n+m)\theta$$

$$\tag{4.24}$$

$$2 \sin m\theta \sin n\theta = \cos(n-m)\theta - \cos(n+m)\theta$$

equation (4.23) can be written as

$$2F_3 = \sum_n [f_n \cos(n-m)\theta + g_n \cos(n+m)\theta] \tag{4.25}$$

where

$$f_n = N^p_{\phi m} k_{\phi n} + N^p_{\theta m} k_{\theta n} + 2N^p_{\phi\theta m} \tau_n$$

$$(4.26)$$

$$g_n = N^p_{\phi m} k_{\phi n} + N^p_{\theta m} k_{\theta n} - 2N^p_{\phi\theta m} \tau_n$$

For an antisymmetric superimposed state, the stability terms have the form

$$F_3 = \sum_n [(N^p_{\phi m} k_{\phi n} + N^p_{\theta m} k_{\theta n}) \cos m\theta \sin n\theta$$

$$+ 2N^p_{\phi\theta m} \tau_n \sin m\theta \cos n\theta] \tag{4.27}$$

and with the use of

$$2\cos m\theta \sin n\theta = \sin(n+m) + \sin(n-m)$$

$$2\sin m\theta \cos n\theta = \sin(n+m) - \sin(n-m) \tag{4.28}$$

we get that

$$2F_3 = \sum_n [f_n \sin(n-m)\theta + g_n \sin(n+m)\theta] \tag{4.29}$$

where

$$f_n = N^p_{\phi m} k_{\phi n} + N^p_{\theta m} k_{\theta n} - 2N^p_{\phi\theta m}\tau_n$$

$$g_n = N^p_{\phi m} k_{\phi n} + N^p_{\theta m} k_{\theta n} + 2N^p_{\phi\theta m}\tau_n \tag{4.30}$$

Let us now rewrite equation (4.25) in the form

$$2F_3 = \sum_{n'=n-m} f_{n'+m} \cos n'\theta + \sum_{n'=n+m} g_{n'-m} \cos n'\theta \tag{4.31}$$

As stated by equation (4.19), the procedure calls for the retention of only those Fourier components of the series of equation (4.31) which are on the list of $n_1, \ldots, n_k$, and the neglect of any other terms. With this approximation, equation (4.31) can be written as

$$2F_3 = \sum_n (f_{n+m} + g_{n-m}) \cos n\theta \tag{4.32}$$

where $n = n_1, \ldots, n_k$.

Since in the solution those terms which are not on the list of the selected wave numbers are zero, then it follows from equation (4.32) that only those wave numbers on the list which are multiples of m will contribute to the stability terms. This makes sense in view of our preceding argument that buckling can only occur in isolated strips having the width of the circumference divided by 2m. If the superimposed state is to be limited to such strips, it should consist of a list of selected wave numbers which could include m and contain only higher multiples of m.

We now have arrived at some procedure for the selection of the list of wave numbers which can be used in the solution. Unless our future experience will prove otherwise, for the stability analysis with nonsymmetric prestress covered in this report, we shall select the list as follows

$$n = jm \tag{4.33}$$

where $j = i_1, \ldots, i_k$ and represents consecutive integers, not necessarily starting with $i_1 = 1$, and k denotes the number of components used in the solution. Then the stability terms which must be used in equations (3.64) can finally be written as

$$F_3 = \sum_j h_{jm} \begin{Bmatrix} \cos jm\theta \\ \sin jm\theta \end{Bmatrix} \tag{4.34}$$

where $j = i_1, \ldots, i_k$, and

$$\begin{aligned} h_{jm} &= N^p_{\phi m}\,[k_{\phi(j+1)m} + k_{\phi(j-1)m}]/2 \\ &+ N^p_{\theta m}\,[k_{\theta(j+1)m} + k_{\theta(j-1)m}]/2 \\ &\pm N^p_{\phi\theta m}\,[\tau_{(j+1)m} - \tau_{(j-1)m}] \end{aligned} \tag{4.35}$$

The upper trigonometric function in equation (4.34) and the upper algebraic sign in equation (4.35) refer to the symmetric superimposed state, while the lower function and sign refer to the antisymmetric state.

It should be clear from this analysis, that the selected Fourier components of the superimposed state are coupled, because the indices are shifted up and down in equation (4.35). Therefore, the system of first-order, ordinary differential equations, defined by equation (4.15b), consists of 8k equations and 8k unknowns.

The actual calculation of the derivatives of the 8k unknowns can be carried out as follows. First, using the fundamental equations of Section III with the prestress terms omitted, the derivatives of each of the fundamental variables, denoted by $y_{jm}(\phi)$, are calculated, in succession for $j = i_1, \ldots, i_k$, and the results are stored in a two-dimensional array. During this calculation the values of $k_{\phi jm}$, $k_{\theta jm}$, and $\lambda_{jm}$ are also found and stored in three one-dimensional arrays. Then, the derivative of $Q_\phi^*$ is augmented by the stability terms $h_{jm}$, given for each value of j by equation (4.35). This procedure will give the proper derivatives for all the 8k unknowns.

The case of the nonseparable solutions also arises in the free-vibration problem with a nonsymmetric prestress. The analysis is identical to that of the stability analysis, except that the inertia terms appearing in the fundamental equations must be included.

# V. SOLUTION OF BOUNDARY VALUE PROBLEMS

## 1. Reduction To Initial Value Problems

As shown in the preceding section, the system of partial differential equations of shell theory can be reduced in various ways to a system of first-order, ordinary differential equations which can be written in the form

$$dy(x)/dx = F(x,y) + b(x) \tag{5.1}$$

where $y(x)$ is an $(m,1)$ column matrix which contains $m$ unknown dependent variables; $F$ denotes $m$ linear functions of the elements of $y$, arranged in a column matrix form; $b(x)$ is an $(m,1)$ column matrix which contains the nonhomogeneous load terms; and $x$ is the independent variable. The solution of the boundary value problem is governed by equation (5.1) in the interval $a \leq x \leq b$, and at the ends of the interval it must satisfy the following boundary conditions

$$\begin{aligned} T_a y(a) &= u_a \\ T_b y(b) &= u_b \end{aligned} \tag{5.2}$$

The elements of the (m,m) matrices, $T_a$ and $T_b$, are specified by the boundary conditions, and m/2 elements of each of $u_a$ and $u_b$ are the prescribed quantities on the boundaries. Equations (5.1) and (5.2) represent a two-point boundary value problem, for which the solution will be found.

Since equation (5.1) consists of a system of linear, ordinary differential equations, its solution can be written as

$$y(x) = W(x)c + d(x) \tag{5.3}$$

where W(x) is an (m,m) matrix whose columns are m independent solutions of the homogeneous equations obtained from equation (5.1); c is an (m,1) column matrix whose elements are arbitrary constants; and d(x) is an (m,1) column matrix which represents a particular solution of equation (5.1).

Evaluation of equation (5.3) at x = a and the solution for c leads to

$$c = W^{-1}(a)y(a) - W^{-1}(a)d(a) \tag{5.4}$$

Substituting c into equation (5.3), gives

$$y(x) = Y(x)y(a) + z(x) \tag{5.5}$$

where

$$Y(x) = W(x)W^{-1}(a)$$

$$z(x) = d(x) - W(x)W^{-1}(a)d(a) \tag{5.6}$$

It should be noted that if the columns of $W(x)$ are homogeneous solutions of equation (5.1), then the columns of $Y(x)$ are linear combinations of $W(x)$ and therefore also homogeneous solutions of equation (5.1). Similarly, $z(x)$ is a particular solution of equation (5.1). Thus, the columns of $Y(x)$, denoted by $Y_n(x)$, satisfy

$$dY_n(x)/dx = F(x,Y_n) \tag{5.7a}$$

where $n = 1,2,\ldots,m$, and

$$dz(x)/dx = F(x,z) + b(x) \tag{5.7b}$$

The initial values for $Y_n(x)$ and $z(x)$ at $x = a$ are obtained from equations (5.6) as

$$Y(a) = I$$

$$z(a) = 0 \tag{5.8}$$

where I is an (m,m) unit matrix.

The solution of equation (5.1) in the whole interval $a \leq x \leq b$ is formally given by equation (5.5), where Y(x) and z(x) are obtained from the m+1 solutions of the initial value problems defined by equations (5.7) and (5.8). In order to make such a solution also satisfy the prescribed boundary conditions, as given by equations (5.2), the following procedure is used.

Evaluation of equation (5.5) at x = b leads to

$$y(b) = Y(b)y(a) + z(b) \tag{5.9}$$

Premultiplication of equation (5.9) by $T_b$ and the use of equations (5.2) to eliminate y(a) and y(b), gives

$$u_b = U(b)u_a + g(b) \tag{5.10}$$

where

$$U(b) = T_b Y(b) T_a^{-1}$$

$$g(b) = T_b z(b)$$

By definition, the column matrices $u_a$ and $u_b$ each contain m/2 known elements, which are the prescribed variables at each end of the interval. It is convenient to arrange the rows of the given boundary condition matrices, $T_a$ and $T_b$, in such a way that the prescribed elements of $u_a$ appear as the first m/2 elements and the prescribed elements of $u_b$ are the last m/2 elements. Such an arrangement permits the partitioning of equation (5.10) in the form

$$\begin{bmatrix} u_{b1} \\ \hline u_{b2} \end{bmatrix} = \left[\begin{array}{c|c} U_1(b) & U_2(b) \\ \hline U_3(b) & U_4(b) \end{array}\right] \begin{bmatrix} u_{a1} \\ \hline u_{a2} \end{bmatrix} + \begin{bmatrix} g_1(b) \\ \hline g_2(b) \end{bmatrix} \tag{5.11}$$

which can be written as

$$u_{b1} = U_1(b)u_{a1} + U_2(b)u_{a2} + g_1(b) \tag{5.12a}$$

$$u_{b2} = U_3(b)u_{a1} + U_4(b)u_{a2} + g_2(b) \tag{5.12b}$$

Since $u_{b2}$ and $u_{a1}$ are known, equation (5.12b) can be used to find $u_{a2}$ in the form

$$u_{a2} = [U_4(b)]^{-1}[u_{b2} - U_3(b)u_{a1} - g_2(b)] \tag{5.13}$$

Having found the unprescribed variables at x = a, the elements of y which appear in equation (5.1) are given at x = a by

$$y(a) = T_a^{-1}\begin{bmatrix} u_{a1} \\ \hline u_{a2} \end{bmatrix} \tag{5.14}$$

Now the solution y(x) can be obtained at any desired output point within the interval $a \leq x \leq b$ by one more initial value integration of equation (5.1), with the initial values given by equation (5.14). Such an integration must give at x = b a solution which satisfies the boundary conditions at x = b exactly. The boundary value problem defined by equations (5.1) and (5.2) can then be regarded as formally solved.

A similar procedure can be employed for the solution of eigenvalue problems, for which in equation (5.1) the functions F depend also on a parameter, say $\omega$, and for which b(x) = 0 and the boundary conditions are homogeneous, i.e., $u_{a1} = u_{b2} = 0$. Then, the solution matrices Y and U depend on $\omega$, and equation (5.12b) can be written as

$$U_4(\omega,b)u_{a2} = 0 \tag{5.15}$$

A nontrivial solution for $u_{a2}$ is possible if the (m/2,m/2) matrix $U_4(\omega,b)$ is such that

$$\det[U_4(\omega,b)] = 0 \tag{5.16}$$

Equation (5.16) is then the characteristic equation of the eigenvalue problem, and the particular value of $\omega$ which satisfies equation (5.16) is an eigenvalue. Once an eigenvalue is found, the corresponding eigenvector, $u_{a2}$, is obtained from

$$u_{a2}^i = d(-1)^{i+1}\det[M_i] \tag{5.17}$$

where $u_{a2}^i$ denotes the i-th element of $u_{a2}$, d is an arbitrary constant, and $M_i$ is an (m/2-1,m/2-1) matrix obtained from $U_4(\omega,b)$ after deleting any one row and then, in succession, the i-th column.

Once the unprescribed elements of $u_{a2}$ are determined from equation (5.17), the solution, corresponding to the eigenvalue which satisfies equation (5.16), can again be obtained by first using equation (5.14) and then performing one initial value integration from x = a to x = b.

While this method of solution is sound in principle, it is not so in practice. If any required number of significant digits were kept in all initial value integrations, matrix inversions and multiplications, then the method would give a correct solution for any size of the interval $a \le x \le b$. However, if only a fixed number of significant digits, such as seven or eight, is used in the calculation, the solution loses all accuracy beyond a certain critical length of the interval.

The inevitable loss of accuracy inherent in this method is not caused by errors introduced through the numerical integration of the initial value problems, but it results from the subtraction of numbers whose significant digits are identical. For example, if seven digits are used for each number, and if at any point during the calculation two numbers with four identical significant digits are subtracted, then the accuracy of subsequent calculations involving these numbers is at most three significant digits.

In particular, such accuracy loss occurs in equation (5.9). It can be illustrated by the following example. When the method is applied to a cylindrical shell, the homogeneous solutions which make up the columns of Y are known to be linear combinations of $e^{ax}$ and $e^{-ax}$. As x is increased, the columns of Y increase in magnitude as $e^{ax}$. Consider, for example, the axisymmetric case when the deformation of the shell is caused by some prescribed edge condition at x = a, say by $M_x(a) = 1$ and $N_x(a) = Q(a) = 0$. As the distance from the loaded end is increased, the solution, according to St. Venant's principle, is supposed to decay. The terms of equation (5.9), then, have the following magnitude as x is increased: (1) y(a) stays about the same; (2) Y(b) increases as $e^x$; (3) y(b) decreases. It is obvious that for a sufficiently long shell all the significant digits of the matrix product Y(b)y(a) will have to be subtracted out to obtain smaller and smaller elements of y(b). This accuracy loss is not limited to self-equilibrated load systems, but occurs also even in the cases when the solution does not decay, simply because initial value solutions of the differential equations of a shell of revolution grow exponentially.

A convenient length factor, borrowed from the solution of a cylindrical shell, which can be used for a rough estimate for the critical length of a shell of revolution, is given by

$$\beta = L/(Rh)^{1/2} \tag{5.18}$$

where L is the length of the meridian of the shell, R is a minimum radius of curvature, and h is some characteristic thickness. If the solutions of the initial value problems, defined by equations (5.7) and (5.8) are obtained with a six-digit accuracy, then the foregoing procedure gives good results in the range $\beta<2$. For practical purposes, however, this restriction limits the method to rather short shells. The way out of this dilemma is to use the multisegment method, given in the following section, which has been developed in References [1] and [2].

## 2. Multisegment Method

The loss of accuracy of the solution can be avoided if the initial value problems of the preceding section are defined over sufficiently short segments of the total interval $a \le x \le b$. The length factor $\beta$ for each of the segments should be approximately one. This multisegment technique can be used for the analysis of shells of revolution with any meridional length.

Let the shell be divided into M segments, denoted by $S_i$, where $i = 1,2,\ldots,M$. The coordinates of the ends of the segments are denoted by $x_i$. The left-hand edge of the shell is at $x = x_1$ and the right-hand edge at $x = x_{M+1}$, as shown in Figure 4. In analogy to equation (5.5) of the preceding section, the solution within each segment $S_i$ is given by

$$y(x) = Y_i(x)y(x_i) + z_i(x) \tag{5.19}$$

where $Y_i(x)$ and $z_i(x)$ denote the matrices corresponding to $Y(x)$ and $z(x)$, but pertaining to the segment $S_i$. The key point in the multisegment technique is that for the calculation of $z_i(x)$ and the columns of $Y_i(x)$ the initial values are reset at the beginning of each segment. This procedure limits the growth of the solution to acceptable magnitudes, and the result is that

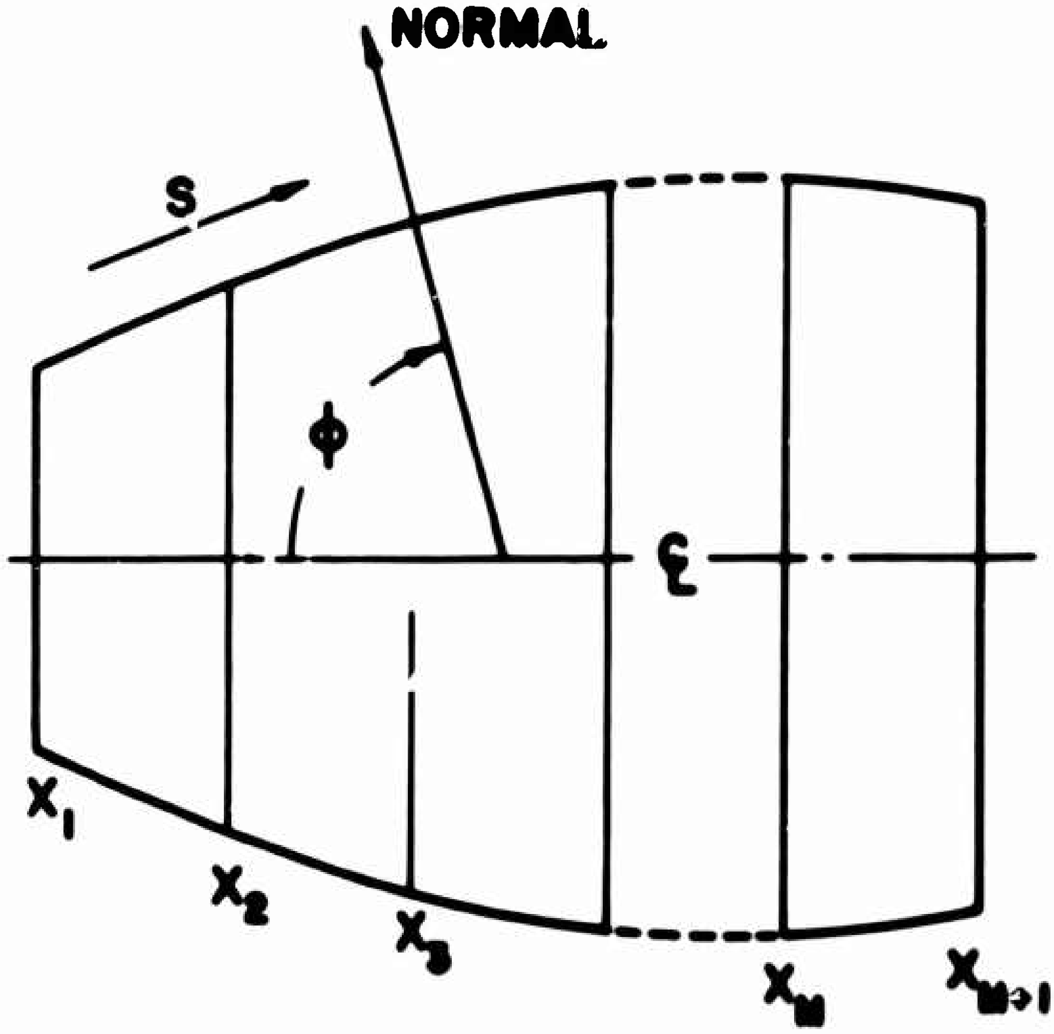


Figure 4. Shell Segments.

the accuracy of the solution can be maintained for long shells.

Thus, the initial value problems, corresponding to equations (5.7) and (5.8), are now defined within the interval $x_i \le x \le x_{i+1}$ by

$$dY_i(x)/dx = F(x, Y_i)$$

$$dz_i(x)/dx = F(x, z_i) + b(x) \tag{5.20}$$

where

$$Y_i(x_i) = I$$

$$z_i(x_i) = 0 \tag{5.21}$$

Requiring the continuity of all elements of $y(x)$ at the inside boundaries of all segments, the following continuity relations are obtained from equation (5.19)

$$y(x_{i+1}) = Y_i(x_{i+1})y(x_i) + z_i(x_{i+1}) \tag{5.22}$$

where $i = 1,2,...,M$. It must be kept in mind, however, that at some points along the shell the variables contained in $y$ may not be continuous. This is the case whenever the normal of the reference surface of the shell changes discontinuously, which happens, for example, at the juncture of a cylindrical and a 45° conical shell. In such cases, the end of a segment should be placed at the point of the discontinuous solution, and the variables of $y$ transformed to new ones, which must be continuous.

For example, if the point of the discontinuous solution is placed at the end of the fourth segment, as shown in Figure 5, then the affected continuity equations are

$$
\begin{aligned}
y'(x_5) &= Y_4(x_5)y(x_4) + z_4(x_5) \\
y(x_6) &= Y_5(x_6)y''(x_5) + z_5(x_6)
\end{aligned}
\qquad (5.23)
$$

The variables given by $y'(x_5)$ belong to the cylindrical shell, while those of $y''(x_5)$ belong the the conical shell, and at the junction, $x = x_5$, they will not be the same.

What must be the same, however, are the displacements and forces at $x = x_5$ along two fixed directions, such as the perpendicular and parallel directions with respect to the axis of symmetry. Thus, the variables $y'(x_5)$ and $y''(x_5)$ must be trans-

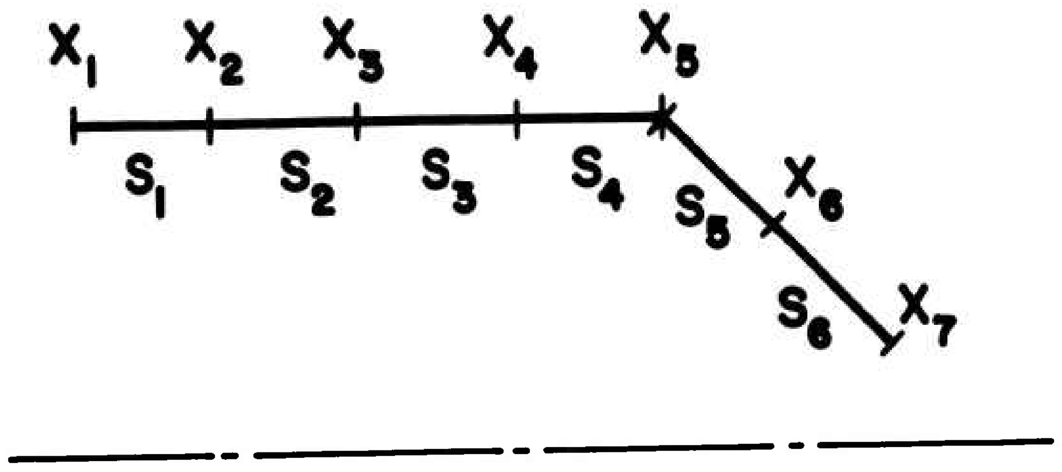


Figure 5. Continuity of Fundemental Variables.

formed to new ones, $u(x_5)$, by means of

$$u(x_5) = T_1 y'(x_5) = T_2 y''(x_5) \tag{5.24}$$

The (m,m) transformation matrices, $T_1$ and $T_2$, are rotation matrices which transform the displacements and forces, contained in y, to those along two fixed directions. Using equations (5.24) in (5.23), we get that

$$\begin{aligned} u(x_5) &= T_1 Y_4(x_5) y(x_4) + T_1 z_4(x_5) \\ y(x_6) &= Y_5(x_6) T_2^{-1} u(x_5) + z_5(x_6) \end{aligned} \tag{5.25}$$

It is seen from equations (5.25) that if a discontinuity in the solution occurs at $x = x_j$, then in the preceding segment, $S_{j-1}$, we must replace the initial value solutions by the rule

$$\begin{aligned} T_1 Y_{j-1}(x_j) &\to Y_{j-1}(x_j) \\ T_1 z_{j-1}(x_j) &\to Y_{j-1}(x_j) \end{aligned} \tag{5.26}$$

and in the segment $S_j$ by

$$Y_j(x_{j+1})T_2^{-1} \rightarrow Y_j(x_{j+1}) \tag{5.27}$$

A similar transformation must be applied at the two outside boundaries of the shell, i.e., at $x = x_1$ and $x = x_{M+1}$. Let us assume that, as in the preceding section, the boundary conditions are given by

$$\begin{aligned} T_a y(x_1) &= u_a \\ T_b y(x_{M+1}) &= u_b \end{aligned} \tag{5.28}$$

where the first m/2 elements of $u_a$ and the last m/2 elements of $u_b$ are prescribed. Then, in analogy to equations (5.26) and (5.27), the initial value solutions in the first segment must be replaced by the rule

$$Y_1(x_2)T_a^{-1} \rightarrow Y_1(x_2) \tag{5.29}$$

and those of the last segment by

$$T_b Y_M(x_{M+1}) \rightarrow Y_M(x_{M+1})$$

$$T_b z_M(x_{M+1}) \rightarrow z_M(x_{M+1}) \tag{5.30}$$

The replacement of the initial value solution matrices at the outside boundaries and at the points of discontinuous solutions is carried out easily by performing the matrix multiplications given by equations (5.26) to (5.30), after the initial value solutions have been obtained within each segment using the regular elements of y. After such a replacement is carried out, the continuity equation (5.22) can be left in the same form, if it is understood that the symbol y at the ends of the shell and at the points of a discontinuous solution really denote the transformed variables, $u_a$, $u_b$, and $u(x_j)$, as defined by equations (5.28) and (5.24), respectively.

The continuity equations can be rewritten as a partitioned matrix product in the form

$$\left[\begin{array}{c} y_1(x_{i+1}) \\ \hline y_2(x_{i+1}) \end{array}\right] = \left[\begin{array}{c|c} Y_i^1(x_{i+1}) & Y_i^2(x_{i+1}) \\ \hline Y_i^3(x_{i+1}) & Y_i^4(x_{i+1}) \end{array}\right] \left[\begin{array}{c} y_1(x_i) \\ \hline y_2(x_i) \end{array}\right] + \left[\begin{array}{c} z_i^1(x_{i+1}) \\ \hline z_i^2(x_{i+1}) \end{array}\right] \tag{5.31}$$

which can be written as

$$Y_i^1(x_{i+1})y_1(x_i) + Y_i^2(x_{i+1})y_2(x_i) - y_1(x_{i+1}) = - z_i^1(x_{i+1})$$

$$Y_i^3(x_{i+1})y_1(x_i) + Y_i^4(x_{i+1})y_2(x_i) - y_2(x_{i+1}) = - z_i^2(x_{i+1}) \tag{5.32}$$

The reason for the partitioning is the need to separate the known from the unknown elements in y at the boundaries of the shell. Recalling that $y(x_1)$ and $y(x_{M+1})$ are really the transformed matrices $u_a$ and $u_b$, and that the transformation matrices $T_a$ and $T_b$ are chosen such that the first m/2 elements of $u_a$ and the last m/2 elements of $u_b$ are prescribed, it follows that $y_1(x_1)$ and $y_2(x_{M+1})$ appearing in equations (5.32) are known. Thus, the 2M linear, algebraic equations (5.32) contain exactly 2M unknowns: $y_1(x_i)$, with $i = 2,3,\ldots, M+1$, and $y_2(x_i)$, with $i = 1,2,\ldots,M$. These unknowns are (m/2,1) matrices and the coefficients are (m/2,m/2) matrices. However, by following the usual rules of matrix algebra, the solution of equations (5.32) can be obtained in the same way as if the unknowns and coefficients were scalars.

The solution of equations (5.32) can be obtained by means of the Gaussian elimination technique. For this purpose, equations (5.32) are written as

$$
\begin{bmatrix}
Y_1^2 & -I & 0 & 0 & 0 & 0 \\
Y_1^4 & 0 & -I & 0 & 0 & 0 \\
0 & Y_2^1 & Y_2^2 & -I & 0 & 0 \\
0 & Y_2^3 & Y_2^4 & 0 & -I & 0 \\
0 & 0 & 0 & Y_M^1 & Y_M^2 & -I \\
0 & 0 & 0 & Y_M^3 & Y_M^4 & 0
\end{bmatrix}
\begin{bmatrix}
y_2(x_1) \\ y_1(x_2) \\ y_2(x_2) \\ y_1(x_M) \\ y_2(x_M) \\ y_1(x_{M+1})
\end{bmatrix}
=
\begin{bmatrix}
- z_1^1 - Y_1^1 y_1(x_1) \\
- z_1^2 - Y_1^3 y_1(x_1) \\
- z_2^1 \\
- z_2^2 \\
- z_M^1 \\
- z_M^2 + y_2(x_{M+1})
\end{bmatrix}
\tag{5.33}
$$

where, for brevity, in place of $Y_i^j(x_{i+1})$ and $z_i^j(x_{i+1})$ the symbols $Y_i^j$ and $z_i^j$ have been used. Using Gaussian elimination, the coefficient matrix is first diagonalized to the form

$$
\begin{bmatrix}
E_1 & -I & 0 & 0 & 0 & 0 \\
0 & C_1 & -I & 0 & 0 & 0 \\
0 & 0 & E_2 & -I & 0 & 0 \\
0 & 0 & 0 & C_2 & -I & 0 \\
0 & 0 & 0 & 0 & E_M & -I \\
0 & 0 & 0 & 0 & 0 & C_M
\end{bmatrix}
\begin{bmatrix}
y_2(x_1) \\ y_1(x_2) \\ y_2(x_2) \\ y_1(x_M) \\ y_2(x_M) \\ y_1(x_{M+1})
\end{bmatrix}
=
\begin{bmatrix}
a_1 \\ b_1 \\ a_2 \\ b_2 \\ a_M \\ b_M + y_2(x_{M+1})
\end{bmatrix}
\tag{5.34}
$$

The diagonal elements are given by

$$E_1 = Y_1^2 \tag{5.35a}$$

$$C_1 = Y_1^4 E_1^{-1} \tag{5.35b}$$

and

$$E_i = Y_i^2 + Y_i^1 C_{i-1}^{-1} \tag{5.35c}$$

$$C_i = (Y_i^4 + Y_i^3 C_{i-1}^{-1}) E_i^{-1} \tag{5 35d}$$

for $i = 2,3,\ldots,M$. The nonhomogeneous terms, $a_i$ and $b_i$, are given by

$$a_1 = - z_1^1 - Y_1^1 y_1(x_1) \tag{5.36a}$$

$$b_1 = - z_1^2 - Y_1^3 y_1(x_1) - C_1 a_1 \tag{5.36b}$$

and by

$$a_i = - z_i^1 - Y_i^1 C_{i-1}^{-1} b_{i-1} \tag{5.36c}$$

$$b_i = - z_i^2 - Y_i^3 C_{i-1}^{-1} b_{i-1} - C_i a_i \tag{5.36d}$$

for $i = 2,3,\ldots,M$.

Once the system of equations is diagonalized and the matrices $E_i$, $C_i$, $a_i$, and $b_i$ stored, the unknowns can be calculated from

$$y_1(x_{M-i+2}) = C_{M-i+1}^{-1} [y_2(x_{M-i+2}) + b_{M-i+1}] \tag{5.37a}$$

$$y_2(x_{M-i+1}) = E_{M-i+1}^{-1} [y_1(x_{M-i+2}) + a_{M-i+1}] \tag{5.37b}$$

where $i = 1,2,\ldots,M$. It should be again recalled that $y_2(x_{M+1})$ represents the $m/2$ known elements contained in $u_b$ as given by equations (5.2).

For eigenvalue problems, when the coefficients of equations (5.32) depend on a parameter, the characteristic equation is obtained from

$$C_M y_1(x_{M+1}) = 0 \tag{5.38}$$

which is the last equation in equations (5.34). If

$$\det[C_M] = 0$$

then a nontrivial solution for $y_1(x_{M+1})$ exists and is given by the same procedure as used in equation (5.17). Other unknowns are then obtained directly from equations (5.37), where all $a_i$ and $b_i$ are zero.

By means of the method given in this section, all the boundary value problems covered in this report will be solved.

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## APPENDIX

# ON THE DERIVATION OF A GENERAL THEORY OF ELASTIC SHELLS*

by

Arturs Kalnins**

## ABSTRACT

A complete and self-contained theory of shells is presented which is based on three and only three assumptions. A first-order system of partial differential equations is given which by means of direct numerical integration can be used to solve a large class of shell problems.

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* This paper has been submitted for publication in the Indian Journal of Mathematics, and was scheduled to be published in Volume 9 of the Journal. It represents Reference [1] in the foregoing PART I.

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# INTRODUCTION

The purpose of this paper is to present a complete and self-contained theory which governs the deformation of an elastic shell. The features of the theory are:

1. It is applicable to orthotropic material.
2. Shell-wall can consist of layers of different material.
3. Reference surface can be any convenient surface and need not be the middle surface of the shell.

In addition to the usual assumptions of the linear theory of elasticity, the shell theory given here is based on the three further assumptions:

1. Points on a normal of a reference surface before deformation remain on a straight line after deformation.
2. Distances between points on a normal do not change during deformation.
3. Stresses are replaced by stress-resultants.

Since the effect of transverse shear strain is not neglected, the theory presented here is equivalent to what might be called a shear theory of shells, which has been considered in the literature before (as, for example, in [1]*). However, the reason for deriving the theory in this paper is not to show how the effect of the transverse shear strain can be included, but rather to show how a complete theory of shells can be derived in a straightforward way from the basic physical concepts and how solutions to such a theory can be obtained.

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*Numbers in brackets designate references at end of paper.

The derived system of equations satisfies the requirement that the shell is stress free when subjected to a rigid-body displacement field, or, equivalently, that the equilibrium equation about the normal is identically satisfied. The system of equations is also in accord with the Principle of Minimum Potential Energy, and its solutions are such that they satisfy an Orthogonality Relation for the modes of free vibration and a Reciprocal Relation.

Because of the attempt to give here a self-contained treatment of the theory, the reader will find portions of this paper in previous publications. For example, the method of deriving the equations of equilibrium in Section III is the same as that used by Reissner [2]. The representation of the strain-displacement equations in vector form in Section II is essentially the same as that employed by Knowles and Reissner [3]. The idea of representing the kinematic constraints on the displacement field by introducing artificial anisotropy in the transverse direction comes from Hildebrand, Reissner, and Thomas [1].

A few remarks on the notation employed in this paper are in order. Symbols are explained in text. Differentiation with respect to a space coordinate is indicated by a comma and with respect to time by a dot. Greek indices can take values 1 and 2, while Latin indices, unless otherwise stated, can be 1, 2, and 3. The usual summation convention of terms with repeated indices, over all values the index can take.

is used in Sections I and II only. A solidus is used in place of a horizontal line to indicate division, so that the expression A/BC means A divided by the product BC.

## I. GEOMETRY OF THE REFERENCE SURFACE OF A SHELL

Consider a curvilinear, three-dimensional coordinate system with coordinates $\xi_i$ and a cartesian coordinate system with coordinates $x_i$ (see Figure 1). It is assumed that the transformation relations of the coordinates of any point in space are uniquely specified by

$$\xi_i = \xi_i(x_1, x_2, x_3) \tag{1.1a}$$

$$x_i = x_i(\xi_1, \xi_2, \xi_3) \tag{1.1b}$$

A shell is defined as a three-dimensional body, whose one dimension is smaller than any other characteristic length. For the descrip ion of the location of the points of the shell, the $\xi_i$ coordinate system is chosen in a special way. A reference surface of the shell is defined as the $(\xi_1, \xi_2)$ coordinate surface at $\xi_3 = 0$, and the $\xi_3$ coordinate curve is taken as a straight line directed along the normal of the reference surface. The reference surface is selected such that the interval in the $\xi_3$ direction, occupied by the shell, equals the smaller dimension of the shell, and the surface should be oriented in such a way that this dimension is as small as possible. The length of the $\xi_3$ coordinate line, lying within the shell, is called the thickness of the shell.

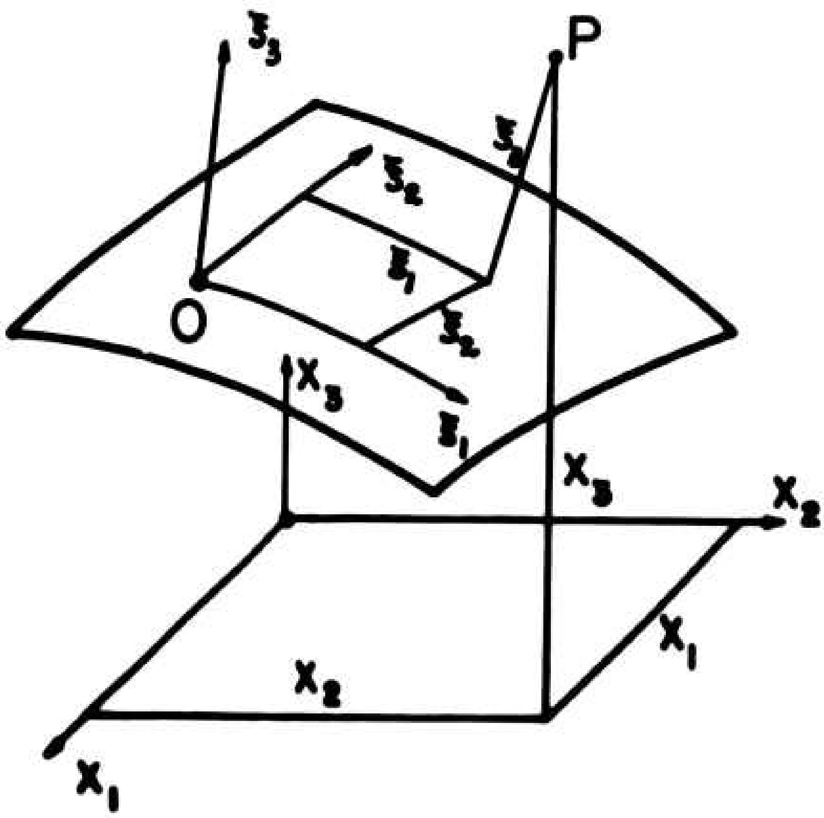


Figure 1. Element of the Reference Surface of the Shell.

The position vector from the origin of the $x_i$-coordinate system to a point P of the shell (Figure 2) is written in the form

$$\underset{\sim}{R}(\xi_1,\xi_2,\xi_3) = \underset{\sim}{r}(\xi_1,\xi_2) + \xi_3\underset{\sim}{t}_3(\xi_1,\xi_2) \qquad (1.2)$$

where $\underset{\sim}{r}$ is the position vector of the point $P_1$ on the reference surface and $\underset{\sim}{t}_3$ is the unit normal vector of the reference surface at $P_1$.

A tangent vector along a given coordinate curve at a point P is obtained by differentiating the position vector of P with respect to the coordinate. For example, the tangent vectors with respect to the coordinate curves of the three-dimensional coordinate system $\xi_i$ are given by

$$\underset{\sim}{g}_i = \underset{\sim}{R}_{,i} \qquad (1.3a)$$

and the tangent vectors along the coordinate curves on the reference surface are given by

$$\underset{\sim}{t}_i = \underset{\sim}{r}_{,i} \qquad (1.3b)$$

In the remainder of this paper, it will be assumed that the coordinate system $\xi_\alpha$ on the reference surface is orthogonal, so that

$$\underset{\sim}{t}_1 \cdot \underset{\sim}{t}_2 = 0 \qquad (1.4)$$

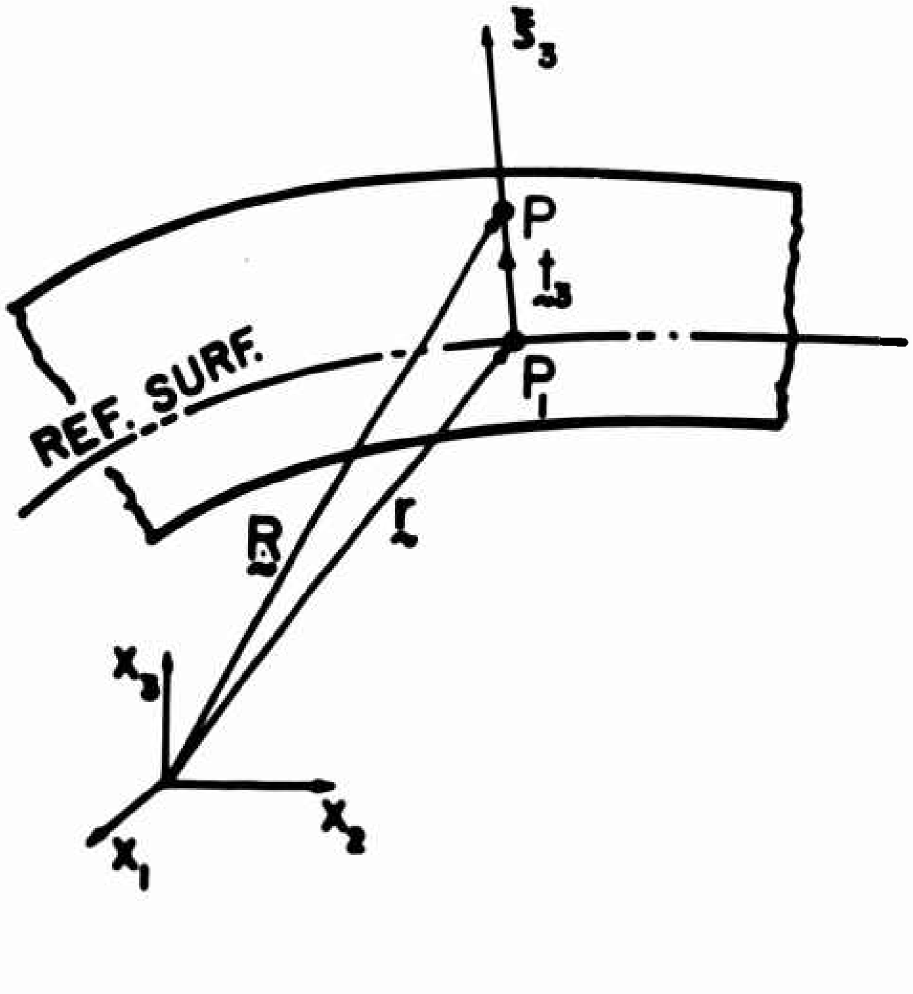


Figure 2. Position Vector of a Point of the Shell.

The nonzero components of the metric tensor of the two-dimensional coordinate system on the reference surface are defined by

$$\alpha_1^2 = \underset{\sim}{a}_1 \cdot \underset{\sim}{a}_1 \tag{1.5a}$$

$$\alpha_2^2 = \underset{\sim}{a}_2 \cdot \underset{\sim}{a}_2 \tag{1.5b}$$

so that we can express the tangent vectors $\underset{\sim}{a}_\lambda$ in terms of unit tangent vectors $\underset{\sim}{t}_1$ and $\underset{\sim}{t}_2$ in the form

$$\underset{\sim}{a}_1 = \alpha_1 \underset{\sim}{t}_1 \tag{1.6a}$$

$$\underset{\sim}{a}_2 = \alpha_2 \underset{\sim}{t}_2 \tag{1.6b}$$

The parameters $\alpha_1$ and $\alpha_2$ are required quantities in the governing equations for a shell. They can be calculated by the following procedure. Starting with the given Eq. (1.1b), with $\zeta_3 = 0$, the position vector $\underset{\sim}{r}$ can be written as

$$\underset{\sim}{r} = x_i(\zeta_1,\zeta_2)\underset{\sim}{e}_i$$

where $\underset{\sim}{e}_i$ are the unit tangent vectors of the cartesian coordinate system. It is convenient to express $\underset{\sim}{r}$ in terms of $\underset{\sim}{e}_i$, because all derivatives of $\underset{\sim}{e}_i$ are zero. The tangent vectors $\underset{\sim}{a}_\lambda$ can then be found in terms of $\underset{\sim}{e}_i$ from Eq. (1.3b) as

$$\underset{\sim}{a}_\lambda = x_{j,\lambda}\underset{\sim}{e}_j$$

and then from Eq. (1.5) it follows that

$$\alpha_\lambda^2 = (x_{1,\lambda})^2 + (x_{2,\lambda})^2 + (x_{3,\lambda})^2 \tag{1.7}$$

Once the reference surface is defined by Eq. (1.1b), with $\xi_3 = 0$, the unit normal vector is also defined and given by

$$\underset{\sim}{t}_3 = \underset{\sim}{a}_1 \times \underset{\sim}{a}_2/\alpha_1\alpha_2$$

Evaluating the vector product, we get, after using Eq. (1.6), that

$$\underset{\sim}{t}_3 = [(x_{2,1}x_{3,2} - x_{3,1}x_{2,2})\underset{\sim}{e}_1 + (x_{3,1}x_{1,2} - x_{1,1}x_{3,2})\underset{\sim}{e}_2 + (x_{1,1}x_{2,2} - x_{2,1}x_{1,2})\underset{\sim}{e}_3]/\alpha_1\alpha_2 \tag{1.8}$$

Consider now the angle between the tangent vectors $\underset{\sim}{g}_i$ of the three-dimensional coordinate system $\xi_i$, with the assumption of Eqs. (1.2) and (1.4). The cosine of the angle between $\underset{\sim}{g}_1$ and $\underset{\sim}{g}_2$ is proportional to their scalar product, and, using Eqs. (1.3a), (1.2), and (1.3b), we obtain

$$\begin{aligned}\underset{\sim}{g}_1 \cdot \underset{\sim}{g}_2 &= (\underset{\sim}{a}_1 + \xi_3\underset{\sim}{t}_{3,1}) \cdot (\underset{\sim}{a}_2 + \xi_3\underset{\sim}{t}_{3,2}) \\ &= \xi_3(\underset{\sim}{t}_{3,1} \cdot \underset{\sim}{a}_2 + \underset{\sim}{t}_{3,2} \cdot \underset{\sim}{a}_1) + \xi_3^2\underset{\sim}{t}_{3,1} \cdot \underset{\sim}{t}_{3,2}\end{aligned} \tag{1.9}$$

Thus, even though the vectors $\underset{\sim}{g}_\alpha$ are orthogonal on the reference surface, they are not orthogonal at points away from the reference surface unless the terms on the right-hand side of Eq. (1.9) are zero.

Let us assume that the coordinate system $\xi_\alpha$ on the reference surface is such that the terms on the right-hand side of Eq. (1.9) are zero, and let us investigate the consequences of

$$\underset{\sim}{t}_{3,1} \cdot \underset{\sim}{a}_2 = 0 \tag{1.10}$$

Referring to Figure 3, this means that the change in the normal along the $\xi_1$-curve is orthogonal to $\underset{\sim}{t}_2$. Because of Eq. (1.4) and the fact that $\underset{\sim}{t}_3$ is a unit vector, Eq. (1.10) also means that $\underset{\sim}{t}_{3,1}$ is parallel to $\underset{\sim}{t}_1$, and, consequently, that the normals at P and $Q_1$ intersect (see Figure 3). The point of intersection, $C_1$, is called the <u>center of curvature</u> of the $\xi_1$ coordinate curve at P, and the distance $C_1P$ is called the <u>principal radius of curvature</u> of the surface at P along the $\xi_1$ coordinate curve and is denoted by $R_1$. From the cross-hatched similar triangles shown in Figure 3, it follows that

$$\underset{\sim}{t}_{3,1} = \alpha_1 \underset{\sim}{t}_1 / R_1 \tag{1.11}$$

where the sign convention has been employed that a radius of curvature at a point on the surface is positive if the normal at that point is directed away from the center of curvature. This convention will be used throughout this paper.

In order to investigate the second term on the right-hand side of Eq. (1.9), the following identities, which are obtained from Eq. (1.3b) and by differentiation of

$$\underset{\sim}{t}_3 \cdot \underset{\sim}{a}_\alpha = 0$$

are required

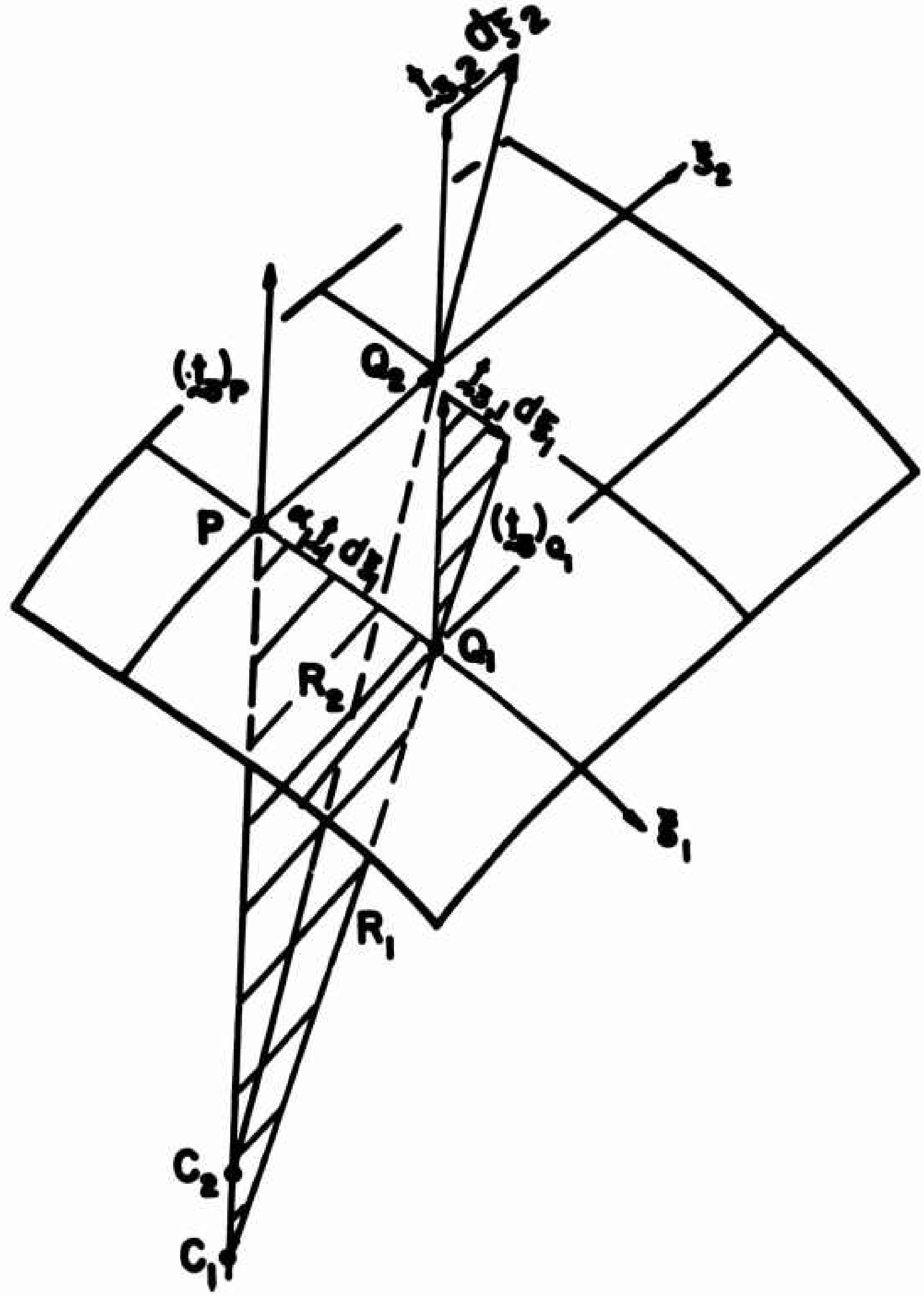


Figure 3. Principal Directions of Reference Surface.

$$\mathbf{t}_{3,1} \cdot \mathbf{a}_2 = - \mathbf{t}_3 \cdot \mathbf{a}_{2,1} \tag{1.12a}$$

$$\mathbf{a}_{2,1} = \mathbf{a}_{1,2} \tag{1.12b}$$

$$\mathbf{t}_3 \cdot \mathbf{a}_{1,2} = - \mathbf{t}_{3,2} \cdot \mathbf{a}_1 \tag{1.12c}$$

These identities imply that if Eq. (1.10) is satisfied, then also the second term in Eq. (1.9) vanishes; i.e.,

$$\mathbf{t}_{3,2} \cdot \mathbf{a}_1 = 0 \tag{1.13}$$

This means that the normals at P and $Q_2$ also intersect. Similarly, the center of curvature, $C_2$, and the principal radius of curvature, $R_2$, with respect to the $\xi_2$ coordinate curve at P are defined, and the relation

$$\mathbf{t}_{3,2} = \alpha_2 \mathbf{t}_2 / R_2 \tag{1.14}$$

is obtained.

At any point on a surface, there are two unique, mutually orthogonal directions along which Eqs. (1.10) and (1.13) hold. These directions are called the <u>principal directions</u> at a point on the surface, and the curves on the surface which are everywhere tangent to the principal directions are called the <u>lines</u> of <u>curvature</u> of the surface. The necessary and sufficient condition that a coordinate system is directed along the lines of curvature is that Eqs. (1.4) and (1.10) are satisfied. In the remainder of this paper, it will be assumed that the coordinate system $\xi_\alpha$

is orthogonal and coincides with the lines of curvature of the reference surface.

In addition to the parameters $\alpha_1$ and $\alpha_2$, which are given by Eqs. (1.7), the only other quantities which are required for a complete description of the reference surface of the shell and its coordinate system are the two principal radii of curvature. They are calculated from Eqs. (1.11) and (1.14) in the form

$$1/R_1 = \underset{\sim}{t}_{3,1} \cdot \underset{\sim}{t}_1/\alpha_1 \tag{1.15a}$$

$$1/R_2 = \underset{\sim}{t}_{3,2} \cdot \underset{\sim}{t}_2/\alpha_2 \tag{1.15b}$$

where the derivatives of $\underset{\sim}{t}_3$ must be obtained by differentiation of Eq. (1.8).

If the $\xi_\alpha$ coordinate curves are the lines of curvature of the surface, then it follows directly from Eqs. (1.11), (1.14), and (1.4) that the last term in Eq. (1.9) is also zero. Consequently, for line-of-curvature coordinates on the reference surface, the three-dimensional coordinate system $\xi_i$ is orthogonal, and we can define the nonzero components of the metric tensor for this coordinate system as

$$\begin{aligned} A_1^2 &= \underset{\sim}{g}_1 \cdot \underset{\sim}{g}_1 \\ A_2^2 &= \underset{\sim}{g}_2 \cdot \underset{\sim}{g}_2 \\ A_3^2 &= \underset{\sim}{g}_3 \cdot \underset{\sim}{g}_3 \end{aligned} \tag{1.16}$$

In view of Eqs. (1.2), (1.3), (1.11), and (1.14), we get that

$$\begin{aligned} \underset{\sim}{g}_1 &= (1 + \xi_3/R_1)\alpha_1 \underset{\sim}{t}_1 \\ \underset{\sim}{g}_2 &= (1 + \xi_3/R_2)\alpha_2 \underset{\sim}{t}_2 \\ \underset{\sim}{g}_3 &= \underset{\sim}{t}_3 \end{aligned} \tag{1.17}$$

and then from Eq. (1.16) it follows that for the special three-dimensional coordinate system employed for a shell, we have

$$\begin{aligned} A_1 &= (1 + \xi_3/R_1)\alpha_1 \\ A_2 &= (1 + \xi_3/R_2)\alpha_2 \\ A_3 &= 1 \end{aligned} \tag{1.18}$$

In the derivation of the governing equations for a shell, relations are required for the derivatives of the unit tangent vectors $\underset{\sim}{t}_\alpha$ in terms of the unit vectors themselves. These relations can be deduced by finding the scalar products of the derivatives with each of the unit vectors.

For example, differentiation of

$$\underset{\sim}{t}_1 \cdot \underset{\sim}{t}_1 = 1$$

with respect to $\xi_1$ gives

$$\underset{\sim}{t}_{1,1} \cdot \underset{\sim}{t}_1 = 0$$

Furthermore, differentiation of Eq. (1.4) with respect to $\xi_1$ and Eq. (1.5a) with respect to $\xi_2$ leads to

$$\underset{\sim}{a}_{1,1} \cdot \underset{\sim}{a}_2 = - \underset{\sim}{a}_1 \cdot \underset{\sim}{a}_{2,1}$$

$$\underset{\sim}{a}_1 \cdot \underset{\sim}{a}_{1,2} = \alpha_1 \alpha_{1,2}$$

Making use of Eqs. (1.12b) and (1.6a), we obtain

$$\underset{\sim}{t}_{1,1} \cdot \underset{\sim}{t}_2 = - \alpha_{1,2}/\alpha_2$$

The $\underset{\sim}{t}_3$ component of $\underset{\sim}{t}_{1,1}$ is found by first forming the scalar product of Eq. (1.11) and $\underset{\sim}{t}_1$ in the form

$$\underset{\sim}{t}_{3,1} \cdot \underset{\sim}{t}_1 = \alpha_1/R_1$$

and then by differentiation of

$$\underset{\sim}{t}_1 \cdot \underset{\sim}{t}_3 = 0$$

with respect to $\xi_1$. This procedure gives

$$\underset{\sim}{t}_{1,1} \cdot \underset{\sim}{t}_3 = - \alpha_1/R_1$$

The $\underset{\sim}{t}_1$ component of $\underset{\sim}{t}_{1,2}$ is found from

$$(\underset{\sim}{t}_1 \cdot \underset{\sim}{t}_1)_{,2} = 2\underset{\sim}{t}_{1,2} \cdot \underset{\sim}{t}_1 = 0$$

and the $\underset{\sim}{t}_3$ component from

$$(\underset{\sim}{t}_1 \cdot \underset{\sim}{t}_3)_{,2} = \underset{\sim}{t}_{1,2} \cdot \underset{\sim}{t}_3 = 0$$

which follows because of Eq. (1.13). Finally, differentiating

Eq. (1.5b) with respect to $\xi_1$ and making use of Eq. (1.12b) leads tc

$$\underset{\sim}{t}_{1,2} \cdot \underset{\sim}{t}_2 = \alpha_{2,1}/\alpha_1$$

From the scalar products given above, and by exchanging the subscripts 1 and 2, the desired formulas are obtained in the form

$$\underset{\sim}{t}_{1,1} = -\alpha_{1,2}\underset{\sim}{t}_2/\alpha_2 - \alpha_1\underset{\sim}{t}_3/R_1 \tag{1.19a}$$

$$\underset{\sim}{t}_{1,2} = \alpha_{2,1}\underset{\sim}{t}_2/\alpha_1 \tag{1.19b}$$

$$\underset{\sim}{t}_{2,2} = -\alpha_{2,1}\underset{\sim}{t}_1/\alpha_1 - \alpha_2\underset{\sim}{t}_3/R_2 \tag{1.19c}$$

$$\underset{\sim}{t}_{2,1} = \alpha_{1,2}\underset{\sim}{t}_1/\alpha_2 \tag{1.19d}$$

Eqs. (1.19) are known as Gauss formulas.

Two more relations of differential geometry will be required which can be obtained from the $\underset{\sim}{t}_3$ component of the identity

$$\underset{\sim}{t}_{1,12} - \underset{\sim}{t}_{1,21} = 0 \tag{1.20}$$

After making use of Eqs. (1.19), the $\underset{\sim}{t}_3$ component of Eq. (1.20) is given by

$$\alpha_{1,2}/R_2 = (\alpha_1/R_1)_{,2} \tag{1.21a}$$

Similarly, by exchanging the indices 1 and 2, we get

$$\alpha_{2,1}/R_1 = (\alpha_2/R_2)_{,1} \tag{1.21b}$$

Eqs. (1.21) are known as the Codazzi formulas.

## II. ANALYSIS OF STRAIN IN A SHELL

Deformation at a point $P_o$ of a continuous medium is characterized by the changes in the lengths of infinitesimal line elements emanating in all directions from $P_o$. One such line element, $P_oQ_o$, which can lie in any desired direction, can be represented in the undeformed body by an increment, $d\underset{\sim}{R}_o$, in the position vector, $\underset{\sim}{R}_o$, of the point $P_o$ (see Figure 4).

At the end of deformation, the point $P_o$ has moved to P, whose position vector is

$$\underset{\sim}{R} = \underset{\sim}{R}_o + \underset{\sim}{v} \tag{2.1}$$

where $\underset{\sim}{v}$ is defined as the displacement vector. The point $Q_o$, which before deformation has the position vector

$$\underset{\sim}{R}_o + d\underset{\sim}{R}_o$$

has moved to Q with the position vector

$$\underset{\sim}{R} + d\underset{\sim}{R} = \underset{\sim}{R}_o + \underset{\sim}{v} + d\underset{\sim}{R}_o + d\underset{\sim}{v} \tag{2.2}$$

The difference in the square of the lengths of the line elements $P_oQ_o$ and PQ is given by

$$ds^2 - ds_o^2 = d\underset{\sim}{R} \cdot d\underset{\sim}{R} - d\underset{\sim}{R}_o \cdot d\underset{\sim}{R}_o \tag{2.3}$$

By means of Eq. (2.2) we get that

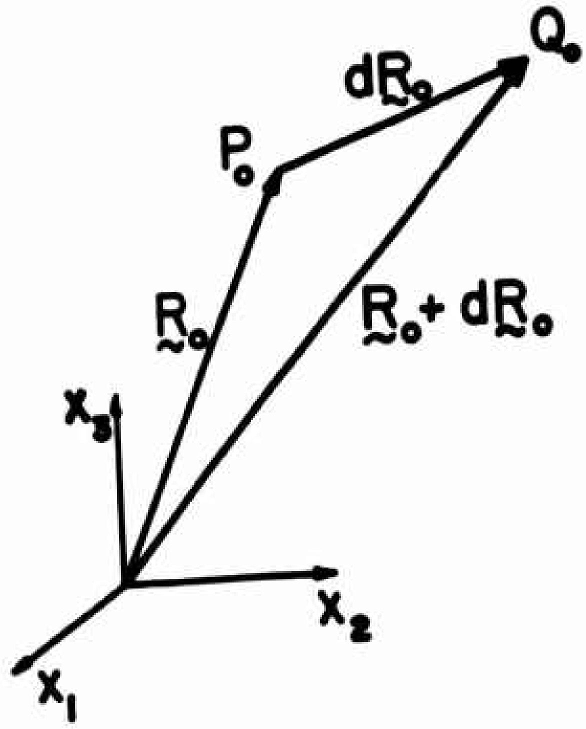


Figure 4. Arbitrary Line Element Through $P_0$.

$$ds^2 - ds_0^2 = 2d\underset{\sim}{R}_0 \cdot d\underset{\sim}{v} + d\underset{\sim}{v} \cdot d\underset{\sim}{v} \tag{2.4}$$

Equation (2.4) is applicable for the calculation of the change in the length of any line element which before deformation is represented by $d\underset{\sim}{R}_0$.

The state of deformation at a point $P_0$ is expressed in terms of the components of a (3,3) matrix which is called the <u>strain matrix</u>. The diagonal elements of the strain matrix represent the elongations of the sides and the off-diagonal elements are the changes in the angles between the sides of a parallelopiped whose diagonal, before deformation, is $d\underset{\sim}{R}_0$, and whose sides are parallel to the tangent vectors at $P_0$ of the coordinate curves of a given coordinate system (see Figure 5). The components of the strain matrix are dependent on the given coordinate system and, if specified at point $P_0$, completely determine the change of the square of the length of any arbitrary line element emanating from $P_0$.

The actual lengths of the sides of the parallelopiped in Figure 5 can be determined by means of the chain-rule expansion

$$d\underset{\sim}{R}_0 = \underset{\sim}{R}_{0,i} d\xi^i = \underset{\sim}{g}_i d\xi^i \tag{2.5}$$

to be $A_1 d\xi^1$, $A_2 d\xi^2$, $A_3 d\xi^3$, where $A_i$ are defined by Eqs. (1.16) and refer to the components of the metric tensor of the coordinate system $\xi_i$ in the undeformed body.

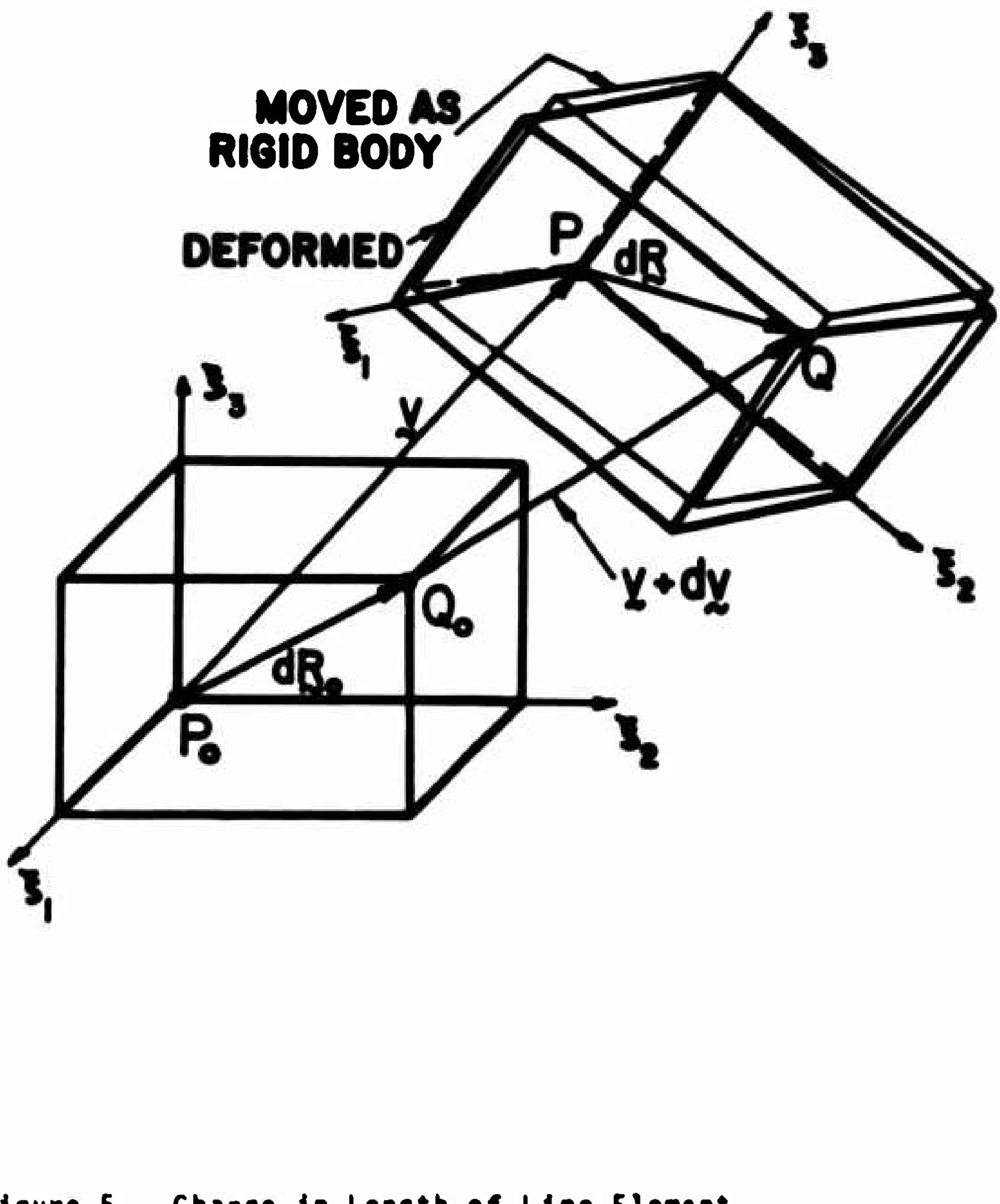


Figure 5. Change in Length of Line Element.

For the purposes of defining the strain matrix for a shell, it is convenient to describe the material points of the shell during deformation by the same coordinates $\xi_i$. This means that all the points on a given coordinate curve before deformation remain on the same coordinate curve after deformation. However, the shapes of the coordinate curves may change during deformation, and, consequently, the tangent vectors $\underset{\sim}{g}_i$ may change in magnitude as well as direction. If such coordinates are employed, then the parallelopiped after deformation with the diagonal $d\underset{\sim}{R}$ will have its sides parallel to the tangent vectors of the coordinate curves $\xi_i$ in the deformed body.

By means of Eq. (2.5) and

$$d\underset{\sim}{v} = \underset{\sim}{v},_i d\xi^i \tag{2.6}$$

Eq. (2.4) can be written as

$$ds^2 - ds_0^2 = (2\underset{\sim}{v},_i \cdot \underset{\sim}{R}_{o,j} + \underset{\sim}{v},_i \cdot \underset{\sim}{v},_j)d\xi^i d\xi^j \tag{2.7}$$

The expression in the parentheses of Eq. (2.7) can be regarded as a displacement gradient matrix and denoted by

$$D_{ij} = 2\underset{\sim}{v},_i \cdot \underset{\sim}{R}_{o,j} + \underset{\sim}{v},_i \cdot \underset{\sim}{v},_j \tag{2.8}$$

It should be noted that in the quadratic form on the right-hand side of Eq. (2.7) only the symmetric part of $D_{ij}$ affects the change in the length of the line element, while the antisymmetric part of $D_{ij}$ subtracts out. In order to determine the meaning of the antisymmetric part of $D_{ij}$, let us write $D_{ij}$

as the sum of a symmetric matrix, $2e_{ij}$, and an antisymmetric matrix, $2\omega_{ij}$, in the form

$$D_{ij} = 2e_{ij} + 2\omega_{ij} \tag{2.9}$$

where $2e_{ij} = \underset{\sim}{v}_{,i} \cdot \underset{\sim}{R}_{o,j} + \underset{\sim}{v}_{,j} \cdot \underset{\sim}{R}_{o,i} + \underset{\sim}{v}_{,i} \cdot \underset{\sim}{v}_{,j}$ (2.10a)

$$2\omega_{ij} = \underset{\sim}{v}_{,i} \cdot \underset{\sim}{R}_{o,j} - \underset{\sim}{v}_{,j} \cdot \underset{\sim}{R}_{o,i} \tag{2.10b}$$

Let us examine the matrices $e_{ij}$ and $\omega_{ij}$ when the displacement vector is that of rigid-body motion. Any such motion can be assumed to consist of a rigid-body translation followed by a single rigid-body rotation through an angle $\theta$ about one axis.

The rigid-body translation can be represented by a constant displacement vector, $\underset{\sim}{v}_o$, which clearly does not affect $D_{ij}$ when substituted into Eqs. (2.10). In order to see the effect of the rigid-body rotation on the symmetric and antisymmetric parts of $D_{ij}$, we can assume that the axis of rotation coincides with the $x_3$ coordinate axis of a cartesian coordinate system. The displacement vector of a point $P_o$ with coordinates $x_1$, $x_2$, 0, which during the rotation moves to P, is shown in Figure 6 and given by

$$\underset{\sim}{v} = - [x_1(1 - \cos\theta) + x_2 \sin\theta]\, \underset{\sim}{e}_1 + [x_1 \sin\theta - x_2(1 - \cos\theta)]\, \underset{\sim}{e}_2 \tag{2.11}$$

Substitution of Eq. (2.11) into (2.10) yields

$$2e_{ij} = 0 \tag{2.12a}$$

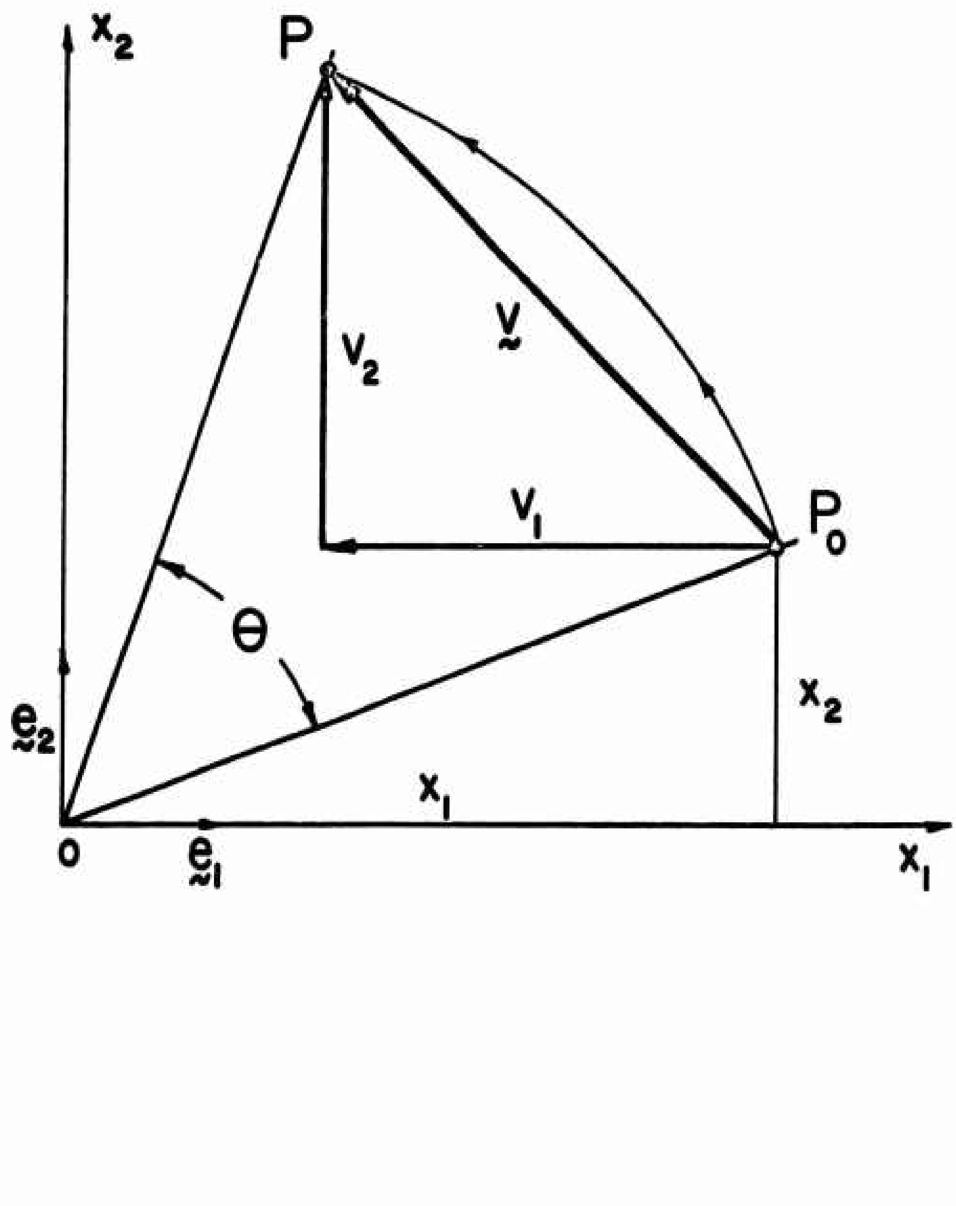


Figure 6. Rigid Body Rotation About $x_3$ Axis.

$$\omega_{12} = - \omega_{21} = \sin \theta \qquad (2.12b)$$

If the components of the symmetric part of $D_{ij}$ are zero in one coordinate system (i.e., in the cartesian system with $x_3$-axis coinciding with the axis of rotation), then they will be zero also in any other coordinate system, because $2e_{ij}$ satisfies the transformation laws of a tensor of order two. Thus, it is clear that the components of the antisymmetric part of $D_{ij}$ merely rotate the parallelopiped as a rigid body, while the symmetric part deforms it (see Figure 5).

Because the state of deformation should be independent of rigid-body motion, it is regarded that the symmetric matrix, $2e_{ij}$, can describe the state of deformation at a point of a continuous medium and, therefore, can be defined as the <u>strain matrix</u>. Because it transforms with respect to coordinate transformations as a second order tensor, it is called the <u>strain tensor</u>.

For the purposes of this paper, it will be assumed that the displacement gradients are infinitesimal, and that the squares of $\underset{\sim}{v}_{,i}$ can be neglected. Moreover, it follows from Eq. (2.7) that if the components of the strain tensor are to have the proper physical dimensions (i.e., dimensionless), then the coordinate increments $d\xi^i$ in Eq. (2.7) must be replaced by the arc-lengths along the coordinate curves, which are given by $A_1 d\xi_1$, $A_2 d\xi_2$, $A_3 d\xi_3$. Consequently, for orthogonal coordinates, the <u>physical components</u> of the strain tensor are defined by

$$2e_{ij} = (\underset{\sim}{v}_{,i} \cdot \underset{\sim}{R}_{o,j} + \underset{\sim}{v}_{,j} \cdot \underset{\sim}{R}_{o,i})/A_i A_j \qquad (2.13)$$

where no summation over i or j is intended. From here on, the symbol $e_{ij}$ will denote the physical components of the strain tensor as given by Eq. (2.13), rather than the tensor components given by Eq. (2.10a).

One of the basic assumptions of the theory of shells derived in this paper is that the displacement vector is linear in $\xi_3$ in the form

$$\underset{\sim}{v}(\xi_1,\xi_2,\xi_3) = \underset{\sim}{u}(\xi_1,\xi_2) + \xi_3\underset{\sim}{\beta}(\xi_1,\xi_2) \tag{2.14}$$

This assumption means that all points on a normal of the reference surface in the undeformed shell are forced to remain on a straight line in the deformed shell and implies a definite constraint on the kinematics of the shell.

While the displacement of the reference surface is expressed in a general form

$$\underset{\sim}{u} = u_1\underset{\sim}{t}_1 + u_2\underset{\sim}{t}_2 + u_3\underset{\sim}{t}_3 \tag{2.15}$$

the vector which describes the rotation of the normal of the reference surface is assumed in the form

$$\underset{\sim}{\beta} = \beta_1\underset{\sim}{t}_1 + \beta_2\underset{\sim}{t}_2 \tag{2.16}$$

Eq. (2.16) implies another assumption used in the paper. Because of the absence of a $\underset{\sim}{t}_3$ component in $\underset{\sim}{\beta}$, it means that the points on a normal do not change their relative positions in the $\xi_3$-direction; i.e., the normals remain the same length.

Substituting Eq. (2.14) into Eq. (2.13) and making use of Eqs. (1.17), strain-displacement relations are obtained in the form

$$2e_{\lambda\delta} = (\underset{\sim}{u},_{\lambda} + \xi_3\underset{\sim}{\beta},_{\lambda}) \cdot \underset{\sim}{t}_{\delta}/(1 + \xi_3/R_{\lambda})\alpha_{\lambda} \qquad (2.17a)$$

$$+ (\underset{\sim}{u},_{\delta} + \xi_3\underset{\sim}{\beta},_{\delta}) \cdot \underset{\sim}{t}_{\lambda}/(1 + \xi_3/R_{\delta})\alpha_{\delta} \qquad (2.17b)$$

$$2e_{33} = 0 \qquad (2.17c)$$

$$2e_{\lambda 3} = (\underset{\sim}{u},_{\lambda} + \xi_3\underset{\sim}{\beta},_{\lambda}) \cdot \underset{\sim}{t}_3/(1 + \xi_3/R_{\lambda})\alpha_{\lambda} + \underset{\sim}{\beta} \cdot \underset{\sim}{t}_{\lambda} \qquad (2.17d)$$

With the use of Eqs. (1.11), (1.14), and (1.19), we find

$$\underset{\sim}{u},_1 = (u_{1,1} + \alpha_{1,2}u_2/\alpha_2 + \alpha_1 u_3/R_1)\underset{\sim}{t}_1$$

$$+ (u_{2,1} - \alpha_{1,2}u_1/\alpha_2)\underset{\sim}{t}_2 + (u_{3,1} - \alpha_1 u_1/R_1)\underset{\sim}{t}_3 \qquad (2.18a)$$

$$\underset{\sim}{\beta},_1 = (\beta_{1,1} + \alpha_{1,2}\beta_2/\alpha_2)\underset{\sim}{t}_1$$

$$+ (\beta_{2,1} - \alpha_{1,2}\beta_1/\alpha_2)\underset{\sim}{t}_2 - \alpha_1\beta_1\underset{\sim}{t}_3/R_1 \qquad (2.18b)$$

and two similar expressions by exchanging the subscripts 1 and 2. Substituting Eqs. (2.18) into Eqs. (2.17), the strain displacement relations for a shell can be written as

$$e_{11} = (\varepsilon_{11} + \xi_3 k_{11})/(1 + \xi_3/R_1) \qquad (2.19a)$$

$$e_{22} = (\varepsilon_{22} + \xi_3 k_{22})/(1 + \xi_3/R_2) \qquad (2.19b)$$

$$e_{33} = 0$$

$$2e_{12} = (\gamma_1 + \xi_3\delta_1)/(1 + \xi_3/R_1)$$

$$+ (\gamma_2 + \xi_3\delta_2)/(1 + \xi_3/R_2) \qquad (2.19c)$$

$$2e_{13} = \gamma_{13}/(1 + \xi_3/R_1) \qquad (2.19d)$$

$$2e_{23} = \gamma_{23}/(1 + \xi_3/R_2) \qquad (2.19e)$$

where we have defined

$$\varepsilon_{11} = \underset{\sim}{u}_{,1} \cdot \underset{\sim}{t}_1/\alpha_1 = u_{1,1}/\alpha_1 + \alpha_{1,2}u_2/\alpha_1\alpha_2 + u_3/R_1 \qquad (2.20a)$$

$$\varepsilon_{22} = \underset{\sim}{u}_{,2} \cdot \underset{\sim}{t}_2/\alpha_2 = u_{2,2}/\alpha_2 + \alpha_{2,1}u_1/\alpha_1\alpha_2 + u_3/R_2 \qquad (2.20b)$$

$$\gamma_1 = \underset{\sim}{u}_{,1} \cdot \underset{\sim}{t}_2/\alpha_1 = u_{2,1}/\alpha_1 - \alpha_{1,2}u_1/\alpha_1\alpha_2 \qquad (2.20c)$$

$$\gamma_2 = \underset{\sim}{u}_{,2} \cdot \underset{\sim}{t}_1/\alpha_2 = u_{1,2}/\alpha_2 - \alpha_{2,1}u_2/\alpha_1\alpha_2 \qquad (2.20d)$$

$$k_{11} = \underset{\sim}{\beta}_{,1} \cdot \underset{\sim}{t}_1/\alpha_1 = \beta_{1,1}/\alpha_1 + \alpha_{1,2}\beta_2/\alpha_1\alpha_2 \qquad (2.20e)$$

$$k_{22} = \underset{\sim}{\beta}_{,2} \cdot \underset{\sim}{t}_2/\alpha_2 = \beta_{2,2}/\alpha_2 + \alpha_{2,1}\beta_1/\alpha_1\alpha_2 \qquad (2.20f)$$

$$\delta_1 = \underset{\sim}{\beta}_{,1} \cdot \underset{\sim}{t}_2/\alpha_1 = \beta_{2,1}/\alpha_1 - \alpha_{1,2}\beta_1/\alpha_1\alpha_2 \qquad (2.20g)$$

$$\delta_2 = \underset{\sim}{\beta}_{,2} \cdot \underset{\sim}{t}_1/\alpha_2 = \beta_{1,2}/\alpha_2 - \alpha_{2,1}\beta_2/\alpha_1\alpha_2 \qquad (2.20h)$$

$$\gamma_{13} = (\underset{\sim}{u}_{,1} \cdot \underset{\sim}{t}_3 + \underset{\sim}{\beta} \cdot \alpha_1\underset{\sim}{t}_1)/\alpha_1$$

$$= u_{3,1}/\alpha_1 - u_1/R_1 + \beta_1 \qquad (2.20i)$$

$$\gamma_{23} = (\underset{\sim}{u}_{,2} \cdot \underset{\sim}{t}_3 + \underset{\sim}{\beta} \cdot \alpha_2\underset{\sim}{t}_2)/\alpha_2$$

$$= u_{3,2}/\alpha_2 - u_2/R_2 + \beta_2 \qquad (2.20j)$$

This completes the derivation of the physical components of the three-dimensional strain tensor for a shell. Clearly, Eqs. (2.19) are no longer exact for a three-dimensional medium, because the restrictive assumptions given by Eqs.(2.14) and (2.16) have been employed.

It is interesting to note that while the components of $e_{ij}$ were shown to be zero for rigid-body motion, the strain measures on the left-hand side of Eqs. (2.20) are not all zero. Using the form of the position vector for a shell given by Eq. (1.2), the rigid-body displacement vector for an infinitesimal rotation $\omega$ and translation $v_0$ is given by

$$\underset{\sim}{v} = \underset{\sim}{v}_0 + \underset{\sim}{\omega} \times \underset{\sim}{r} + \xi_3 \underset{\sim}{\omega} \times \underset{\sim}{t}_3 \tag{2.21}$$

which shows that for rigid-body motion the linear displacement field of the form of Eq. (2.14) is exact. Substituting

$$\underset{\sim}{u} = \underset{\sim}{v}_0 + \underset{\sim}{\omega} \times \underset{\sim}{r} \tag{2.22a}$$

$$\underset{\sim}{\beta} = \underset{\sim}{\omega} \times \underset{\sim}{t}_3 \tag{2.22b}$$

into Eqs. (2.20), we find that all are zero except

$$\gamma_1 = \underset{\sim}{\omega} \cdot \underset{\sim}{t}_3 \tag{2.23a}$$

$$\gamma_2 = -\underset{\sim}{\omega} \cdot \underset{\sim}{t}_3 \tag{2.23b}$$

$$\delta_1 = \underset{\sim}{\omega} \cdot \underset{\sim}{t}_3/R_1 \tag{2.23c}$$

$$\delta_2 = -\underset{\sim}{\omega} \cdot \underset{\sim}{t}_3/R_2 \tag{2.23d}$$

It should be remarked that at this point it is not necessary that the strain measures defined by Eqs. (2.20) be zero. It is important, however, that the components of the strain tensor, as expressed by Eqs. (2.19), be zero, which by means of Eqs. (2.23) is easily shown to be true.

## III. STRESS-RESULTANTS AND EQUILIBRIUM

The stress field at a point P of a three-dimensional continuous medium is characterized by means of the stress vector given on three mutually orthogonal planes passing through P. Assuming that the three mutually orthogonal planes coincide with the tangent planes of the $\xi_i$=constant coordinate surfaces at P, the stress vector, $\underset{\sim}{\sigma}_i$, on the tangent plane to the $\xi_i$=constant coordinate surface is defined by

$$\underset{\sim}{\sigma}_i = \lim_{\Delta A \to 0} \Delta \underset{\sim}{R} / \Delta A \tag{3.1}$$

where $\Delta A$ is an element of area in the tangent plane, containing P, and $\Delta \underset{\sim}{R}$ is the resultant force exerted on $\Delta A$ by the neighboring element lying in the positive coordinate direction from P.

By definition of a shell, the thickness is restricted to be smaller than ony other characteristic length. Because of this property, the stress vector on the faces of the element shown in Figure 7 is replaced in the theory of shells by a statically equivalent* force-couple system attached at the reference surface. In addition to Eqs. (2.14) and (2.16), this replacement

*The term "statically equivalent" is used in this section in the same sense as in statics of rigid bodies. Of course, the replacement of the stresses by their resultant force and couple will not produce identical effects in a deformable medium.

of the stresses by the resultant forces and moments is another basic assumption used in the theory of shells. All of these assumptions introduce errors in the solutions of the equations of shell theory, when compared to the solutions of the three-dimensional theory of elasticity, and these errors are directly proportional to the ratio of the thickness to any other characteristic length of the shell.

Consider an element of the shell which is bounded by the normals of the $\xi_\alpha$ coordinate curves on the reference surface and the two bounding surfaces of the shell, defined by $\xi_3 = z_1$ and $\xi_3 = z_2$ (see Figure 7). The sides of the reference surface, included in the element, have the lengths

$$ds_1 = \alpha_1 d\xi_1$$

$$ds_2 = \alpha_2 d\xi_2$$

and are, therefore, infinitesimals which can be taken as small as desired. The height of the element is finite and equal to the thickness of the shell at point P. This means that the stress field on the faces of the element can be assumed constant in the $\xi_\alpha$ directions and equal to that on the normal at point P. Clearly, the stresses have finite variations in the $\xi_3$ direction. Since the $\xi_\alpha$ coordinate curves are assumed to coincide with the lines of curvature of the reference surface, the four faces of the element are planes and the normals at the edges of a face intersect.

Consider the face $A_1$ shown in Figure 7, which is defined by $\xi_1$=constant. An element of area on this face is given by

$$dA_1 = d\xi_3 dS_2 \tag{3.2}$$

From the ratio of the sides of similar triangles shown in Figure 7, it follows that

$$dS_2 = (1 + \xi_3/R_2)ds_2 \tag{3.3}$$

so that the area element can be expressed in terms of the length of the reference surface on the face $A_1$ in the form

$$dA_1 = (1 + \xi_3/R_2)ds_2 d\xi_3 \tag{3.4}$$

The resultant force, per unit length of the reference surface, produced by the stresses on the face $A_1$ is given by

$$\underset{\sim}{N}_1 = \int \underset{\sim}{\sigma}_1(1 + \xi_3/R_2)d\xi_3 \tag{3.5a}$$

where the integration with respect to $\xi_3$ is carried out from $\xi_3 = z_1$ to $\xi_3 = z_2$*. The resultant moment about point $0_1$, per unit length of the reference surface, is given by

$$\underset{\sim}{M}_1 = \int \underset{\sim}{\varsigma} \times \underset{\sim}{\sigma}_1(1 + \xi_3/R_2)d\xi_3 \tag{3.5b}$$

where the position vector $\underset{\sim}{\varsigma}$, from $0_1$ to any point on the face $A_1$, is defined by

$$\underset{\sim}{\varsigma} = \epsilon \underset{\sim}{t}_2 + \xi_3 \underset{\sim}{t}_3 \tag{3.6a}$$

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*Unless otherwise specified, the limits of all integrals with respect to $\xi_3$ will be understood to be $z_1$ and $z_2$; i.e., the coordinates of the two bounding surfaces of the shell.

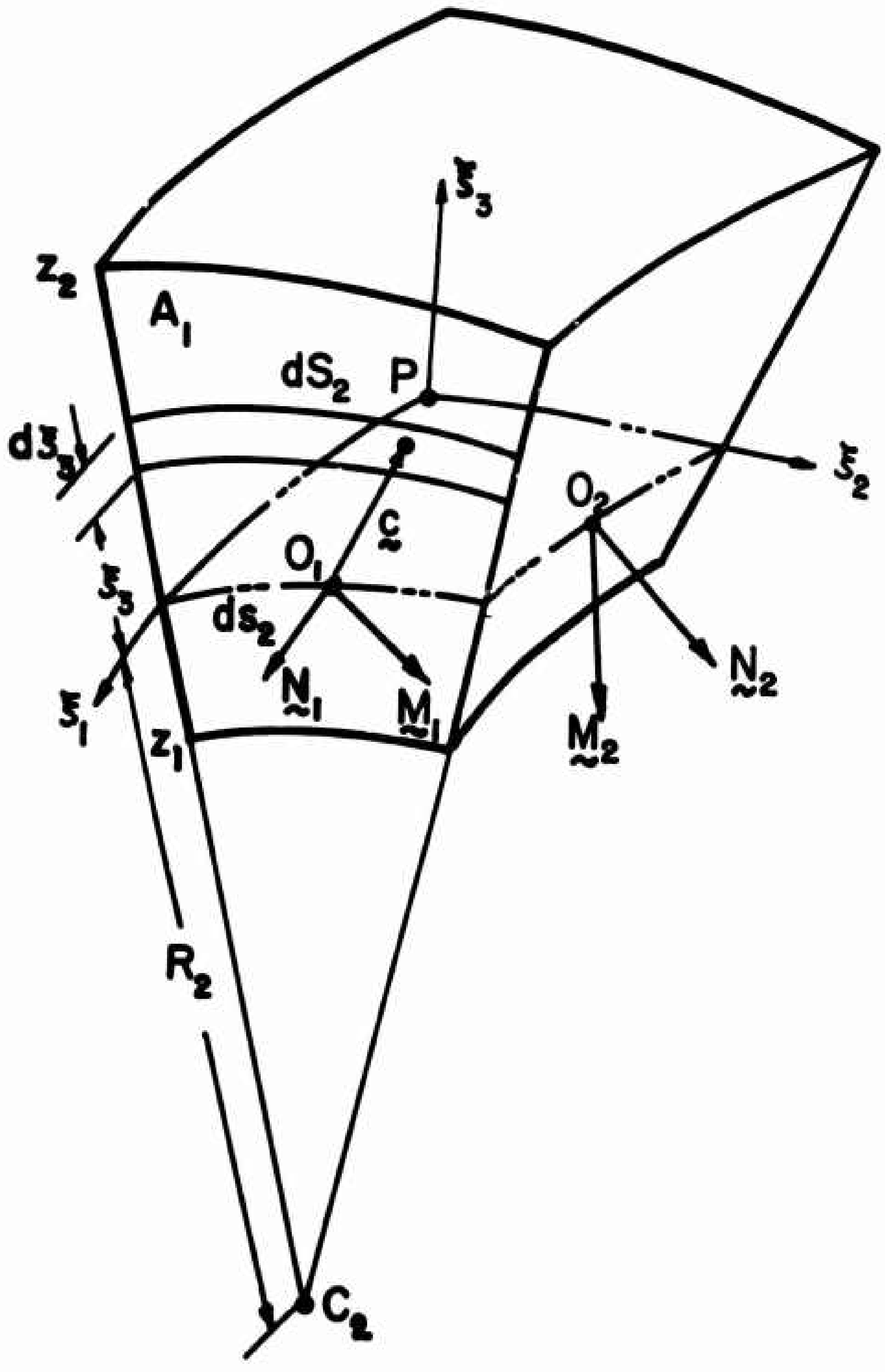


Figure 7. Shell Element.

However, since $2\varepsilon \leq ds_2$ and $ds_2$ is an infinitesimal, the $\underset{\sim}{t}_2$ component of $\underset{\sim}{c}$ can be made as small as desired, and therefore neglected. For this reason, in the following equations the expression

$$\underset{\sim}{c} = \xi_3 \underset{\sim}{t}_3 \tag{3.6b}$$

will be employed.

The stress resultants, $\underset{\sim}{N}_1$ and $\underset{\sim}{M}_1$, can be resolved along the unit vectors in the form

$$\underset{\sim}{N}_1 = N_{11}\underset{\sim}{t}_1 + N_{12}\underset{\sim}{t}_2 + Q_1\underset{\sim}{t}_3 \tag{3.7a}$$

$$\underset{\sim}{M}_1 = -M_{12}\underset{\sim}{t}_1 + M_{11}\underset{\sim}{t}_2 \tag{3.7b}$$

Similarly, the resultant force and moment about $O_2$, produced by the stresses on the face $A_2$, are given by

$$\underset{\sim}{N}_2 = N_{21}\underset{\sim}{t}_1 + N_{22}\underset{\sim}{t}_2 + Q_2\underset{\sim}{t}_3 \tag{3.8a}$$

$$\underset{\sim}{M}_2 = -M_{22}\underset{\sim}{t}_1 + M_{21}\underset{\sim}{t}_2 \tag{3.8b}$$

Substituting the stress vectors on the faces $A_1$ and $A_2$, expressed in terms of their physical components in the form

$$\underset{\sim}{\sigma}_1 = \sigma_{11}\underset{\sim}{t}_1 + \sigma_{12}\underset{\sim}{t}_2 + \sigma_{13}\underset{\sim}{t}_3 \tag{3.9a}$$

$$\underset{\sim}{\sigma}_2 = \sigma_{21}\underset{\sim}{t}_1 + \sigma_{22}\underset{\sim}{t}_2 + \sigma_{23}\underset{\sim}{t}_3 \tag{3.9b}$$

into Eqs. (3.5) and two similar equations with the subscripts 1 and 2 exchanged, and comparing the result to Eqs. (3.7) and (3.8), we find that

$$N_{11} = \int \sigma_{11}(1 + \xi_3/R_2)d\xi_3 \tag{3.10a}$$

$$N_{12} = \int \sigma_{12}(1 + \xi_3/R_2)d\xi_3 \tag{3.10b}$$

$$N_{21} = \int \sigma_{21}(1 + \xi_3/R_1)d\xi_3 \tag{3.10c}$$

$$N_{22} = \int \sigma_{22}(1 + \xi_3/R_1)d\xi_3 \tag{3.10d}$$

$$M_{11} = \int \sigma_{11}\xi_3(1 + \xi_3/R_2)d\xi_3 \tag{3.10e}$$

$$M_{12} = \int \sigma_{12}\xi_3(1 + \xi_3/R_2)d\xi_3 \tag{3.10f}$$

$$M_{21} = \int \sigma_{21}\xi_3(1 + \xi_3/R_1)d\xi_3 \tag{3.10g}$$

$$M_{22} = \int \sigma_{22}\xi_3(1 + \xi_3/R_1)d\xi_3 \tag{3.10h}$$

$$Q_1 = \int \sigma_{13}(1 + \xi_3/R_2)d\xi_3 \tag{3.10i}$$

$$Q_2 = \int \sigma_{23}(1 + \xi_3/R_1)d\xi_3 \tag{3.10j}$$

The quantities $N_{\alpha\beta}$ are called membrane stress resultants, $Q_\alpha$ are transverse shear resultants, and $M_{\alpha\beta}$ are moment resultants. They represent forces and couples per unit length of the reference surface in the directions as specified by Eqs. (3.7) and (3.8) and shown in Figure 8.

The surface loads on the two bounding surfaces of the element of the shell are replaced by a statically equivalent force-couple system attached at the reference surface. Let us denote by $\underset{\sim}{p}_1, \underset{\sim}{m}_1$ and $\underset{\sim}{p}_2, \underset{\sim}{m}_2$ the resultant forces and couples, per unit area of the reference surface, which are prescribed on the bounding surfaces of the element, defined by $\xi_3 = z_1$ and $\xi_3 = z_2$, respectively. Then the statically equivalent

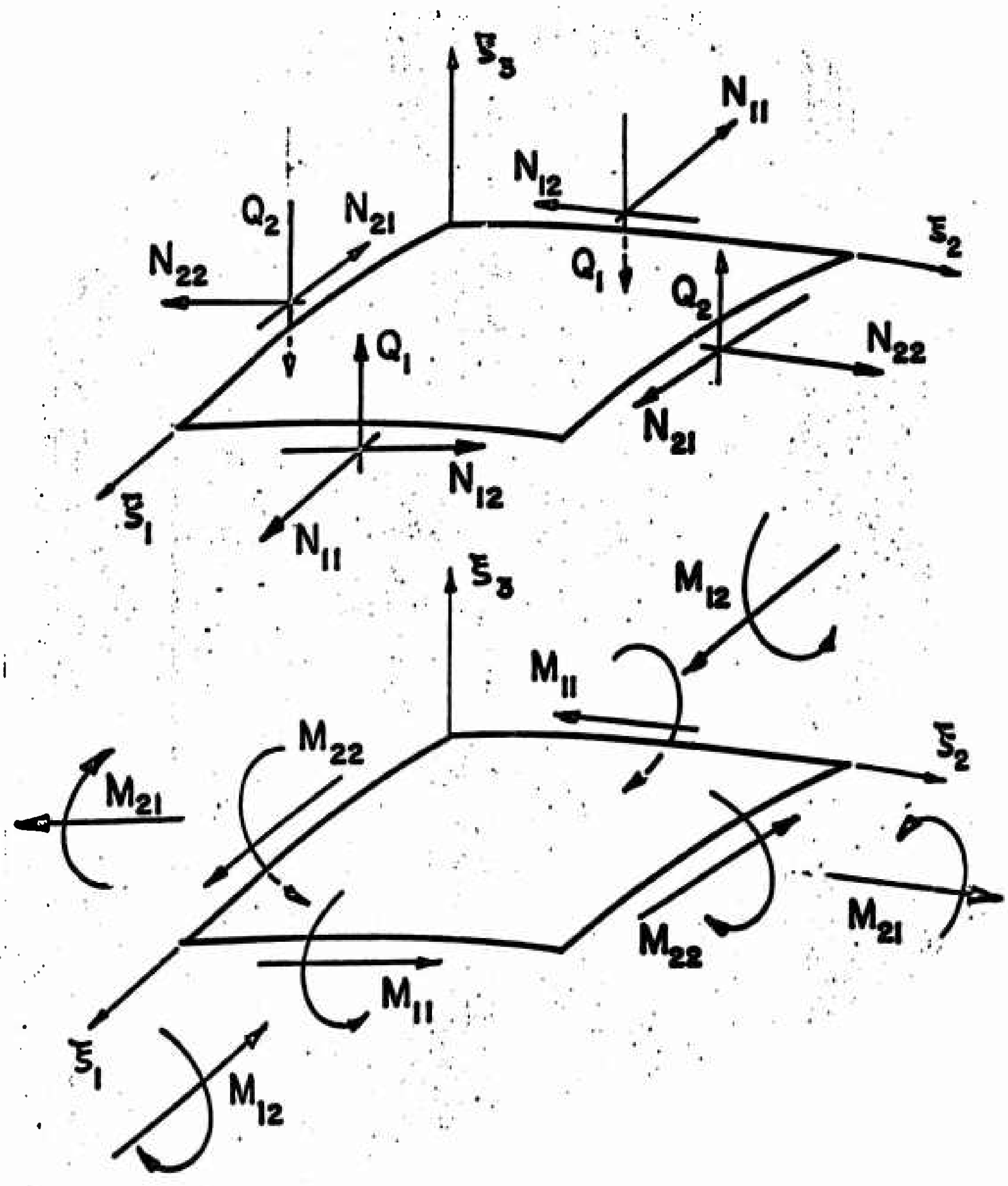


Figure 8. Stress-resultants of a Shell.

force-couple system, attached at the reference surface, consists of a resultant force and couple given by

$$\underset{\sim}{p} = \underset{\sim}{p}_1 + \underset{\sim}{p}_2 \tag{3.11a}$$

$$\underset{\sim}{m} = \underset{\sim}{m}_1 + \underset{\sim}{m}_2 + z_2\underset{\sim}{t}_3 \times \underset{\sim}{p}_2 + z_1\underset{\sim}{t}_3 \times \underset{\sim}{p}_1 \tag{3.11b}$$

which can be resolved along the unit tangent vectors in the form

$$\underset{\sim}{p} = p_1\underset{\sim}{t}_1 + p_2\underset{\sim}{t}_2 + p_3\underset{\sim}{t}_3 \tag{3.12a}$$

$$\underset{\sim}{m} = -m_2\underset{\sim}{t}_1 + m_1\underset{\sim}{t}_2 + m_3\underset{\sim}{t}_3 \tag{3.12b}$$

If acceleration of the element is present, then the inertia forces must also be replaced by a statically equivalent force-couple system attached at the reference surface. Considering a volume element in the form

$$dV = dS_1 dS_2 d\xi_3 = (1 + \xi_3/R_1)(1 + \xi_3/R_2)\alpha_1\alpha_2 d\xi_1 d\xi_2 d\xi_3$$

the resultant force, per unit area of the reference surface, which is caused by the inertia forces, is given by

$$\underset{\sim}{F} = -\int \rho(\ddot{\underset{\sim}{u}} + \xi_3\ddot{\underset{\sim}{\beta}})(1 + \xi_3/R_1)(1 + \xi_3/R_2)d\xi_3 \tag{3.13a}$$

where $\rho$ denotes the mass density of the material. The resultant moment about any point on the reference surface, per unit area of the surface, is given by

$$\underset{\sim}{M} = -\int \xi_3\underset{\sim}{t}_3 \times \rho(\ddot{\underset{\sim}{u}} + \xi_3\ddot{\underset{\sim}{\beta}})(1 + \xi_3/R_1)(1 + \xi_3/R_2)d\xi_3 \tag{3.13b}$$

Eqs. (3.13) can be written as

$$\underset{\sim}{F} = -b_1\ddot{\underset{\sim}{u}} - b_2\ddot{\underset{\sim}{\beta}} \tag{3.14a}$$

$$\underset{\sim}{M} = -b_2\ddot{\underset{\sim}{U}} - b_3\ddot{\underset{\sim}{B}} \tag{3.14b}$$

where

$$\underset{\sim}{U} = \underset{\sim}{t}_3 \times \underset{\sim}{u} \tag{3.15a}$$

$$\underset{\sim}{B} = \underset{\sim}{t}_3 \times \underset{\sim}{\beta} \tag{3.15b}$$

and

$$b_1 = \int \rho(1 + \xi_3/R_1)(1 + \xi_3/R_2)d\xi_3 \tag{3.16a}$$

$$b_2 = \int \rho\xi_3(1 + \xi_3/R_1)(1 + \xi_3/R_2)d\xi_3 \tag{3.16b}$$

$$b_3 = \int \rho\xi_3^2(1 + \xi_3/R_1)(1 + \xi_3/R_2)d\xi_3 \tag{3.16c}$$

Consider now the equilibrium of an element of the shell whose reference surface, together with all the resultant forces and couples acting on this element, is shown in Figure 9. If the element is to remain in equilibrium, then the sum of all resultant forces and moments about one point must be zero.

After division by $d\xi_1 d\xi_2$, the sum of all forces is zero if

$$(\alpha_2\underset{\sim}{N}_1)_{,1} + (\alpha_1\underset{\sim}{N}_2)_{,2} + \alpha_1\alpha_2\underset{\sim}{p} = \alpha_1\alpha_2(b_1\ddot{\underset{\sim}{u}} + b_2\ddot{\underset{\sim}{\beta}}) \tag{3.17}$$

In order to calculate the resultant moment about, say, point P (see Figure 9), the moment of the resultant forces on the four

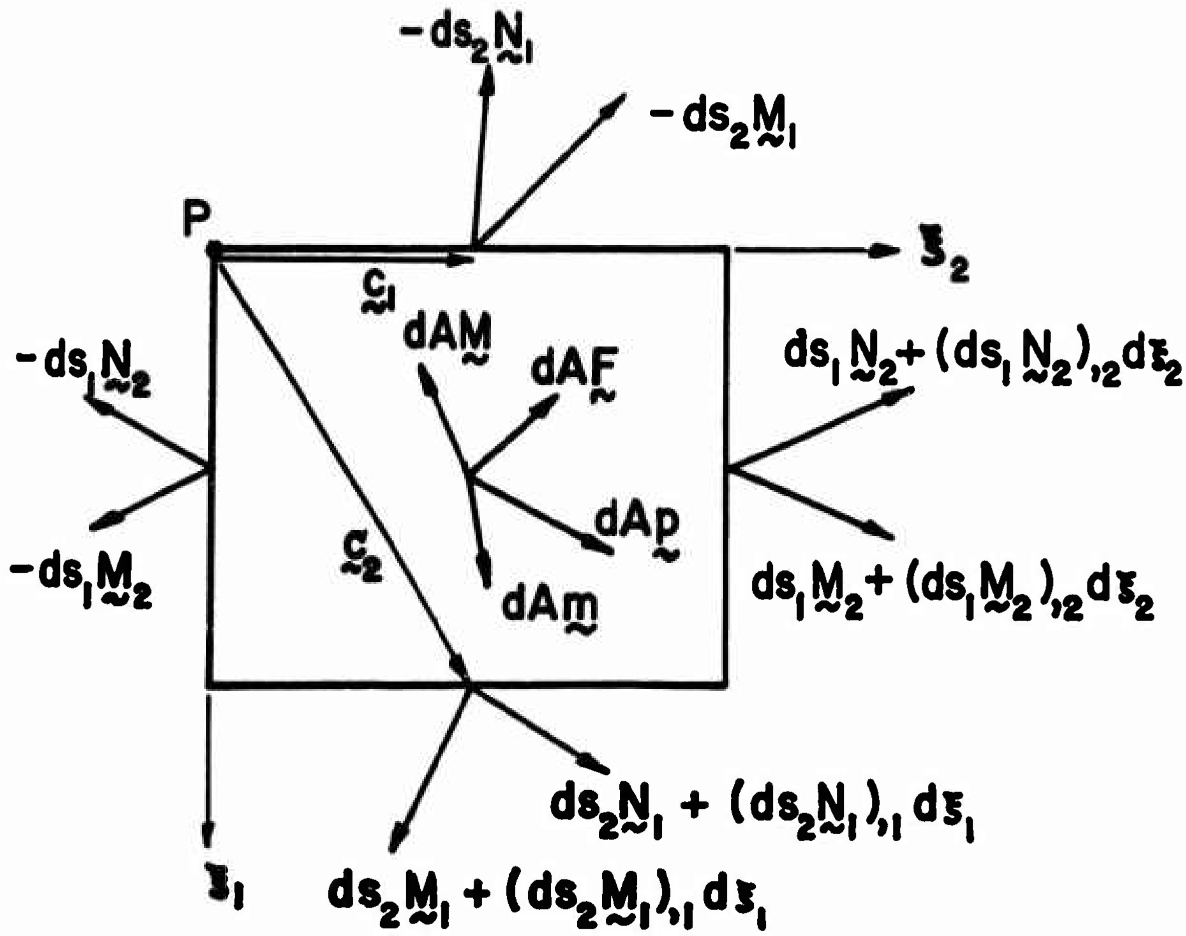


Figure 9. Forces and Couples Acting on an Element of the Shell.

faces about P must also be included. Consider first the two faces defined by $\xi_1$=constant. The moment of the forces on these faces about P is given by

$$-\underset{\sim}{c}_1 \times \alpha_2 d\xi_2 \underset{\sim}{N}_1 + \underset{\sim}{c}_2 \times [\alpha_2 \underset{\sim}{N}_1 + (\alpha_2 \underset{\sim}{N}_1)_{,1} d\xi_1] d\xi_2 \qquad (3.18)$$

where

$$\underset{\sim}{c}_1 = \alpha_2 d\xi_2 \underset{\sim}{t}_2/2$$

$$\underset{\sim}{c}_2 = \alpha_1 d\xi_1 \underset{\sim}{t}_1 + \alpha_2 d\xi_2 \underset{\sim}{t}_2/2$$

After cancelling like terms and neglecting infinitesimals of third order, Eq. (3.18) becomes

$$\alpha_1 \alpha_2 d\xi_1 d\xi_2 \underset{\sim}{t}_1 \times \underset{\sim}{N}_1 \qquad (3.19a)$$

Similarly, the moment about P of the forces on the faces $\xi_2$=constant is given by

$$\alpha_1 \alpha_2 d\xi_1 d\xi_2 \underset{\sim}{t}_2 \times \underset{\sim}{N}_2 \qquad (3.19b)$$

The moment about P of the surface load and inertia forces contains infinitesimals of third order, and need not be considered.

Thus, the resultant moment about P of all the force-couple systems acting on the element is zero if

$$(\alpha_2 \underset{\sim}{M}_1)_{,1} + (\alpha_1 \underset{\sim}{M}_2)_{,2} + \alpha_1 \alpha_2 (\underset{\sim}{t}_1 \times \underset{\sim}{N}_1 + \underset{\sim}{t}_2 \times \underset{\sim}{N}_2)$$

$$+ \alpha_1 \alpha_2 \underset{\sim}{m} = \alpha_1 \alpha_2 (b_2 \ddot{\underset{\sim}{U}} + b_3 \ddot{\underset{\sim}{B}}) \qquad (3.20)$$

where again the common multiplier $d\xi_1 d\xi_2$ has been cancelled and infinitesimals of third order neglected.

Substituting the stress-resultant, surface load, and displacement vectors, resolved along the unit tangent vectors $\underset{\sim}{t}_i$, into Eqs. (3.17) and (3.20), making use of Eqs. (1.19), and then setting each component of Eqs. (3.17) and (3.20) along the unit tangent vectors equal to zero, six equations of equilibrium are obtained which can be written as

$$(\alpha_2 N_{11})_{,1} + (\alpha_1 N_{21})_{,2} + \alpha_{1,2} N_{12} - \alpha_{2,1} N_{22}$$
$$+ \alpha_1\alpha_2 Q_1/R_1 + \alpha_1\alpha_2 p_1 = \alpha_1\alpha_2(b_1\ddot{u}_1 + b_2\ddot{\beta}_1) \qquad (3.21a)$$

$$(\alpha_2 N_{12})_{,1} + (\alpha_1 N_{22})_{,2} + \alpha_{2,1} N_{21} - \alpha_{1,2} N_{11}$$
$$+ \alpha_1\alpha_2 Q_2/R_2 + \alpha_1\alpha_2 p_2 = \alpha_1\alpha_2(b_1\ddot{u}_2 + b_2\ddot{\beta}_2) \qquad (3.21b)$$

$$(\alpha_2 Q_1)_{,1} + (\alpha_1 Q_2)_{,2} - \alpha_1\alpha_2(N_{11}/R_1 + N_{22}/R_2)$$
$$+ \alpha_1\alpha_2 p_3 = \alpha_1\alpha_2 b_1 \ddot{u}_3 \qquad (3.21c)$$

$$(\alpha_2 M_{11})_{,1} + (\alpha_1 M_{21})_{,2} + \alpha_{1,2} M_{12} - \alpha_{2,1} M_{22}$$
$$- \alpha_1\alpha_2 Q_1 + \alpha_1\alpha_2 m_1 = \alpha_1\alpha_2(b_2\ddot{u}_1 + b_3\ddot{\beta}_1) \qquad (3.21d)$$

$$(\alpha_2 M_{12})_{,1} + (\alpha_1 M_{22})_{,2} + \alpha_{2,1} M_{21} - \alpha_{1,2} M_{11}$$
$$- \alpha_1\alpha_2 Q_2 + \alpha_1\alpha_2 m_2 = \alpha_1\alpha_2(b_2\ddot{u}_2 + b_3\ddot{\beta}_2) \qquad (3.21e)$$

$$N_{12} - N_{21} + M_{12}/R_1 - M_{21}/R_2 + m_3 = 0 \qquad (3.22)$$

A remark is in order regarding the distributed moment intensity, $m_3$, which represents the applied couple about the normal, measured per unit area of the reference surface. The order of the differential equations of shell theory is such that $m_3$ cannot be prescribed arbitrarily. The problem is solved without involving Eq. (3.22). After the stress resultants at every point on the reference surface are calculated, then $m_3$ can be found from Eq. (3.22), and that value of $m_3$, wanted or not, represents the applied couple about the normal at every point of the shell.

As will be shown in the following section, $N_{12}$, $N_{21}$, $M_{12}$, $M_{21}$ can be defined in such a way that $m_3$ is automatically zero. If, however, a shell theory is used in which the left-hand side of Eq. (3.22) does not vanish identically, then the effect of the applied $m_3$ must be considered. In the simplest version of the classical theory of shells, it is assumed that $N_{12} = N_{21}$ and $M_{12} = M_{21}$, so that

$$m_3 = M_{12}(1/R_1 - 1/R_2) \tag{3.23}$$

Therefore, any shell which is solved by such a theory will be subjected to an extraneous applied couple about the normal. To be sure, in most cases the effect of such a couple will be small. However, for torsion and for bending of a long shell of revolution, the effect of $m_3$ can become appreciable.

## IV. RELATIONS BETWEEN STRESS-RESULTANTS AND STRAIN

Let us assume that the material of the shell is elastic and possesses at every point three mutually orthogonal planes of elastic symmetry which coincide with the tangent planes of the coordinate surfaces. Such a material is called orthotropic (see, for example, [4], p. 67). In the presence of a temperature increment T, measured from a reference temperature at which the shell is stress-free, the relations between the components of strain and stress in an orthotropic, elastic material, according to generalized Hooke's law ([4], p. 63), are given by

$$e_{11} = \sigma_{11}/E_1 - \nu_{12}\sigma_{22}/E_2 - \nu_{13}\sigma_{33}/E_3 + a_1T \qquad (4.1a)$$

$$e_{22} = -\nu_{21}\sigma_{11}/E_1 + \sigma_{22}/E_2 - \nu_{23}\sigma_{33}/E_3 + a_2T \qquad (4.1b)$$

$$e_{33} = -\nu_{31}\sigma_{11}/E_1 - \nu_{32}\sigma_{22}/E_2 + \sigma_{33}/E_3 + a_3T \qquad (4.1c)$$

$$2e_{12} = \sigma_{12}/G_{12} \qquad (4.2a)$$

$$2e_{23} = \sigma_{23}/G_{23} \qquad (4.2b)$$

$$2e_{13} = \sigma_{13}/G_{13} \qquad (4.2c)$$

where $E_i$ denotes Young's modulus in the $\xi_i$ direction; $\nu_{ij}$ designates the contraction (Poisson's ratio) in the $\xi_i$ direction, caused by a positive normal stress in the $\xi_j$ direction; $G_{ij}$ is the shear modulus in a plane which is tangent to the $(\xi_i, \xi_j)$ coordinate surface; and $a_i$ is the coefficient of thermal expansion ([4], p. 359) in the $\xi_i$ direction prcduced by the temperature increment. Because the strain-energy density function ([4], p. 60) is quadratic in the components of strain, the elastic parameter matrix which relates the components of stress and strain in Eqs. (4.1) is symmetric, and therefore we have that

$$\nu_{12}/E_2 = \nu_{21}/E_1 \tag{4.3a}$$

$$\nu_{13}/E_3 = \nu_{31}/E_1 \tag{4.3b}$$

$$\nu_{23}/E_3 = \nu_{32}/E_2 \tag{4.3c}$$

Owing to the restriction on the kinematics of the shell introduced by Eq. (2.16), which means that the points on a normal cannot change their relative positions, the stress-strain law for the shell must be such that the stress components, regardless how large, cannot produce the transverse normal strain, $e_{33}$. Since this must be true for any state of stress, it follows from Eq. (4.1c) that

$$\nu_{31} = \nu_{32} = 1/E_3 = a_3 = 0 \tag{4.4}$$

It should be emphasized that the "anisotropy" introduced by Eqs. (4.4) has no connection with the material of the shell, but is the result of the assumption made by Eq. (2.16). This means that regardless of what material the shell is made, Poisson's ratios, Young's modulus, and the coefficient of thermal expansion along the normal cannot be prescribed, but are automatically assigned the values given by Eqs. (4.4).

After using Eqs. (4.4), Eqs. (4.1) can be solved for the normal stress components and written as

$$\sigma_{11} = B_{11}e_{11} + B_{12}e_{22} + A_1 T \tag{4.5a}$$

$$\sigma_{22} = B_{12}e_{11} + B_{22}e_{22} + A_2 T \tag{4.5b}$$

where

$$B_{11} = E_1/(1 - \nu_{12}\nu_{21}) \tag{4.6a}$$

$$B_{12} = \nu_{12}E_1/(1 - \nu_{12}\nu_{21}) = \nu_{21}E_2/(1 - \nu_{12}\nu_{21}) \tag{4.6b}$$

$$B_{22} = E_2/(1 - \nu_{12}\nu_{21}) \tag{4.6c}$$

$$A_1 = a_1 + \nu_{12}a_2 \tag{4.6d}$$

$$A_2 = a_2 + \nu_{21}a_1 \tag{4.6e}$$

The transverse normal stress, $\sigma_{33}$, cannot be determined from the stress-strain law, and, although it can be calculated from the stress equilibrium equation in the $\xi_3$ direction, it must be admitted that after the assumption of Eq. (2.16), such a calculation is not meaningful.

The relations between the stress-resultants and the strain can now be derived by substituting the components of strain defined by Eqs. (2.19) into Eqs. (4.5) and (4.2), and the components of stress into the definitions of stress-resultants given by Eqs. (3.10). This procedure yields

$$N_{11} = C_{11}\varepsilon_{11} + C_{12}\varepsilon_{22} + E_{11}k_{11} + E_{12}k_{22} + H_1 \tag{4.7a}$$

$$N_{22} = C_{12}\varepsilon_{11} + C_{22}\varepsilon_{22} + E_{12}k_{11} + E_{22}k_{22} + H_2 \tag{4.7b}$$

$$M_{11} = E_{11}\varepsilon_{11} + E_{12}\varepsilon_{22} + D_{11}k_{11} + D_{12}k_{22} + H_3 \tag{4.7c}$$

$$M_{22} = E_{12}\varepsilon_{11} + E_{22}\varepsilon_{22} + D_{12}k_{11} + D_{22}k_{22} + H_4 \tag{4.7d}$$

$$N_{12} = F_{11}\gamma_1 + F_{12}\gamma_2 + J_{11}\delta_1 + J_{12}\delta_2 \tag{4.8a}$$

$$N_{21} = F_{12}\gamma_1 + F_{22}\gamma_2 + J_{12}\delta_1 + J_{22}\delta_2 \tag{4.8b}$$

$$M_{12} = J_{11}\gamma_1 + J_{12}\gamma_2 + K_{11}\delta_1 + K_{12}\delta_2 \tag{4.8c}$$

$$M_{21} = J_{12}\gamma_1 + J_{22}\gamma_2 + K_{12}\delta_1 + K_{22}\delta_2 \tag{4.8d}$$

$$Q_1 = L_1\gamma_{13} \tag{4.9a}$$

$$Q_2 = L_2\gamma_{23} \tag{4.9b}$$

where

$$C_{11} = \int B_{11}S_{21}d\xi_3$$

$$C_{12} = \int B_{12}d\xi_3$$

$$C_{22} = \int B_{22}S_{12}d\xi_3$$

$$
\begin{aligned}
E_{11} &= \int B_{11}S_{21}\xi_3 d\xi_3 \\
E_{12} &= \int B_{12}\xi_3 d\xi_3 \\
E_{22} &= \int B_{22}S_{12}\xi_3 d\xi_3 \\
D_{11} &= \int B_{11}S_{21}\xi_3^2 d\xi_3 \\
D_{12} &= \int B_{12}\xi_3^2 d\xi_3 \\
D_{22} &= \int B_{22}S_{12}\xi_3^2 d\xi_3 \\
F_{11} &= \int G_{12}S_{21} d\xi_3 \\
F_{12} &= \int G_{12} d\xi_3 \qquad\qquad (4.10) \\
F_{22} &= \int G_{12}S_{12} d\xi_3 \\
J_{11} &= \int G_{12}S_{21}\xi_3 d\xi_3 \\
J_{12} &= \int G_{12}\xi_3 d\xi_3 \\
J_{22} &= \int G_{12}S_{12}\xi_3 d\xi_3 \\
K_{11} &= \int G_{12}S_{21}\xi_3^2 d\xi_3 \\
K_{12} &= \int G_{12}\xi_3^2 d\xi_3 \\
K_{22} &= \int G_{12}S_{12}\xi_3^2 d\xi_3 \\
L_1 &= \int G_{13}S_{21} d\xi_3 \\
L_2 &= \int G_{23}S_{12} d\xi_3
\end{aligned}
$$

where the notation

$$
\begin{aligned}
S_{12} &= (1 + \xi_3/R_1)/(1 + \xi_3/R_2) \\
S_{21} &= (1 + \xi_3/R_2)/(1 + \xi_3/R_1)
\end{aligned}
\tag{4.11}
$$

has been employed. These quantities can be regarded as the nonzero elements of a (10,10) elastic parameter matrix for a shell which connects the ten stress resultants defined by Eqs. (3.10) with the ten strain measures defined by Eqs. (2.20). The effect of the temperature increment is represented by

$$
\begin{aligned}
H_1 &= \int A_1 T(1 + \xi_3/R_2)d\xi_3 \\
H_2 &= \int A_2 T(1 + \xi_3/R_1)d\xi_3 \\
H_3 &= \int A_1 T(1 + \xi_3/R_2)\xi_3 d\xi_3 \\
H_4 &= \int A_2 T(1 + \xi_3/R_1)\xi_3 d\xi_3
\end{aligned}
\tag{4.12}
$$

Eqs. (4.7), (4.8), and (4.9) complete the system of equations which represents the mathematical model used for a shell. Together with Eqs. (2.20) and (3.21), there are 25 equations containing 25 unknowns, which consist of 10 stress-resultants, 10 strains, and 5 displacement quantities. Only three assumptions have been used in the derivation of these equations: (1) linear displacement field, as given by Eq. (2.14); (2) normals remain the same length, as specified by Eq. (2.16); and (3) replacement of the stress field by stress resultants.

It is of interest to examine the stress-resultants when the shell undergoes rigid-body motion. Substituting in Eqs.(4.7)-(4.9) the expressions for the nonzero strain measures given by Eqs. (2.23), we conclude that all stress-resultants are zero for rigid-body motion if

$$
\begin{aligned}
F_{11} - F_{12} + J_{11}/R_1 - J_{12}/R_2 &= 0 \\
F_{12} - F_{22} + J_{12}/R_1 - J_{22}/R_2 &= 0 \\
J_{11} - J_{12} + K_{11}/R_1 - K_{12}/R_2 &= 0 \\
J_{12} - J_{22} + K_{12}/R_1 - K_{22}/R_2 &= 0
\end{aligned}
\qquad (4.13)
$$

From the definition of the elastic parameters given by Eqs. (4.10), it is easily seen that Eqs. (4.13) are identically satisfied. Moreover, substitution of Eqs. (4.8) into Eq. (3.22) reveals that if Eqs. (4.13) hold, then also the moment equilibrium equation about the normal, as given by Eq. (3.22) with $m_3=0$, is identically satisfied.

In the case when the elastic parameters $B_{\alpha\beta}$ and $G_{\alpha i}$ do not vary with respect to $\xi_3$, then the integrals which appear in Eqs. (4.10) can be evaluated exactly and expressed in terms of the following relations

$$\int S_{12} d\xi_3 = Z_1 R_2/R_1 + R_2(1 - R_2/R_1)Z \qquad (4.14a)$$

$$
\begin{aligned}
\int S_{12}\xi_3 d\xi_3 = {} & Z_1 R_2(1 - R_2/R_1) + Z_2 R_2/R_1 \\
& - R_2^2(1 - R_2/R_1)Z
\end{aligned}
\qquad (4.14b)
$$

$$\int S_{12}\xi_3^2 d\xi_3 = Z_1 R_2^2(1 - R_2/R_1) + Z_2 R_2(1 - R_2/R_1)$$
$$+ Z_3 R_2/R_1 + R_2^3(1 - R_2/R_1)Z \qquad (4.14c)$$

where

$$Z = \log[(1 + z_2/R_2)/(1 + z_1/R_2)]$$

and

$$Z_n = (z_2^n - z_1^n)/n$$

for $n = 1,2,\ldots$ . The corresponding integrals of $S_{21}$ can be obtained from Eqs. (4.14) by exchanging the subscripts 1 and 2 for the radii of curvature.

Eqs. (4.14), although formally exact, are not suitable for the calculation of the integrals, because the leading terms of the power series expansion of the logarithmic term, Z, subtract out the larger of the other terms. Upon using the expansion

$$Z = \sum_{n=1}^{\infty} (-1)^{n+1} Z_n/R_2^n \qquad (4.15)$$

in Eqs. (4.14) and cancelling like terms, the integrals can be written as

$$\int S_{12} d\xi_3 = Z_1 + (1 - R_2/R_1) \sum_{n=2}^{\infty} (-1/R_2)^{n-1} Z_n \qquad (4.16a)$$
$$\int S_{12}\xi_3 d\xi_3 = Z_2 + (1 - R_2/R_1) \sum_{n=3}^{\infty} (-1/R_2)^{n-2} Z_n \qquad (4.16b)$$
$$\int S_{12}\xi_3^2 d\xi_3 = Z_3 + (1 - R_2/R_1) \sum_{n=4}^{\infty} (-1/R_2)^{n-3} Z_n \qquad (4.16c)$$

Before the actual calculation of the integrals by means of Eqs. (4.16), the decision must be made on the number of terms which are to be retained in the infinite series. It should be borne in mind, however, that the three assumptions which have already been used in the derivation of the theory will introduce definite errors in the solution regardless how many terms are used in the calculation of the integrals of $S_{12}$ and $S_{21}$. On the other hand, if too few terms in Eqs. (4.16) are retained, then the errors introduced in the solution through Eqs. (4.16) will exceed the errors of the other three assumptions. Thus, there must be an optimum number of terms which should be retained in the infinite series, so that the truncation errors match those of the other three assumptions. Unfortunately, no reliable estimate of such an optimum number of terms for arbitrary shells with arbitrary loads is available, and one is forced to make the decision on the truncation of the series without regard to the other three assumptions. When the series in Eqs. (4.16) are truncated, it should be done in such a way that Eqs. (4.13) are satisfied. This can be shown to be the case when the last terms retained in Eqs. (4.16) have the same value for the index n.

# V. INTEGRAL IDENTITIES IN SHELL THEORY

Solutions of the equations of shell theory can be shown to satisfy a certain identity from which important relations can be obtained. In the theory of partial differential equations, this identity is called Green's Formula, and it applies to systems of differential operators, such as those derived for shells in the preceding sections.

Consider a shell with a reference surface S, boundary* B, and a coordinate system $\xi_\lambda$. Consider also two sets of shell-variables distinguished by a prime or the absence of a prime. Let the unprimed variables be the stress-resultants $\underset{\sim}{N}_\alpha, \underset{\sim}{M}_\alpha$, surface loads $\underset{\sim}{p}, \underset{\sim}{m}$, and acceleration vectors $\ddot{\underset{\sim}{u}}, \ddot{\underset{\sim}{\beta}}, \ddot{\underset{\sim}{U}}, \ddot{\underset{\sim}{B}}$, which satisfy the equilibrium Eqs. (3.17) and (3.20). The primed variables consist of a displacement field, denoted by $\underset{\sim}{u}'$ and $\underset{\sim}{B}'$, whose derivatives, in analogy to Eqs. (2.18), can be written as

$$
\begin{aligned}
\underset{\sim}{u}'_{,1} &= \alpha_1\epsilon'_{11}\underset{\sim}{t}_1 + \alpha_1\gamma'_1\underset{\sim}{t}_2 + \alpha_1(\gamma'_{13} - \beta'_1)\underset{\sim}{t}_3 \\
\underset{\sim}{u}'_{,2} &= \alpha_2\gamma'_2\underset{\sim}{t}_1 + \alpha_2\epsilon'_{22}\underset{\sim}{t}_2 + \alpha_2(\gamma'_{23} - \beta'_2)\underset{\sim}{t}_3 \\
\underset{\sim}{B}'_{,1} &= -\alpha_1\delta'_1\underset{\sim}{t}_1 + \alpha_1 k'_{11}\underset{\sim}{t}_2 + \alpha_1\beta'_2\underset{\sim}{t}_3/R_1 \\
\underset{\sim}{B}'_{,2} &= -\alpha_2 k'_{22}\underset{\sim}{t}_1 + \alpha_2\delta'_2\underset{\sim}{t}_2 + \alpha_2\beta'_1\underset{\sim}{t}_3/R_2
\end{aligned}
\qquad (5.1)
$$

*For simplicity of this discussion, it is assumed that the boundary of S is defined by coordinate curves $\xi_\lambda$=constant. All conclusions reached in this section apply also to an arbitrary boundary.

It should be emphasized that for the derivation of Green's Formula neither the primed nor the unprimed variables are required to be those of a solution state which would satisfy all equations of shell theory. It is merely required that the unprimed variables be such that they satisfy the equations of equilibrium. The primed displacement field need not be related in any way to the unprimed variables.

Green's Formula for the shell-equations is derived by forming the scalar product of the equilibrium Eqs. (3.17) and (3.20) with $\underset{\sim}{u}'$ and $\underset{\sim}{B}'$, respectively, integrating with respect to $d\xi_1 d\xi_2$ over the reference surface of the shell, using integration by parts of the form

$$\iint_S (\alpha_2 \underset{\sim}{N}_1)_{,1} \cdot \underset{\sim}{u}' d\xi_1 d\xi_2 = \int \alpha_2 \underset{\sim}{N}_1 \cdot \underset{\sim}{u}' d\xi_2 - \iint_S \alpha_2 \underset{\sim}{N}_1 \cdot \underset{\sim}{u}'_{,1} d\xi_1 d\xi_2 \tag{5.2}$$

and, finally, making use of Eqs. (3.7), (3.8), and (5.1). The final result is the identity

$$\begin{aligned} &\iint_S (\underset{\sim}{P} \cdot \underset{\sim}{u}' + \underset{\sim}{M} \cdot \underset{\sim}{B}')ds \\ &= \iint_S (N_{11}\epsilon'_{11} + N_{12}\gamma'_1 + N_{21}\gamma'_2 + N_{22}\epsilon'_{22} + M_{11}k'_{11} \\ &\qquad + M_{12}\delta'_1 + M_{21}\delta'_2 + M_{22}k'_{22} + Q_1\gamma'_{13} + Q_2\gamma'_{23})dS \\ &- \int_{B_1} (\underset{\sim}{N}_1 \cdot \underset{\sim}{u}' + \underset{\sim}{M}_1 \cdot \underset{\sim}{B}')\alpha_2 d\xi_2 \\ &- \int_{B_2} (\underset{\sim}{N}_2 \cdot \underset{\sim}{u}' + \underset{\sim}{M}_2 \cdot \underset{\sim}{B}')\alpha_1 d\xi_1 \end{aligned} \tag{5.3}$$

where

$$\underset{\sim}{P} = \underset{\sim}{p} - b_1\ddot{\underset{\sim}{u}} - b_2\ddot{\underset{\sim}{\beta}}$$

$$\underset{\sim}{M} = \underset{\sim}{m} - b_2\ddot{\underset{\sim}{U}} - b_3\ddot{\underset{\sim}{B}}$$

$$dS = \alpha_1\alpha_2 d\xi_1 d\xi_2$$

The symbol $B_1$ means that the integration is performed on the boundary $\xi_1$=constant, and $B_2$ has a similar meaning.

For static problems, Eq. (5.3) can be regarded as a proof that the equilibrium Eqs. (3.17) and (3.20) can be derived from the Principle of Minimum Potential Energy. In order to see this, consider a shell which is subjected to surface loads $\underset{\sim}{p}$,$\underset{\sim}{m}$, and edge loads $\underset{\sim}{N}^*_\lambda$,$\underset{\sim}{M}^*_\lambda$ on a boundary $\xi_\lambda$=constant. If we define the strain-energy density function by

$$2W = N_{11}\epsilon_{11} + N_{12}\gamma_1 + N_{21}\gamma_2 + N_{22}\epsilon_{22} + M_{11}k_{11}$$
$$+ M_{12}\delta_1 + M_{21}\delta_2 + M_{22}k_{22} + Q_1\gamma_{13} + Q_2\gamma_{23} \qquad (5.4)$$

where the stress-resultants are expressed in terms of strains by means of Eqs. (4.7)-(4.9), and regard the primed variables as variations produced by a geometrically admissible variation of the displacement field, then Eq. (5.3) expresses the fact that the variation of the potential energy, defined by

$$V = \iint_S W dS - \iint_S(\underset{\sim}{p} \cdot \underset{\sim}{u} + \underset{\sim}{m} \cdot \underset{\sim}{B})dS$$
$$- \int_{B_1}(\underset{\sim}{N}^*_1 \cdot \underset{\sim}{u} + \underset{\sim}{M}^*_1 \cdot \underset{\sim}{B})\alpha_2 d\xi_2 - \int_{B_2}(\underset{\sim}{N}^*_2 \cdot \underset{\sim}{u} + M^*_2 \cdot B)\alpha_1 d\xi_1 \quad (5.5)$$

is zero. Taking the variation of Eq. (5.5), it can be shown by retracing the steps backwards from Eq. (5.3) that the Euler equations of the variational problem are Eqs. (3.17) and (3.20), and that the boundary conditions on an edge $\xi_\lambda$ = constant must be such that they satisfy

$$\int_{B_\lambda} [(\underset{\sim}{N}^*_\lambda - \underset{\sim}{N}_\lambda) \cdot \underset{\sim}{u}' + (\underset{\sim}{M}^*_\lambda - \underset{\sim}{M}_\lambda) \cdot \underset{\sim}{B}']ds = 0 \qquad (5.6)$$

Equation (5.3) has other interpretations. For example, consider that the primed and unprimed variables in Eq. (5.3) represent two complete solution states which satisfy the boundary conditions given by Eq. (5.6) and are produced by primed and unprimed surface and edge loads, respectively. Then it can be shown, with the use of Eqs. (4.7)-(4.9), that the first integral on the right-hand side is symmetric in the primed and unprimed variables. Consequently, an equation similar to Eq. (5.3) can be written with the primes and unprimes exchanged, and, upon subtraction, the following Reciprocal Relation obtained

$$\int_S\int (\underset{\sim}{P} \cdot \underset{\sim}{u}' - \underset{\sim}{P}' \cdot \underset{\sim}{u} + \underset{\sim}{M} \cdot \underset{\sim}{B}' - \underset{\sim}{M}' \cdot \underset{\sim}{B})ds$$

$$+ \sum_{\lambda=1}^{2} \int_{B_\lambda} (\underset{\sim}{N}_\lambda \cdot \underset{\sim}{u}' - \underset{\sim}{N}'_\lambda \cdot \underset{\sim}{u} + \underset{\sim}{M}_\lambda \cdot \underset{\sim}{B}' - \underset{\sim}{M}'_\lambda \cdot \underset{\sim}{B})ds = 0 \qquad (5.7)$$

Moreover, for free-vibration problems of shells, Eq. (5.3) can be used to obtain the Orthogonality Relation for the modes associated with two different natural frequencies. Let us assume that the primed and unprimed variables represent two different solutions of the form

$$U(\xi_1,\xi_2,t) = U_i(\xi_1,\xi_2)\ e^{i\omega_i t}$$

which satisfy all homogeneous shell-equations and boundary conditions. Then the Orthogonality Relation follows directly from Eq.(5.7) in the form

$$(\omega_i^2 - \omega_j^2)\iint_S [b_1 \underset{\sim}{u}_i \cdot \underset{\sim}{u}_j + b_2(\underset{\sim}{\beta}_i \cdot \underset{\sim}{u}_j + \underset{\sim}{\beta}_j \cdot \underset{\sim}{u}_i) + b_3 \underset{\sim}{\beta}_i \cdot \underset{\sim}{\beta}_j]dS = 0 \tag{5.8}$$

where the solution with a subscript i is associated with the natural frequency $\omega_i$, and the solution with a subscript j goes with $\omega_j$. In deriving Eq. (5.8), the identities

$$\underset{\sim}{U} \cdot \underset{\sim}{B} = \underset{\sim}{u} \cdot \underset{\sim}{\beta}$$

$$\underset{\sim}{B} \cdot \underset{\sim}{B} = \underset{\sim}{\beta} \cdot \underset{\sim}{\beta}$$

have been employed which follow from the vector identity

$$(\underset{\sim}{a} \times \underset{\sim}{b}) \cdot (\underset{\sim}{c} \times \underset{\sim}{d}) = (\underset{\sim}{a} \cdot \underset{\sim}{c})(\underset{\sim}{b} \cdot \underset{\sim}{d}) - (\underset{\sim}{a} \cdot \underset{\sim}{d})(\underset{\sim}{b} \cdot \underset{\sim}{c})$$

and Eqs. (3.15).

## VI. BOUNDARY CONDITIONS

On physical grounds, the boundary conditions on an edge of a shell, say $\xi_1$=constant, must be such that either the components of $\underset{\sim}{N}_1$ and $\underset{\sim}{M}_1$ along a unit tangent vector or the components of $\underset{\sim}{u}$ and $\underset{\sim}{B}$ along the same tangent vector are prescribed. It would simply be unreasonable to prescribe both, say $N_{11}$ and $u_1$, because it takes a certain force $N_{11}$, which must be given by the solution, to produce a specified $u_1$.

Thus, the boundary conditions on an edge $\xi_1$=constant can be stated as follows:

1. Either $N_{11}$ or $u_1$ prescribed
2. Either $N_{12}$ or $u_2$ prescribed
3. Either $M_{11}$ or $\beta_1$ prescribed
4. Either $M_{12}$ or $\beta_2$ prescribed
5. Either $Q_1$ or $u_3$ prescribed

Such boundary conditions are applicable to any other edge if we regard that the $\xi_1$ coordinate direction coincides with the normal and the $\xi_2$ direction with the tangent of the boundary of the shell.

These boundary conditions coincide with those which could have been derived from the Principle of Minimum Potential Energy and are given by Eq. (5.6). Since the primed variables represent a geometrically admissible variation of the displacement field, they must vanish wherever displacements are prescribed. Wherever the displacements are not described, it follows from Eq. (5.6) that the components of the stress resultants must be specified.

Because they are derivable from a variational problem, such boundary conditions as given above can be called the natural boundary conditions. Together with the governing equations of the theory of shells, Eqs. (2.20), (3.21), and (4.7)-(4.9), they constitute a properly posed boundary value problem.

## VII. LAYERED SHELLS

In addition to the geometric parameters, $\alpha_1, \alpha_2, R_1, R_2$, which are given with the reference surface, the governing equations also contain the material parameters which are defined by Eqs. (3.16), (4.10), and (4.12). The calculation of the material parameters can be simplified if it is assumed that the material properties of the shell, represented by $\rho$, $B_{\alpha\beta}$, $G_{\alpha i}$, and $A_\alpha$, are piecewise constant with respect to $\xi_3$ (i.e., constant in finite subintervals of the interval from $\xi_3 = z_1$ to $\xi_3 = z_2$). This assumption amounts to the introduction of layers in the shell-wall within which the properties of the material do not change in the $\xi_3$ direction.

Let us assume that the shell-wall consists of m layers (see Figure 10) whose bounding surfaces are defined by $\xi_3 = z_1, z_2, \ldots, z_m, z_{m+1}$, where each $z_i$ (with $i = 1,2,\ldots,m+1$) can be a function of $\xi_1$ and $\xi_2$. It is assumed that the layers are perfectly joined at the bounding surfaces, so that the displacement vector is continuous within the shell-wall. It is also assumed that for layers of variable thickness, the angle (in radians) between the normal of a bounding surface of a layer and the normal at the corresponding point on the reference surface is much smaller than unity.

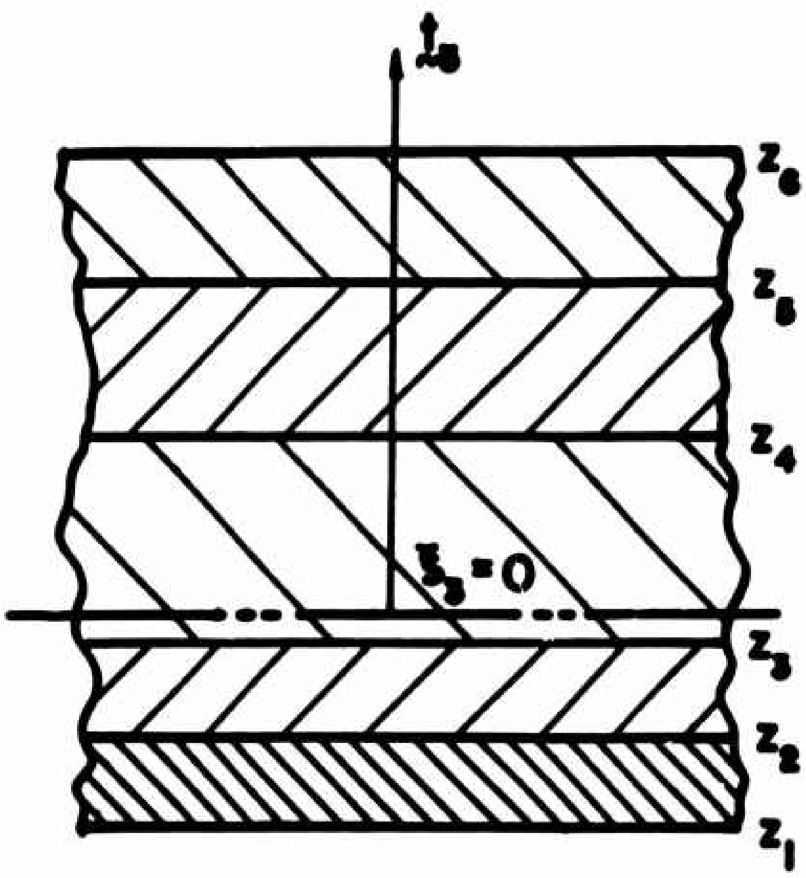


Figure 10. Layered Shell Element.

In the calculation of the material parameters from Eqs. (3.16), (4.10), and (4.12) for such a layered shell, the integrals on the right-hand side of these equations can be replaced by a sum of integrals which can be evaluated exactly over every layer. For example, when the density of the layers does not vary with $\zeta_3$ within each layer, then the inertia parameters, defined by Eqs. (3.16), are given by

$$b_1 = \sum_{i=1}^{m} \rho^i [Z_1^i + (1/R_1 + 1/R_2) Z_2^i + Z_3^i/R_1R_2] \tag{7.1a}$$

$$b_2 = \sum_{i=1}^{m} \rho^i [Z_2^i + (1/R_1 + 1/R_2) Z_3^i + Z_4^i/R_1R_2] \tag{7.1b}$$

$$b_3 = \sum_{i=1}^{m} \rho^i [Z_3^i + (1/R_1 + 1/R_2) Z_4^i + Z_5^i/R_1R_2] \tag{7.1c}$$

where $\rho^i$ denotes the density of the i-th layer, and

$$Z_n^i = (z_{i+1}^n - z_i^n)/n \tag{7.2}$$

for $n = 1,2,\ldots$ .

For the calculation of the parameters defined by Eqs. (4.10), the exact integrals are given by Eqs. (4.16). For example, for the first parameter of Eqs. (4.10), we have

$$\begin{aligned} C_{11} &= \sum_{i=1}^{m} B_{11}^i \int_{z_i}^{z_{i+1}} S_{21} d\zeta_3 \\ &= \sum_{i=1}^{m} B_{11}^i Z_1^i + (1 - R_1/R_2) \sum_{n=2}^{\infty} (-1/R_1)^{n-1} \sum_{i=1}^{m} B_{11}^i Z_n^i \end{aligned} \tag{7.3}$$

where the superscript i denotes the properties of the i-th layer. The other parameters are evaluated by following the same procedure. With the notation

$$B_{\alpha\beta n} = \sum_{i=1}^{m} B^{i}_{\alpha\beta} Z^{i}_{n} \tag{7.4a}$$

$$G_{\alpha j n} = \sum_{i=1}^{m} G^{i}_{\alpha j} Z^{i}_{n} \tag{7.4b}$$

the parameters defined by Eqs. (4.10) can be written for a layered shell in the form

$$
\begin{aligned}
C_{11} &= B_{111} + (1 - R_1/R_2) \sum_{n=2}^{\infty} (-1/R_1)^{n-1} B_{11n} \\
C_{12} &= B_{121} \\
C_{22} &= B_{221} + (1 - R_2/R_1) \sum_{n=2}^{\infty} (-1/R_2)^{n-1} B_{22n} \\
E_{11} &= B_{112} + (1 - R_1/R_2) \sum_{n=3}^{\infty} (-1/R_1)^{n-2} B_{11n} \\
E_{12} &= B_{122} \\
E_{22} &= B_{222} + (1 - R_2/R_1) \sum_{n=3}^{\infty} (-1/R_2)^{n-2} B_{22n} \\
D_{11} &= B_{113} + (1 - R_1/R_2) \sum_{n=4}^{\infty} (-1/R_1)^{n-3} B_{11n} \\
D_{12} &= B_{123} \\
D_{22} &= B_{223} + (1 - R_2/R_1) \sum_{n=4}^{\infty} (-1/R_2)^{n-3} B_{22n} \\
F_{11} &= G_{121} + (1 - R_1/R_2) \sum_{n=2}^{\infty} (-1/R_1)^{n-1} G_{12n} \\
F_{12} &= G_{121} \qquad (7.5) \\
F_{22} &= G_{121} + (1 - R_2/R_1) \sum_{n=2}^{\infty} (-1/R_2)^{n-1} G_{12n}
\end{aligned}
$$

$$J_{11} = G_{122} + (1 - R_1/R_2)\sum_{n=3}^{\infty}(-1/R_1)^{n-2}G_{12n}$$

$$J_{12} = G_{122}$$

$$J_{22} = G_{122} + (1 - R_2/R_1)\sum_{n=3}^{\infty}(-1/R_2)^{n-2}G_{12n}$$

$$K_{11} = G_{123} + (1 - R_1/R_2)\sum_{n=4}^{\infty}(-1/R_1)^{n-3}G_{12n}$$

$$K_{12} = G_{123}$$

$$K_{22} = G_{123} + (1 - R_2/R_1)\sum_{n=4}^{\infty}(-1/R_2)^{n-3}G_{12n}$$

$$L_1 = G_{131} + (1 - R_1/R_2)\sum_{n=2}^{\infty}(-1/R_1)^{n-1}G_{13n}$$

$$L_2 = G_{231} + (1 - R_2/R_1)\sum_{n=2}^{\infty}(-1/R_2)^{n-1}G_{23n}$$

In order to simplify the calculation of the thermal parameters given by Eqs. (4.12), some assumption must be made on the distribution of the temperature within the shell in the $\xi_3$ direction. A simple form of the temperature field that can be assumed is one which is linear in $\xi_3$ and in the i-th layer can be written as

$$T^i(\xi_1,\xi_2,\xi_3) = T_A^i(\xi_1,\xi_2) + \xi_3 T_M^i(\xi_1,\xi_2) \qquad (7.6)$$

Eq. (7.6) implies that the flux of heat in the $\xi_3$ direction, given by

$$f_3^i = - K_i \partial T^i/\partial\xi_3 = - K_i T_M^i \qquad (7.7)$$

is constant with respect to $\xi_3$ within a layer. In Eq. (7.7), $K_i$ denotes the thermal conductivity of the i-th layer in the $\xi_3$ direction.

If it is assumed that no heat sources exist within the shell-wall and that the thermal boundary conditions are applied only to the bounding surfaces of the shell, we must require that the temperature and the flux of heat along the normal of the reference surface are continuous on the bounding surfaces of each layer within the shell. These continuity conditions are given by

$$T_A^{i+1} + z_{i+1}T_M^{i+1} = T_A^i + z_{i+1}T_M^i \tag{7.8a}$$

$$K_{i+1}T_M^{i+1} = K_i T_M^i \tag{7.8b}$$

where $i = 1,2,\ldots,m-1$. The boundary conditions at $\zeta_3 = z_1$ and $\zeta_3 = z_{m+1}$ can be either

$$T_A^1 + z_1 T_M^1 = T_L \tag{7.9a}$$

or

$$K_1 T_M^1 = F_L \tag{7.9b}$$

and either

$$T_A^m + z_{m+1}T_M^m = T_U \tag{7.9c}$$

or

$$K_m T_M^m = F_U \tag{7.9d}$$

where $T_L, F_L$ and $T_U, F_U$ denote prescribed temperature and heat flux on the lower ($\zeta_3 = z_1$) and upper ($\zeta_3 = z_{m+1}$) bounding surfaces of the shell, respectively. Together with two of Eqs. (7.9), Eqs. (7.8) constitute a system of 2m equations from which the 2m unknowns, $T_A^i$ and $T_M^i$, can be uniquely determined at every point of the reference surface.

Thus, by assuming a linear distribution of temperature within the shell-wall, the temperature field is completely determined from the boundary and continuity conditions. In general, this temperature field will violate the heat-conduction equation, and the use of Eq. (7.6) will introduce errors in the solutions, which, just as the other assumptions already employed in this theory, will be directly proportional to the ratio of the thickness of the shell over some characteristic length.

Assuming that the quantities $T_A^i$ and $T_M^i$ have been determined in every layer at every point on the reference surface, the thermal parameters for a layered shell are given by

$$
\begin{aligned}
H_1 &= \sum_{i=1}^{m} A_1^i[T_A^i Z_1^i + (T_M^i + T_A^i/R_2)Z_2^i + T_A^i Z_3^i/R_2] \\
H_2 &= \sum_{i=1}^{m} A_2^i[T_A^i Z_1^i + (T_M^i + T_A^i/R_1)Z_2^i + T_A^i Z_3^i/R_1] \\
H_3 &= \sum_{i=1}^{m} A_1^i[T_A^i Z_2^i + (T_M^i + T_A^i/R_2)Z_3^i + T_A^i Z_4^i/R_2] \\
H_4 &= \sum_{i=1}^{m} A_2^i[T_A^i Z_2^i + (T_M^i + T_A^i/R_1)Z_3^i + T_A^i Z_4^i/R_1]
\end{aligned}
\qquad (7.10)
$$

Thus it has been shown that if the material properties of the shell are piecewise constant with respect to $\xi_3$, then the material parameters can be calculated from the relatively simple formulas given by Eqs. (7.1), (7.5), and

(7.10). Except for the additional assumption of a linear temperature field, these equations are still based only on the three basic assumptions. No further approximations have been employed.

Once the boundary value problem of a layered shell is solved in terms of the stress-resultants, strains, and displacements, the stresses in the i-th layer are calculated from Eqs. (4.5) and (4.2) as

$$\sigma^{i}_{11} = B^{i}_{11}e_{11} + B^{i}_{12}e_{22} + A^{i}_{1}(T^{i}_{A} + \xi_{3}T^{i}_{M}) \tag{7.11a}$$

$$\sigma^{i}_{22} = B^{i}_{12}e_{11} + B^{i}_{22}e_{22} + A^{i}_{2}(T^{i}_{A} + \xi_{3}T^{i}_{M}) \tag{7.11b}$$

$$\sigma^{i}_{12} = G^{i}_{12}2e_{12} \tag{7.11c}$$

$$\sigma^{i}_{23} = G^{i}_{23}2e_{23} \tag{7.11d}$$

$$\sigma^{i}_{13} = G^{i}_{13}2e_{13} \tag{7.11e}$$

where the components of strain are given by Eqs. (2.19).

Clearly, if the elastic properties in the layers are not the same, then the stresses will not be continuous on the bounding surfaces of the layers. This is acceptable as far as the in-plane stress components, $\sigma_{11}$, $\sigma_{22}$, and $\sigma_{12}$, are concerned. However, a discontinuity of the transverse shear stresses, $\sigma_{23}$ and $\sigma_{13}$, violates the condition that the stress vector must be continuous across a contact surface. This violation is a direct consequence of the assumption that the stress field can be replaced by stress-resultants. For this reason, the transverse shear stresses, as given by Eq. (7.11 d,e), must be regarded as the average stresses within a layer.

# VIII. A METHOD OF SOLUTION OF SHELL EQUATIONS

In order to solve the boundary-value problem of a shell, defined by the system of Eqs. (2.20), (3.21), (4.7)-(4.9), and the boundary conditions of Section VI, two approaches can be employed. One approach uses elimination of the dependent variables from the system of equations, so that one or more equations, each of which contains only one unknown, are obtained. The solution of the boundary-value problem is based on the solutions of these uncoupled equations. Unfortunately, such uncoupled equations have been derived only for a few simple shell-configurations, which are limited to constant material properties over the reference surface of the shell.

The second approach is applicable to a much larger class of shell-configurations, for which the geometrical and material properties as well as the applied surface loads can vary in any manner over an arbitrarily selected reference surface. This approach is based on a method of solution (see [5]) of boundary-value problems which are governed in a two-dimensional region S, defined by $a_1 \leq \xi_1 \leq b_1$ and $a_2 \leq \xi_2 \leq b_2$, by a system of differential equations stated in the form

$$\partial y/\partial \xi_1 = F(\xi_1, \xi_2, y, \partial y/\partial \xi_2, \partial^2 y/\partial \xi_2^2, \ldots) \qquad (8.1)$$

In Eq. (8.1), $y = y(\xi_1,\xi_2)$ denotes a $(k,1)$ column matrix whose elements are the k unknown dependent variables, and F denotes k linear functions in y and its derivatives with respect to $\xi_2$, arranged as elements of a column matrix. In this formulation, $\xi_1$ is a preferred coordinate, along which the solution is expected to vary more rapidly than along the $\xi_2$ coordinate.

Although the second approach admits any natural boundary conditions on the edges of S, it has been successfully applied to cases ([5], [6], and [7]) when the boundary conditions can be stated in the form

$$T_a(\xi_2)y(a_1,\xi_2) = u_a(\xi_2) \tag{8.2a}$$

$$T_b(\xi_2)y(b_1,\xi_2) = u_b(\xi_2) \tag{8.2b}$$

$$y(\xi_1,a_2) = y(\xi_1,b_2) \tag{8.2c}$$

The elements of the $(k,k)$ matrices, $T_a$ and $T_b$, are specified by the statement of the boundary conditions on the coordinate curves $\xi_1 = a_1$ and $\xi_2 = a_2$, respectively, and $u_a, u_b$ are $(k,1)$ column matrices which contain k/2 prescribed elements. The last condition, Eq. (8.2c), is a continuity condition on the coordinate curves $\xi_2 = a_2$ and $\xi_2 = b_2$, respectively.

As explained in [5], the solution of the boundary-value problem is reduced to the solution of certain initial value problems which are governed by the system of Eqs. (8.1).

The object of this section is to show how Eqs. (8.1) can be derived for the shell theory given in this paper.

First of all, it is necessary to decide how many of the 25 variables appearing in the governing equations should be included in Eqs. (8.1). This can be answered by noting that since five boundary conditions must be prescribed on one edge of the shell, the governing system of equations is of tenth order. This means that the system of Eqs. (8.1) must represent ten equations in ten unknowns.

It is convenient to choose the ten unknowns, henceforth called fundamental variables, as those quantities which appear in the natural boundary conditions on an edge $\xi_1 =$ constant, because then the boundary conditions are stated exclusively in terms of the fundamental variables. Thus, for the shell theory of this paper, the following fundamental variables are selected: $u_1, u_2, u_3, \beta_1, \beta_2, N_{11}, N_{12}, Q_1, M_{11}, M_{12}$.

For the solution of the initial-value problems, which are defined in [5], the system of Eqs. (8.1) must be in a form from which the derivatives of each of the fundamental variables with respect to $\xi_1$ can be calculated at any point in S from known fundamental variables, their derivatives with respect to $\xi_2$, surface loads, and geometrical and material properties of the shell. The calculation of such derivatives of the fundamental variables with respect to $\xi_1$

can be carried out by means of the following five steps:

1. Find $\epsilon_{22}$, $\gamma_2$, $k_{22}$, $\delta_2$, $\gamma_{23}$ from Eqs. (2.20b,d,f,h,j).

2. Solve for $\epsilon_{11}$, $k_{11}$, $\gamma_1$, $\delta_1$, $\gamma_{13}$ from Eqs. (4.7a,c), (4.8a,c), (4.9a).

3. Find $N_{22}$, $M_{22}$, $N_{21}$, $M_{21}$, $Q_2$ from Eqs. (4.7b,d), (4.8b,d), (4.9b).

4. Find the derivatives of $u_1$, $u_2$, $\beta_1$, $\beta_2$, $u_3$ from Eqs. (2.20a,c,e,g,i).

5. Find the derivatives of $N_{11}$, $N_{12}$, $Q_1$, $M_{11}$, $M_{12}$ from Eqs. (3.21a,b,c,d,e).

Thus it is seen that the governing equations, as given by Eqs. (2.20), (3.21), and (4.7)-(4.9), are already in a form which is equivalent to Eqs. (8.1), and the suggested five steps merely provide a systematic sequence for carrying out the computation of the derivatives of fundamental variables. If the calculation is arranged in this order, then at every step the quantities needed in the equations are either known or have been calculated at a preceding step. It should be noted that all twenty-five equations have been utilized, and that since no additional differentiation has been applied to the equations, no derivatives of the material properties of the shell or the radii of curvature are needed in the calculation.

## IX. CLASSICAL THEORY OF SHELLS

Perhaps the most commonly used theory of shells in engineering practice is one which in addition to the three assumptions employed in the preceding sections also assumes the following:

1. Points on a normal of a reference surface before deformation remain on a normal of the deformed reference surface.
2. The terms $\xi_3/R_\alpha$, appearing in Eqs. (2.19), (3.16), (4.11), and (4.12), are set equal to zero.

Such a theory is called the <u>classical theory of shells</u>.

The first assumption means that the transverse shear stress, $\sigma_{13}$, $\sigma_{23}$, regardless how large, cannot produce any transverse shear strain, $e_{13}$, $e_{23}$. Within the concept of our mathematical model for an orthotropic material, as given by Eqs. (4.1) and (4.2), this can be achieved by setting

$$1/G_{13} = 1/G_{23} = 0 \tag{9.1}$$

Again, it must be emphasized that Eq. (9.1) has nothing to do with the actual material of the shell, but is imposed by the kinematic constraint used in the theory. Consequently, it follows from Eqs. (4.2b,c) that

$$e_{13} = e_{23} = 0$$

and from Eqs. (2.19d,e) and (2.20i,j) that

$$u_{3,1}/\alpha_1 - u_1/R_1 + \beta_1 = 0 \tag{9.2a}$$

$$u_{3,2}/\alpha_2 - u_2/R_2 + \beta_2 = 0 \tag{9.2b}$$

The second assumption means that the ratio of the thickness of the shell over the minimum radius of curvature must be negligible with respect to one. For a layered shell, this means that in the calculation of the material parameters only the first terms on the right-hand side of Eqs. (7.1), (7.5), and the first two terms in Eqs. (7.10) are retained.

Thus, it follows from Eqs. (4.10) that

$$\begin{aligned} F_{11} &= F_{12} = F_{22} \\ J_{11} &= J_{12} = J_{22} \\ K_{11} &= K_{12} = K_{22} \end{aligned} \tag{9.3}$$

and from Eqs. (4.8) that

$$\begin{aligned} N_{12} &= N_{21} \\ M_{12} &= M_{21} \end{aligned} \tag{9.4}$$

Consequently, the second assumption will make the stress-resultants such that they may not satisfy the sixth equation of equilibrium, Eq. (3.22) with $m_3 = 0$. Similarly, the stress resultants may not vanish for rigid-body motion. Although in most cases the effects of these violations will be small, the presence of an extraneous surface couple should be understood.

The first assumption has a direct effect on the boundary conditions which must be prescribed on an edge of the shell. For example, if on the edge $\xi_1$ = constant $u_2$ and $u_3$ are prescribed, then it is obvious from Eq. (9.2b) that $\beta_2$ cannot be prescribed independently. Thus, according to the classical theory, only four boundary conditions can be prescribed on an edge. This means that the system of equations is of eighth order and that the boundary-value problem can be stated in terms of eight first-order differential equations and eight fundamental variables.

It can be shown on physical grounds ([8], pp. 55-58) that according to the mathematical model used in the classical theory, the following boundary conditions are appropriate:

1. Either $N_{11}$ or $u_1$ prescribed

2. Either $N_{12}^*$ or $u_2$ prescribed

3. Either $Q_1^*$ or $u_3$ prescribed

4. Either $M_{11}$ or $\beta_1$ prescribed

The <u>effective shear resultants</u>, $N_{12}^*$ and $Q_1^*$, are defined by

$$N_{12}^* = N_{12} + M_{12}/R_2 \qquad (9.5a)$$

$$Q_1^* = Q_1 + M_{12,2}/\alpha_2 \qquad (9.5b)$$

The same conclusion can be obtained from Eq. (5.6) with $\lambda = 2$ by replacing $\beta_2'$ from Eq. (9.2b) and integrating the $u_{3,2}'$ term by parts with respect to $d\xi_2$. After collecting the factors of $u_3'$ and $u_2'$, Eqs. (9.5) are obtained.

It is also customary in classical theory to say that the "rotatory inertia" term [the terms with $\ddot{\underset{\sim}{\beta}}$ in Eq. (3.17) and $\ddot{\underset{\sim}{B}}$ in Eq. (3.20)] and the surface moment vector, $\underset{\sim}{m}$, are negligible. This is a reasonable assumption but is applicable only when the reference surface is the "middle" surface of the shell. For an arbitrary reference surface, however, in order to have the effect of the load and inertia terms independent of the location of the reference surface, these terms should not be neglected.

The system of Eqs. (8.1) which is needed for the solution of the boundary-value problem of a shell by classical theory can be formulated in terms of the following eight fundamental variables: $u_1$, $u_2$, $u_3$, $\beta_1$, $N_{11}$, $N_{12}^*$, $Q_1^*$, $M_{11}$. In order to calculate the derivatives of these variables with respect to $\xi_1$, the procedure given in the preceding section must be modified.

Since $N_{12}$, $M_{12}$ and $\beta_2$ are no longer among the fundamental variables, we must first combine Eqs. (4.8a) and (4.8c) in the form

$$N_{12}^* = (F_{11} + J_{11}/R_2)\gamma_1 + (F_{12} + J_{12}/R_2)\gamma_2$$
$$+ (J_{11} + K_{11}/R_2)\delta_1 + (J_{12} + K_{12}/R_2)\delta_2 \qquad (9.6)$$

Then, differentiation of

$$\beta_2 = u_2/R_2 - u_{3,2}/\alpha_2 \qquad (9.7)$$

with respect to $\xi_1$, use of the Codazzi Eq. (1.21b), and substitution into Eq. (2.20g) yields

$$\delta_1 = u_{2,1}/\alpha_1 R_2 + \delta_3 \qquad (9.8a)$$

where

$$\alpha_1\alpha_2\delta_3 = \alpha_{2,1}(1/R_1 - 1/R_2)u_2$$
$$-u_{3,21} + \alpha_{2,1}u_{3,2}/\alpha_2 - \alpha_{1,2}\beta_1 \qquad (9.8b)$$

Now, the substitution of $\gamma_1$ from Eq. (2.20c) and $\delta_1$ from Eq. (9.8a) into Eq. (9.6), and the solution for $u_{2,1}$ gives

$$u_{2,1} = A[N_{12}^* + (F_{11} + J_{11}/R_2)\alpha_{1,2}u_1/\alpha_1\alpha_2$$
$$- (J_{11} + K_{11}/R_2)\delta_3 - (F_{12} + J_{12}/R_2)\gamma_2$$
$$- (J_{12} + K_{12}/R_2)\delta_2] \qquad (9.9)$$

where

$$A = \alpha_1/(F_{11} + 2J_{11}/R_2 + K_{11}/R_2^2)$$

The equations of equilibrium must also be modified, so that the derivatives of the effective shear resultants are obtained. We must first solve Eq. (3.21e) for $Q_2$, and eliminate it from Eqs. (3.21b,c). After making use of Eq. (1.21b), the final expressions are given by

$$\alpha_2 N^*_{12,1} = \alpha_{2,1}(1/R_1 - 1/R_2)M_{12} - 2\alpha_{2,1}N^*_{12} - (\alpha_1 N_{22})_{,2}$$

$$+ \alpha_{1,2}N_{11} - (\alpha_1 M_{22})_{,2}/R_2 + \alpha_{1,2}M_{11}/R_2$$

$$- \alpha_1\alpha_2[p_2 + m_2/R_2 - (b_1 + b_2/R_2)\ddot{u}_2 - (b_2 + b_3/R_2)\ddot{\beta}_2] \tag{9.10a}$$

$$\alpha_2 Q^*_{1,1} = - \alpha_{2,1}Q^*_1 + \alpha_1\alpha_2(N_{11}/R_1 + N_{22}/R_2)$$

$$- B_{,2} - \alpha_1\alpha_2 p_3 + \alpha_1\alpha_2 b_1\ddot{u}_3 \tag{9.10b}$$

where

$$\alpha_2 B = 2\alpha_{2,1}M_{12} + (\alpha_1 M_{22})_{,2} - \alpha_{1,2}M_{11}$$

$$+ \alpha_1\alpha_2 m_2 - \alpha_1\alpha_2(b_2\ddot{u}_2 + b_3\ddot{\beta}_2) \tag{9.11}$$

The sequence of calculation of the derivatives of the fundamental variables for the classical theory is as follows:

1. Find $\varepsilon_{22}$, $\gamma_2$, $\beta_2$, $k_{22}$, $\delta_2$ from Eqs. (2.2ob,d) (9.7), (2.20f,h).

2. Find $\varepsilon_{11}$, $k_{11}$, $\delta_3$ from Eqs. (4.7a,c), (9.8b).

3. Find $u_{2,1}$ from Eq. (9.9).

4. Find $\gamma_1$, $\delta_1$ from Eqs. (2.20c), (9.8a).

5. Find $u_{1,1}$, $\beta_{1,1}$, $u_{3,1}$ from Eqs. (2.20a,e), (9.2a).

6. Find $N_{22}$, $M_{22}$, $M_{12}$ from Eqs. (4.7b,d), (4.8c).

7. Find $N_{12}$, $Q_1$, B from Eqs. (9.5), (9.11).

8. Find $N_{11,1}$, $M_{11,1}$, $N^*_{12,1}$, $Q^*_{1,1}$ from Eqs. (3.21a,d), (9.11).

After eliminating the derivatives with respect to $\xi_2$ either by separation of variables [5] or by substitution of finite-difference expressions [7], the derivatives of the fundamental variables with respect to $\xi_1$ can be used to solve initial-value problems from $\xi_1 = a_1$ to $\xi_1 = b_1$. Then, on the basis of the method explained in [5], the solution to the boundary-value problem of a shell can be obtained.

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## PART II. COMPUTER PROGRAMS FOR THE ANALYSIS OF THIN, ELASTIC SHELLS OF REVOLUTION.

## I. OPERATION OF COMPUTER PROGRAMS

### 1. Introduction

Three separate computer programs for the analysis of thin, elastic shells of revolution will be described in this part of the report:

1. Static Program, for stress analysis of shells of revolution subjected to arbitrary surface, edge, and /or thermal loads.
2. Axisymmetric Eigenvalue Program, for free-vibration and stability analysis of shells of revolution subjected to axisymmetric prestress.
3. Nonsymmetric Eigenvalue Program, for free-vibration and stability analysis of shells of revolution subjected to nonsymmetric prestress.

The governing equations employed in all three of the computer programs are Eqs. (3.36) - (3.64) listed in Section 3, Chapter II, of Part I of this report. The multisegment, direct numerical integration method, described in Chapter V of Part I of this report, is used to solve all boundary value problems arising in the analysis.

The computer programs are applicable to the analysis of thin, elastic shells which are symmetric about one straight axis, henceforth called the axis of symmetry. The requirement of symmetry include: all geometric as well as physical parameters of the shell. All properties of the shell must be identical at all points on any one latitude circle.

which is defined as the intersection of the shell with a plane perpendicular to the axis of symmetry. However, the properties of the shell can have arbitrary variations (including discontinuities) along the meridian of the shell, which is the curve of intersection of the reference surface of the shell and a plane passing through the axis of symmetry.

The reference surface of the shell need not be the middle surface, but can be any arbitrary, continuous surface of revolution. The shell material can be distributed about the reference surface in any arbitrary manner. One admissible geometry of the shell is shown in Figure 1.

The Static and Axisymmetric Eigenvalue Programs can admit discontinuities in the normal of the reference surface, when proceeding along the meridian. An example of such a discontinuous normal is the joint A of the cylindrical and conical shell, as shown in Figure 1. The Nonsymmetric Eigenvalue Program cannot admit such a discontinuous normal.

The material of the shell can be either isotropic or orthotropic, with the principal axes of elastic symmetry coinciding with the principal axes of the reference surface. The wall of the shell can consist of a number of layers having different physical properties, such as the elastic parameters, coefficients of thermal expansion, and mass density.

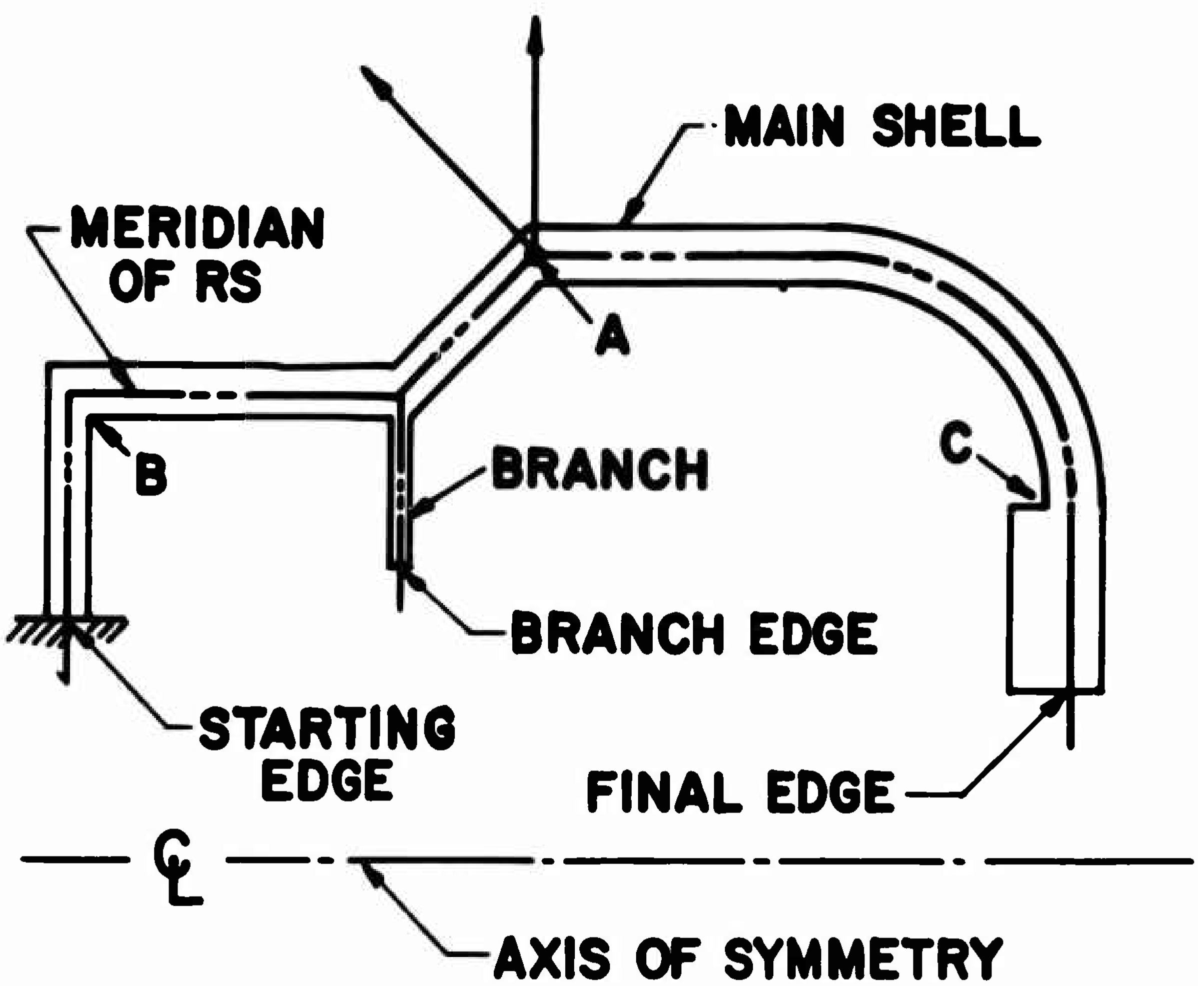



FIGURE 1. EXAMPLE OF SHELL ANALYZED BY PROGRAMS

The Static and the Axisymmetric Eigenvalue Programs can admit axisymmetric skirts, henceforth called <u>Branches</u>, which are attached to a <u>Main Shell</u>. The Nonsymmetric Eigenvalue Program cannot admit any Branches. A typical Branch and Main Shell are shown in Figure 1.

## 2. <u>Procedure for Selecting Input Data</u>

Before the preparation of the input data, the following items should be considered.

### 1. <u>Reference Surface</u>

After the meridional section of a shell of revolution is given, a <u>Reference Surface</u>, henceforth called the RS, must be chosen. The significance of the RS for our analysis lies in the fact that the stress resultants are attached to and the loads are applied at the RS, and the displacement components, calculated by the programs, are those of the RS.

The RS can be chosen as any convenient surface of revolution. It must be continuous for the whole shell. It should be selected in such a way that the interval along the normal of the RS which lies wholly within the shell material is as small as possible. The RS should be situated in such a way that it lies as close to the midpoint between the two bounding surfaces of the shell as possible.

The most important feature of having an arbitrary RS, which need not be the middle surface of the shell, is that the RS for various portions of the shell can be defined as algebraically describable surfaces for which all the geometric properties can be automatically calculated by the program. For example, if the meridional section of the shell is as shown in Figure 2, it is much easier to draw the meridian of the RS as consisting of a straight line and a circle, which correspond to cylindrical and toroidal surfaces, rather than to find the "middle surface" of such a shell. The shell thickness, of course, would be variable along the meridian of the RS, but that can be easily measured and input into the program.

At the present time, the geometrical properties of seven shell types are automatically calculated by the program:

1. Cylindrical
2. Spherical
3. Paraboloidal
4. Ellipsoidal
5. Conical (including a flat plate)
6. Toroidal
7. Hyperboloidal

Any other types, for which formulas for the radii of curvature are known, could be easily added. If these shell types are used to construct the shell, then the input data is very simple, because only a few constants and the beginning and the end of the shell must be defined.

Our approach practically eliminates the need for a

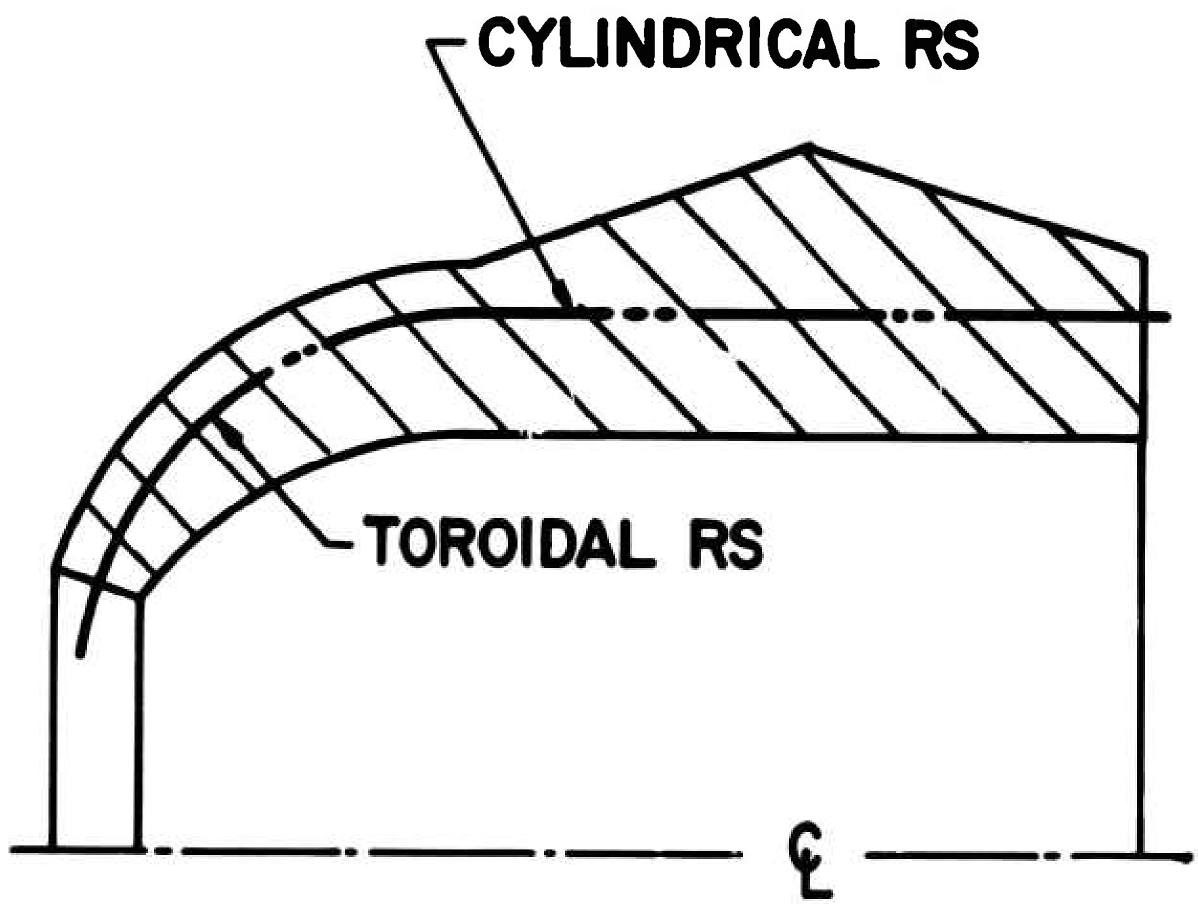



FIGURE 2. EXAMPLE FOR SELECTING REFERENCE SURFACE

"general shell of revolution", for which the geometrical properties must be calculated and input at a discrete number of pivotal points along the meridian. Not only would such a procedure make the input much more complicated, but the geometric properties between the pivotal points would have to be obtained by means of interpolation, and therefore they would not describe the shell exactly. However, the programs also include the provision for a General Shell, just in case a need for it arises.

## 2. Main Shell and Branches

When the meridian of the axially symmetric RS is known, a continuous portion of it, bounded by two latitude circles, must be selected as the Main Shell, as shown in Figure 1. The skirts, which can be attached at one point of the Main Shell and must terminate with their own latitude circles, are then designated as Branches. A number of Branches are allowed. Every one of them must be attached to the Main Shell at one point, but not to each other. It should be remembered that the Nonsymmetric Eigenvalue Program does not admit any Branches.

## 3. Shell Edges

The RS of the shell must be bounded by a number of circular edges. Two of them belong to the Main Shell and the rest to Branches. One of the edges of the Main Shell is designated as the Starting Edge, from which the integration is begun. As a rule, the Starting Edge must be that one which can restrict the shell from rigid-body motion. This means that for axisymmetric cases, when the Fourier wave number n=0, it must not be possible to move the Starting Edge parallel to the axis of symmetry without any resistance. Similarly, for bending cases, when n=1, it must not be possible to rotate the Starting Edge about any line perpendicular to the axis of symmetry without any resistance. For n≥2, no rigid body requirements have to be met.

Once a Starting Edge is selected, the other edge of the Main Shell is named the Final Edge. The edges of the Branches are called Branch Edges.

## 4. Shell Parts

The meridian of the Main Shell and Branches is divided into a number of Parts, which are selected in such a way that some shell properties, shell type, and/or loads are constant over each Part. For example, portions of the shell which are cylindrical, spherical, paraboloidal, ellipsoidal, conical, toroidal, or hyperboloidal should always be named as separate Parts, because formulas for their radii of curvature are already in the program and need not be input at every point along the meridian. Any portions of the shell with different but constant parameters, such as thickness, elastic properties, or loads, should be made separate Parts.

The division into Parts permits easier description of input parameters, because within each Part some of the properties (shell type, thickness, loads, etc.) could be constant, but different from another Part. The input cards for each Part are read for one Part at a time, and data cards describing each Part have identical formats.

A Branch must be always made a separate group of Parts.

The shell is made up of all the Parts, put together in the order in which they are read in. Part No. 1 begins with the Starting Edge of the Main Shell. Last Part ends with the Final Edge of the Main Shell. When a Branch is reached, the Parts of the Branch must come first, and then the Main Shell continued.

## 5. Shell Segments

The multisegment method of direct, numerical integration requires the integration of initial value problems over sufficiently short meridional segments of the shell. We have already achieved some segmentation through the introduction of Parts. However, the meridian of each Part may still be too long, so that each Part must be further subdivided into Segments. The lengths of the Segments within each Part are the same. In order to vary Segment lengths over the shell, separate Parts must be defined.

The lengths of Segments must be estimated only approximately,and it is advisable to use shorter Segments rather than longer ones. An estimate of the length $\ell$ of a Segment is approximately given by

$$\ell = \sqrt{Rh}$$

where h is the average wall thickness, and R is the average minimum radius of curvature.

After the solution is calculated, two items indicate whether the Segments have been chosen too long:

1. The solution at the end of a Segment does not match the solution at the beginning of the following Segment within a desired degree of accuracy.

2. The number of integration points, printed out together with the initial value integration results, exceeds 250. This number is only printed if NPRT=1.

The first item reflects also general overall accuracy of the solution, which is as accurate as the continuity between segments and the satisfaction of the boundary conditions.

## 6. Boundary Conditions

The boundary conditions at the circular edges of the shell are given by code numbers which identify the prescribed fundamental variables. The code numbers are:

1 for $w$; 2 for $Q$; 3 for $u_\phi$; 4 for $N_\phi$

5 for $\beta_\phi$; 6 for $M_\phi$; 7 for $u_\theta$; 8 for $N$

where $w$, $u_\phi$, and $u_\theta$ denote the displacement components of the RS in the normal, meridional, and circumferential directions, respectively; $Q$, $N_\phi$, and $N$ are forces per unit length of circumference in the normal, meridional, and circumferential directions, respectively; $\beta_\phi$ designates the angle of rotation, in radians, of the normal in the meridional direction; and $M_\phi$ is the meridional bending moment, per unit length of circumference. Positive directions on leading and trailing edges of all fundamental variables are shown in Figure 3. According to shell theory, the boundary conditions must be such that either $w$ or $Q$, $u_\phi$ or $N_\phi$, $\beta_\phi$ or $M_\phi$, and $u_\theta$ or $N$ are prescribed on each edge.

In some shell problems the prescribed variables on the edges are not the displacement and force components along the normal and meridional tangent of the RS, but rather the corresponding components rotated within the meridional plane; i.e., the plane which contains the axis of symmetry. The Static and Axisymmetric Eigenvalue Programs admit such rotated boundary conditions, while the Nonsymmetric Eigenvalue Program does not.

If rotated boundary conditions on an edge are employed, then the code numbers for boundary variables are assigned as follows:

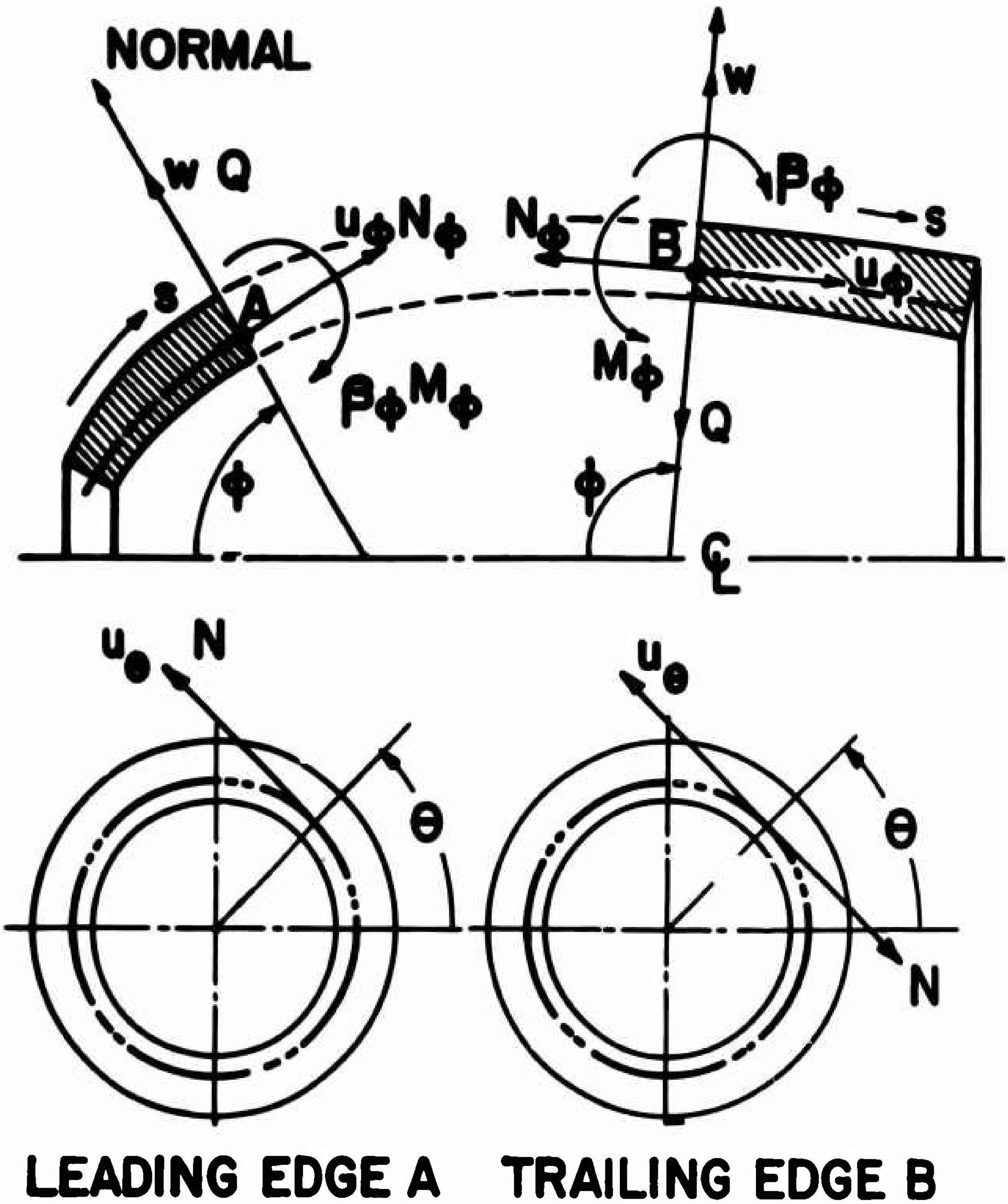



FIGURE 3. POSITIVE DIRECTIONS OF FUNDAMENTAL VARIABLES

1 for $u_1$; 2 for $Q_1$; 3 for $u_2$; 4 for $Q_2$

5 for $\beta_\phi$; 6 for $M_\phi$; 7 for $u_\theta$; 8 for N

where $u_1$, $Q_1$ and $u_2$, $Q_2$ are the displacement and force components along two mutually orthogonal directions in the meridional plane.

## 7. Meridional Coordinate

The coordinate for identifying points on the meridian of the RS of each Part can be either the arclength s,measured along the meridian,or the angle $\phi$ between the normal of the RS and the axis of symmetry. The type of the shell within a Part determines which one of the two coordinates must be used. For example, a conical shell uses s, while a toroidal shell uses $\phi$.

The meridional coordinate in each Part can have a different origin. The Parts are automatically joined together in the order in which they are read in on the input cards, without any regard to the meridional coordinate in each Part. As a rule, the meridional coordinate, s or $\phi$, must be always positive, and at the beginning of each Part it must have a smaller value than at the end. The integration can only be carried out in the positive direction of the meridional coordinate.

For those Parts for which $\phi$ is the meridional coordinate, it may be sometimes necessary, owing to the orientation of the Part, to integrate from a larger $\phi$ to a smaller $\phi$. According to the above rule, this cannot be done. To admit such cases, a Direction Index is introduced, which, when set equal to -1.0, reverses the direction of measuring $\phi$ for that particular Part. This results in a smaller value of $\phi$ at the beginning edge of the Part, and the integration can be carried out.

## 8. Loads

The external loads which produce stresses and displacements in the shell can consist of mechanical loads, applied either on the RS or the edges of each Part, and/or thermal loads, which are defined by a prescribed temperature distribution over the two bounding surfaces of the shell. The edge loads on the terminal edges of the shell appear as prescribed stress resultants, Q, $N_\phi$, $M_\phi$, N, in the boundary conditions, while the loads at the end of each Part are specified as Ring Loads on a special input card.

The Ring loads, if given nonzero values for a Part, mean that there will be a discontinuity in the stress resultants between the end of the Part and the beginning of the following Part. The discontinuity is equivalent to the effect of an imaginary ring which is inserted at the end of the Part and subjected to the stress resultants Q, $N_\phi$, $M_\phi$, and N, regarded now as Ring Loads. The positive signs of these Ring Loads are the same as the positive signs of Q, $N_\phi$, $M_\phi$, and N at the end of the Part for which they are given.

The Ring Loads can only be applied at the end of a Part. If necessary, the point on the meridian at which Ring Loads are applied should be made the end of a Part. The loads at the end of the last Part of either a Main Shell or a Branch should not be specified as Ring Loads, but rather as prescribed boundary conditions.

External loads appear cnly in the Static Program. The shell is free of any external loads in the cases considered by the Eigenvalue Programs.

If the loads have circumferential variation, they must be first expanded in a Fourier series, and the problem is solved for each set of Fourier coefficients separately. Thus, the surface loads must be expanded as

$$\begin{bmatrix} p \\ p_\phi \\ T_U \\ T_L \end{bmatrix} = \sum_n \begin{bmatrix} p_n(\phi) \\ p_{\ n}(\phi) \\ T_{Un}(\phi) \\ T_{Ln}(\phi) \end{bmatrix} \begin{Bmatrix} \cos n\theta \\ \sin n\theta \end{Bmatrix}$$

$$p_\theta = \sum_n p_{\theta n}(\phi) \begin{Bmatrix} \sin n\theta \\ \cos n\theta \end{Bmatrix}$$

and the edge loads as

$$\begin{bmatrix} Q \\ N_\phi \\ M_\phi \end{bmatrix} = \sum_n \begin{bmatrix} Q_n(\phi) \\ N_{\phi n}(\phi) \\ M_{\phi n}(\phi) \end{bmatrix} \begin{Bmatrix} \cos n\theta \\ \sin n\theta \end{Bmatrix}$$

$$N = \sum_n N_n(\phi) \begin{Bmatrix} \sin n\theta \\ \cos n\theta \end{Bmatrix}$$

For the definition of the load parameters, the reader should consult Section 2, Chapter II, of Part I of this report.

All the load series are summed over some selected list of integer circumferential wave numbers, denoted by $n = n_1, n_2, \ldots, n_k$. The list should be such that the series reasonably represent the given circumferential variations of each load at a selected number of discrete points on the meridian. The top and bottom trigonometric functions in the curly brackets indicate that each of the series can have cosine as well as sine terms.

The set of Fourier coefficients of all the loads, corresponding to one wave number $n_i$ and either the top or bottom trigonometric function, constitute the external loads for one separate problem. If such external loads for one wave number $n_i$ are used as input, the output represents the Fourier coefficients of each fundamental variable with the same wave number $n_i$.

As many such separate problems must be solved as the largest number of terms appearing in the load series. After the solutions for all the wave numbers are calculated, the complete solution of the problem is given by the following Fourier series expressions

$$\begin{bmatrix} w \\ Q \\ u_\phi \\ N_\phi \\ \beta_\phi \\ M_\phi \end{bmatrix} = \sum_n \begin{bmatrix} w_n(\phi) \\ Q_n(\phi) \\ u_{\phi n}(\phi) \\ N_{\phi n}(\phi) \\ \beta_{\phi n}(\phi) \\ M_{\phi n}(\phi) \end{bmatrix} \begin{Bmatrix} \cos n\theta \\ \sin n\theta \end{Bmatrix}$$

$$\begin{bmatrix} u_\theta \\ N \end{bmatrix} = \sum_n \begin{bmatrix} u_{\theta n}(\phi) \\ N_n(\phi) \end{bmatrix} \begin{Bmatrix} \sin n\theta \\ \cos n\theta \end{Bmatrix}$$

It should be understood that the Fourier coefficients of the loads produce the corresponding coefficients of the solution, with the same circumferential wave number $n_i$ and either the top or bottom trigonometric function. The solution is not automatically added together by the program, but such an addition can be easily carried out after each Fourier component of the solution is calculated and printed out along the meridian. It should be also remembered that the program does not find the Fourier coefficients of a given load distribution along the latitude circles. This must be done separately, before using the Static Program.

## 9. Rules for Selecting Shell Parameters

Owing to the many kinds of combinations of shell types which may be used to construct a shell, certain rules must be observed for the selection of the normal, the direction of $\phi$, and the sign of the meridional radius of curvature $R_\phi$.

The following steps are recommended for the selection of shell parameters:

1. After the Starting Edge, Main Shell, and Branches are determined, the direction of integration should be indicated and the Parts numbered for the construction of the shell. Part No. 1 must start with the Starting Edge. When meeting a Branch, the Parts of the Branch must come first, and only then the Main Shell continued. The direction of integration begins at the Starting Edge and proceeds continuously over the whole shell.

2. The direction of the normal should be selected in Part No. 1. Then the normal in the remaining Parts must be such that $\beta_\phi$ is positive in the same direction for all Parts at every juncture. (See Figure 3 for the sign of $\beta_\phi$.)

3. The angle $\phi$ must be measured in all Parts from a line which is parallel to the axis of symmetry, and it must be measured always in the same direction, either clockwise or counterclockwise.

4. The algebraic sign of $R_\phi$ should be selected according to the following rule: if the normal points away

from the center of curvature of the meridian, then $R_\phi$ is positive; if it points toward the center of curvature, then $R_\phi$ is negative.

The geometry of the shell, as used in the program, requires that the following two rules always be satisfied:

1. If the integration over a region of a Part proceeds toward the axis of symmetry, then $\cos\phi$ over that region must be negative; if it proceeds away from the axis of symmetry, then $\cos\phi$ must be positive.

2. If the normal points away from the axis of symmetry, then $\sin\phi$ must be positive; if it points toward the axis of symmetry, then $\sin\phi$ must be negative.

After the direction of normal and $\phi$ for each Part are selected, these two rules must be checked. If they are not satisfied, the results will not be correct.

An example of properly selected normals and the measurement of $\phi$ is shown in Figure 4.

## 10. Direction Index

Whenever the direction of integration in a Part is such that the meridional coordinate $\phi$ goes from a larger value to a smaller value, then the Direction Index must be set equal to -1.0. Such a situation occurs in Part No. 5 for the shell shown in Figure 4. This Part is a toroidal shell, and it starts at $\phi = 270°$ and ends at $\phi = 180^\circ$. Such a situation is not allowed, because integration must proceed in the positive direction of $\phi$.

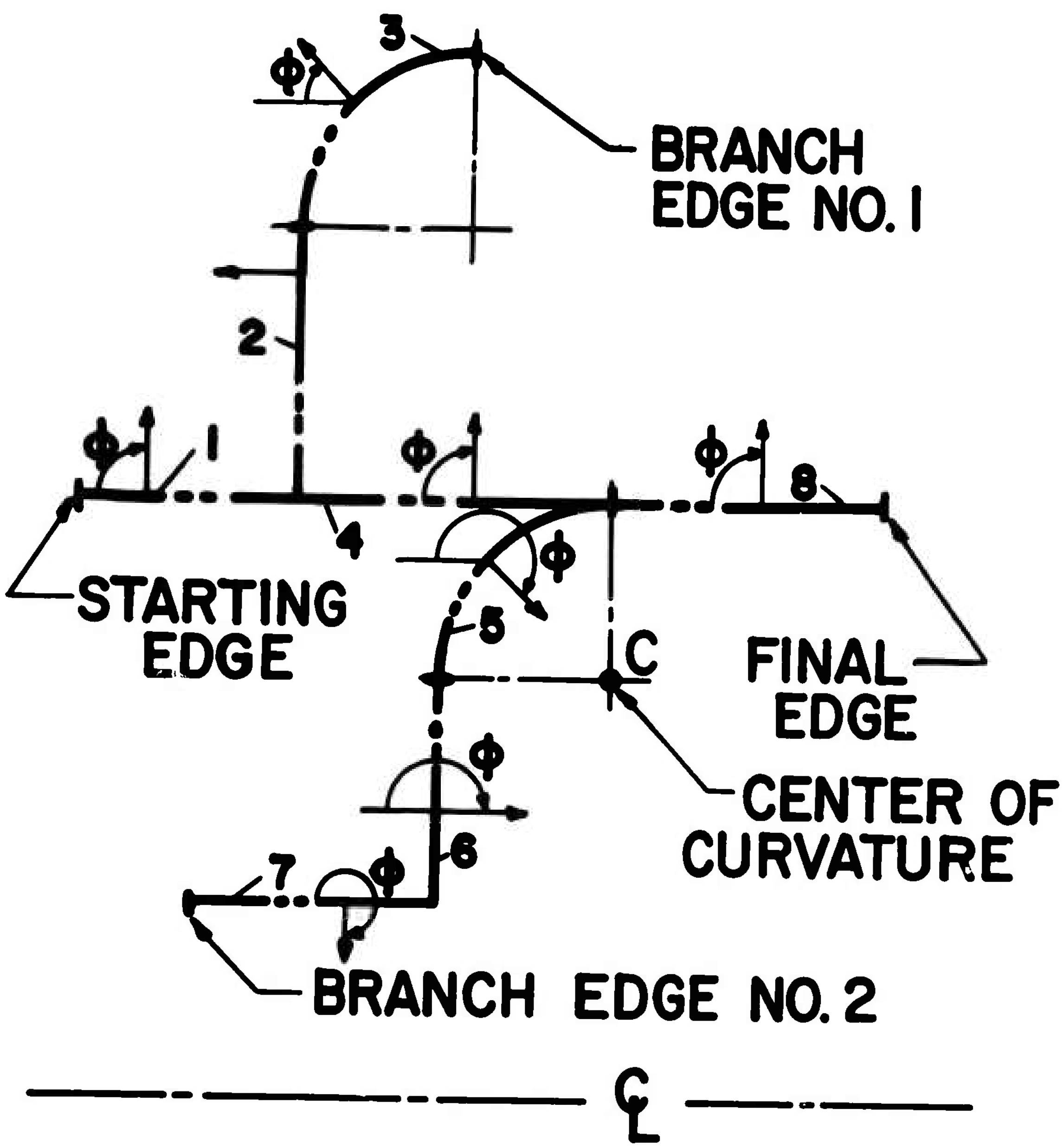



FIGURE 4. EXAMPLE FOR SELECTING SHELL PARAMETERS

By setting the Direction Index equal to -1.0, the direction of measuring $\phi$ is reversed for the purpose of defining the beginning and end of the Part. Thus, for Part No. 5 in Figure 4, the beginning coordinate is $\phi=90°$ and the final coordinate is $\phi=180°$, if the Direction Index is -1.0.

It should be emphasized that the Direction Index affects only the meridional coordinate $\phi$ for defining the two edges of the Part. It does not affect the angle $\phi$ which is used to calculate $\sin\phi$ and $\cos\phi$. For Part No. 5 in Figure 4, $\phi$ must still be regarded as being between 270° and 180°, where $\sin\phi$ and $\cos\phi$ are both negative. Since the integration does proceed toward the axis of symmetry and the normal does point toward the center of curvature (Point C in Figure 4), negative signs for both $\sin\phi$ and $\cos\phi$ are correct. It should also be remembered that $R_\phi$ for Part No. 5 must be input as a negative number.

## 11. Solution Near the Axis of Symmetry

The point where the meridian crosses the axis of symmetry is a singular point in the differential equations, and it cannot be included in any Part. In order to analyze a Part which is actually closed on the axis, a small hole must be inserted about the axis of symmetry. The opening of the hole should be about three thicknesses of the Part, or approximately 2 degrees.

## 12. Initial Selection of Eigenvalue Parameter

The Eigenvalue Programs find for one given wave number all eigenvalues within a specified interval of the eigenvalue parameter, which is input through the Eigenvalue Card. A characteristic determinant is evaluated within this interval at a given number of steps, and the eigenvalues, at which the determinant is zero, are found.

Our procedure requires the approximate estimate of where the eigenvalues are for each wave number. If no such estimate is available, the following procedure for the free vibration analysis is recommended. Two values of the eigenvalue parameter are selected and the solution obtained at these values. The mode shapes are always printed out for each value of the eigenvalue parameter, and they represent the solution to a forced, steady state vibration problem, where the forcing function is the first prescribed variable at the Final Edge, which is only zero at an eigenvalue. All other boundary conditions are zero automatically for any value of the eigenvalue parameter.

Such a forced vibration solution can reveal the location of the eigenvalue parameter with respect to the eigenvalues of the system. The simplest way to estimate the location is to count the number of nodes in the normal displacement w. For example, if for a given eigenvalue parameter the forced vibration solution shows three nodes in w, then that particular eigenvalue parameter lies between the third and fourth mode.

This procedure is not foolproof, because in the vicinity of longitudinal modes the node count in w starts again from zero. However, if several eigenvalue parameters are tried, this procedure will provide a good estimate on the location of the eigenvalue parameter.

The same procedure can be employed for the stability problems, where, of course, only the lowest eigenvalue for each wave number is of interest. It should be pointed out that the lowest buckling mode could have either zero or one node in w, depending on the problem. A safe procedure is to examine the whole range of the eigenvalue parameter below an accepted eigenvalue.

Moreover, for stability problems the object is to find also that wave number at which the eigenvalue is the lowest one. This means that a selected list of wave numbers must be tried until a minimum is found. The selection of this list must be based on experience, and no definite procedure can be recommended at this time.

## 3. Description of Input Data Cards

Schematic diagrams of the arrangement of data cards for each of the three computer programs are shown in Figures 5 and 6. The input cards are identified by name. In the descriptions of each data card, included in this section, it is indicated in which program the card is applicable. For brevity, the Static Program is designated as SP, the Axisymmetric Eigenvalue Program as AEP, and the Nonsymmetric Eigenvalue Program as NEP.

The F format in the following descriptions of the data cards is indicated as F10.0, which does not correspond exactly with that in the program. This difference is of little significance, because in most computer systems the decimal point for F formats can be put anywhere as long as the floating point number fits within the total field, which in this case is 10.

The word "case" pertains to one shell geometry and one given prestress. Several "subcases" can be run from the same geometry and prestress. For the Static Program, each subcase is defined as the solution for one set of Fourier coefficients of all loads, including one set of boundary conditions, corresponding to one given wave number. For the Axisymmetric Eigenvalue Program, a subcase is the solution for one wave number and a set of boundary conditions. For the Nonsymmetric Eigenvalue Program, a subcase is the solution for one set of

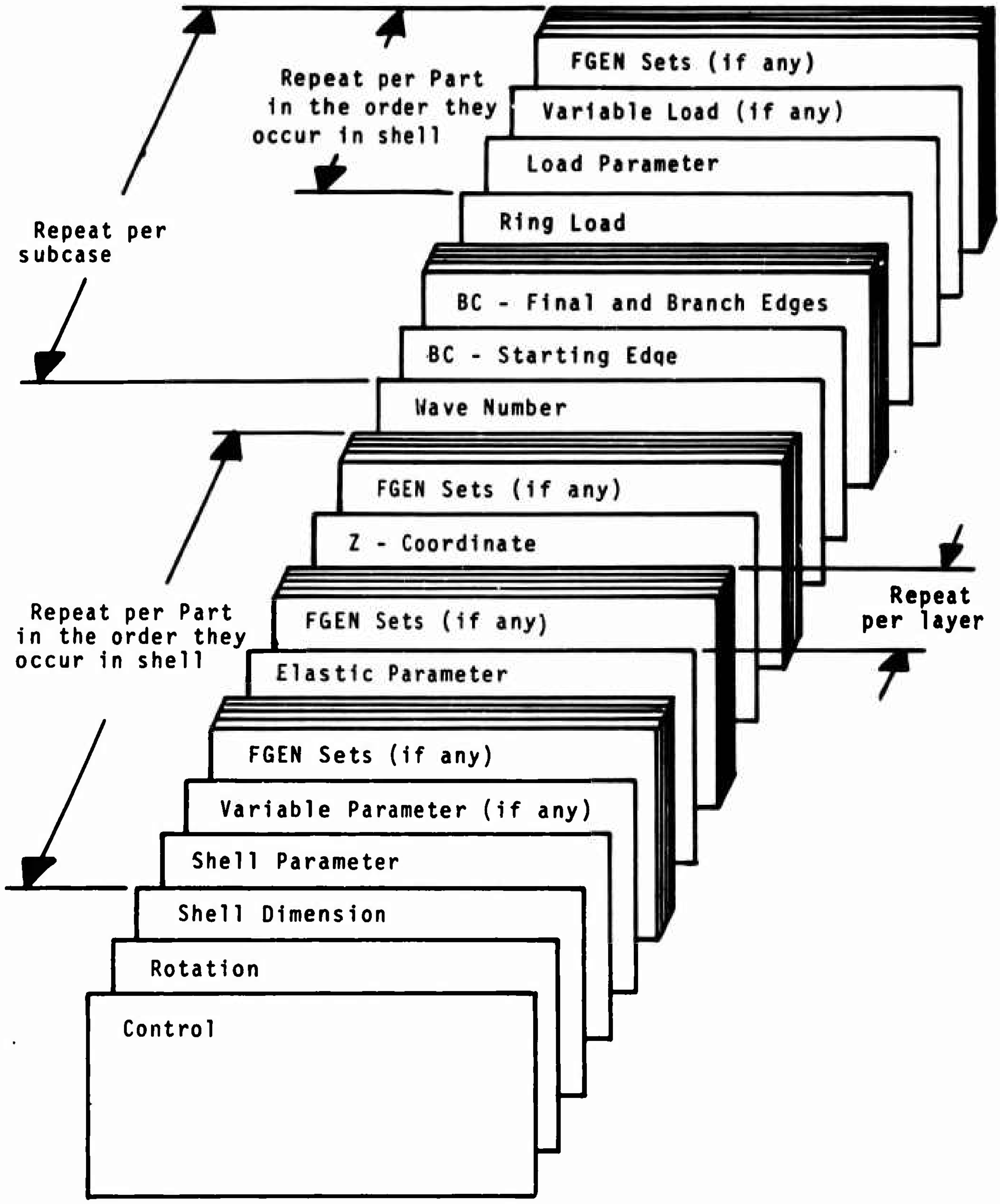



FIGURE 5. SCHEMATIC OF DATA CARDS FOR ONE CASE FOR STATIC PROGRAM

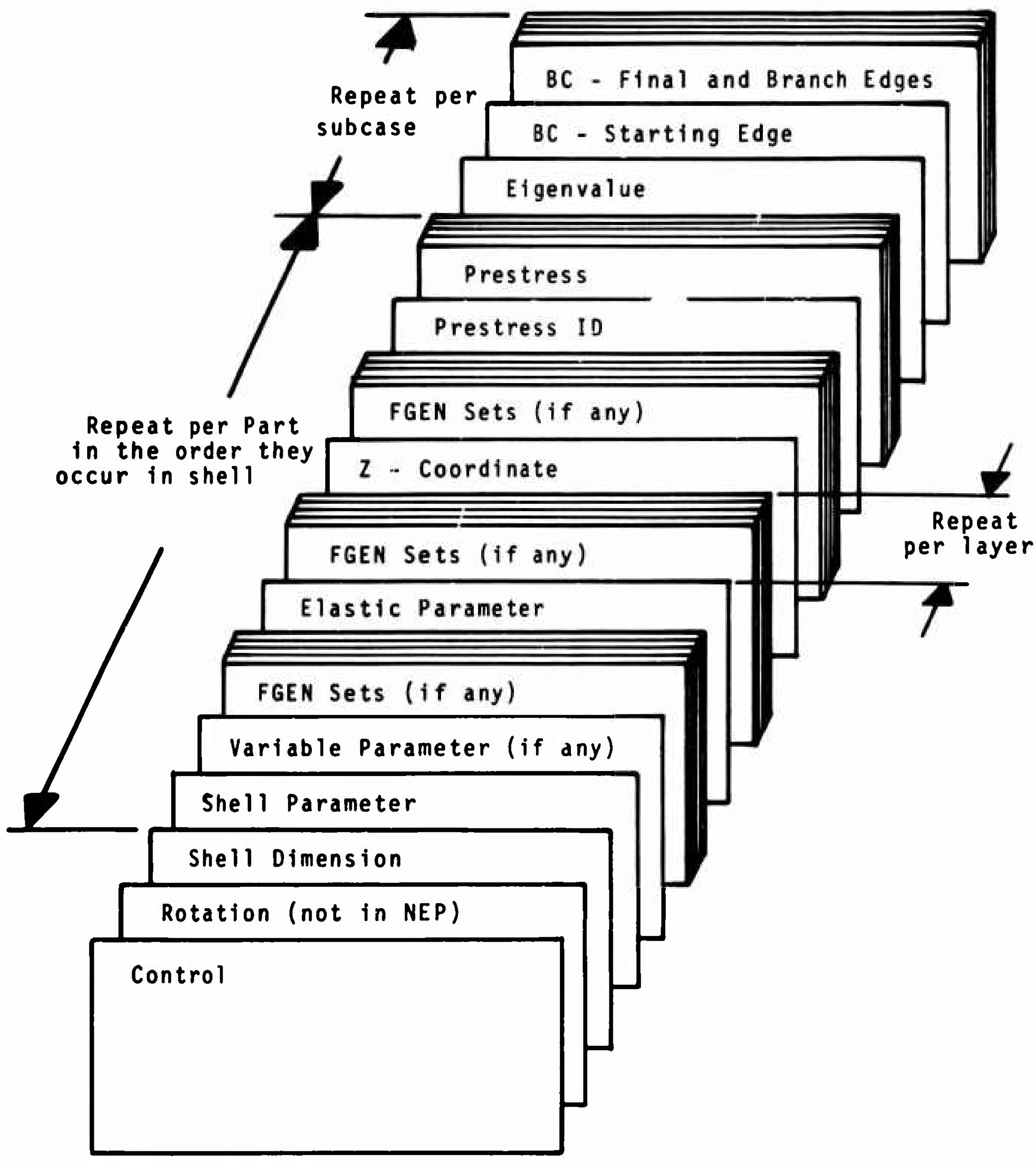



FIGURE 6. SCHEMATIC OF DATA CARDS FOR ONE CASE FOR EIGENVALUE PROGRAMS

wave numbers, which are selected to participate in the solution, and one set of boundary conditions.

The input card descriptions, given next, are followed by the illustrations of the eight shell types from which the whole shell can be constructed, and the definitions of shell parameters pertaining to each shell type.

## Control Card

Application - SP, AEP, NEP.

Number of consecutive cards = one per case.

| Variables | Format | Columns | Description |
|---|---|---|---|
| IBRM | I5 | 1-5 | Total number of Parts (not more than 20). |
| ISTK | I5 | 6-10 | If ISTK=1, stress resultants are printed. If ISTK=0, stresses are printed. If ISTK=2, both are printed. Not used in NEP, where stresses are not calculated. |
| NBR | I5 | 11-15 | Total number of Branches (not more than 3). Not used in NEP. |
| NXT | I5 | 16-20 | Total number of subcases for this case. See discussion elsewhere in this Section. |
| IVB | I5 | 21-25 | For stability analysis, set IVB=1. For free vibration, set IVB=0. Not used in SP. |
| NPRT | I5 | 26-30 | If NPRT=0, then intermediate results are not printed. If NPRT=1, they are printed. Ordinarily, set NPRT=0. See Comments. |
| NPRE | I5 | 31-35 | Wave number of prestressed state. For NEP only. |

Comments:

1. The intermediate results consist of all parameters at the beginning and end of each Part, initial value integrations, determinants of all inverted matrices, and the values of the fundamental variables at ends of segments. These items are useful only if something has gone wrong, and their evaluation requires the understanding of the theory of Chapters IV and V of Part I.

## Rotation Card

Application - SP, AEP.

Number of consecutive cards = one per case.

| Variables | Format | Columns | Description |
|---|---|---|---|
| ALFL | F10.0 | 1-10 | Angle of rotation, in degrees, of displacements and forces prescribed on Starting Edge. |
| ALFR(I) | 4F10.0 | 11-50 | Same on Final and Branch Edges. |

Comments:

1. If this card is left blank, then prescribed displacements and forces on edges are normal and tangential.
2. ALFR(1) refers to Final Edge, ALFR(2) to Branch Edge No. 1, ALFR(3) to Branch Edge No. 2, etc.
3. For rotated boundary conditions, the prescribed variables are either $u_1$ or $Q_1$, and $u_2$ or $Q_2$, as shown in Figure 7. See also the discussion in Item 6, Section 2, Chapter I, of Part II of this report.
4. The rotated variables on a given edge are related to the fundamental variables by the equations

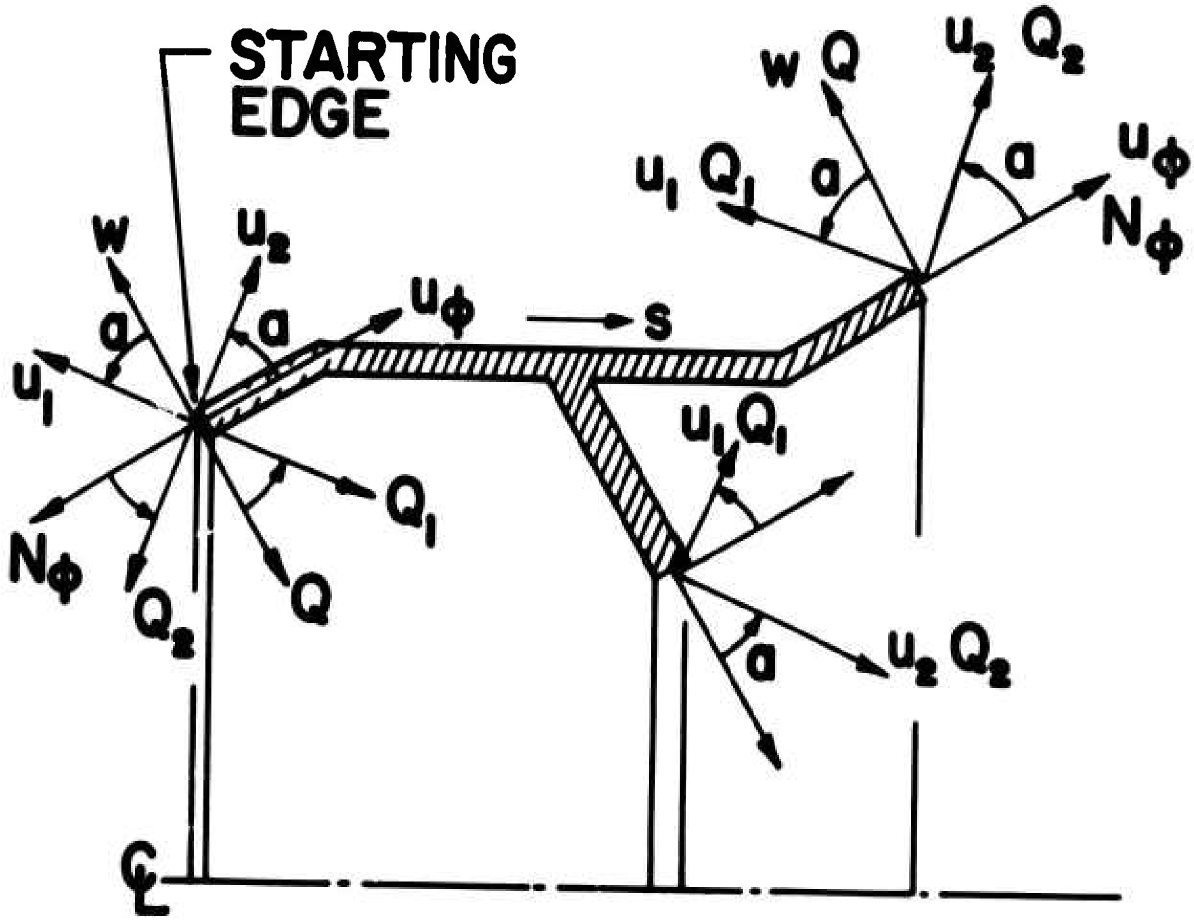
$$u_1 = w\cos a - u_\phi \sin a$$

$$u_2 = w\sin a + u_\phi \cos a$$

$$Q_1 = Q\cos a - N_\phi \sin a$$

$$Q_2 = Q\sin a + N_\phi \cos a$$

where a is the angle of rotation given on this card.




a = ANGLE OF ROTATION

FIGURE 7. ROTATED VARIABLES USED IN BOUNDARY CONDITIONS

## Shell Dimension Card

Application - SP, AEP, NEP.

Number of consecutive cards = one per Part.

| Variables | Format | Columns | Description |
|---|---|---|---|
| SI | F10.0 | 1-10 | Meridional coordinate of beginning of this Part. |
| SX | F10.0 | 11-20 | Meridional coordinate of end of this Part. |
| IPAR | I5 | 21-25 | Number of Segments. See Comments. |
| ING | I5 | 26-30 | Number of output points within one Segment. |
| ISS | I5 | 31-35 | Shell type code number. See Shell Parameter Card description. |
| NTP | I5 | 36-40 | Identifies whether this Part belongs to a Main Shell or Branch. Not used in NEP. |
| MLY | I5 | 41-45 | Number of layers (not more than 4). |

Comments:

1. Set NTP=0, if this Part belongs to Main Shell. Set NTP=1, if it belongs to a Branch. For the last Part in a Branch, set NTP=2. If Branch has only one Part, set NTP=2.

2. The maximum number of segments, over all Parts, is now 100. There is no specific limit on the number of segments in any one Part, as long as 100 over all Parts is not exceeded.

## Shell Parameter Card

Application - SP, AEP, NEP.

Number of consecutive cards = one per Part.

| Variables | Format | Columns | Description |
|---|---|---|---|
| VN(I,1) | F10.0 | 1-10 | Thickness, for reference only. Not used in any calculations. |
| VN(I,2) | F10.0 | 11-20 | |
| VN(I,3) | F10.0 | 21-30 | Shell parameters. See definitions for each type of shell below. |
| VN(I,4) | F10.0 | 31-40 | |
| IL2 | I5 | 41-45 | If IL2=0, then all shell parameters are constant along meridian. Otherwise, IL2=number of variable parameters. Not used in NEP, where all shell parameters must be constant. |

Comments:

1. Shell type code numbers and shell parameters for each type of shell are assigned as follows:

| Shell Type | ISS | VN(I,2) | VN(I,3) | VN(I,4) |
|---|---|---|---|---|
| Cylindrical | 2 | R | $\phi$ deg. | Blank |
| Spherical | 3 | R | Blank | Direction Index |
| Paraboloidal | 4 | 2P | Blank | Direction Index |
| Ellipsoidal | 5 | A | B | Direction Index |
| Conical | 6 | $\phi$ deg. | A | Blank |
| Toroidal | 7 | A | B | Direction Index |
| General | 8 | $1/R_\phi$ | R | $\phi$ deg. |
| Hyperboloidal | 9 | A | B | Direction Index |

2. For the symbols appearing in above Table, see Shell Type Descriptions in this Section. For Direction Index, see Item 10, Section 2, Chapter I, of Part II.

3. When using spinning shell, set INORM=1 and give mass density, through RHO, on Elastic Parameter Card.

4. The only variable shell parameters are those for a General Shell No. 8.

5. Cylindrical Shell No. 2 should only be used if its thickness and elastic parameters are constants over this Part. If they are not, use General Shell No. 8, with the following parameters:

$1/R_\phi=0$; R=mean radius; $\phi=90°$ or $270°$

## Variable Parameter Card

Application - SP, AEP,

Number of consecutive cards = one per Part. This card should be inserted only if IL2>0 on Shell Parameter Card.

| Variables | Format | Columns | Description |
|---|---|---|---|
| IFG(I,1) | I5 | 1-5 | Code number of first variable shell parameter. See Comments. |
| IFG(I,2) | I5 | 6-10 | Same of second variable parameter. |
| IFG(I,3) | I5 | 11-15 | Same of third variable parameter. |
| IL1 | I5 | 16-20 | Code number of FGEN set used for first variable shell parameter. |

Comments:

1. The only possible variable shell parameters are for General Shell No. 8. The code numbers are as follows:

   2 for $1/R_\phi$; 3 for r; 4 for $\phi$

2. As many IFG's must be given as IL2 on Shell Parameter Card. Example: if r and $\phi$ vary along the meridian, then IL2=2 on Shell Parameter Card, and IFG(I,1)=3, IFG(I,2)=4. IFG(I,3) is left blank.

3. IL1 designates the code number of the first of IL2 FGEN sets, by which the IL2 variable shell parameters are read, immediately following the Variable Parameter Card.

4. The values of the variable parameters, entered on Shell Parameter Card, are used for reference only.

## Elastic Parameter Card

Application - SP, AEP, NEP

Number of consecutive cards = as many as the number of layers (MLY), starting with Layer No. 1. One such set per Part.

| Variables | Format | Columns | Description |
|---|---|---|---|
| B11 | F10.0 | 1-10 | Isotropic - Young's modulus<br>Orthotropic - $E_\phi/(1-\nu_\phi\nu_\theta)$ |
| B12 | F10.0 | 11-20 | Isotropic - Poisson's ratio<br>Orthotropic - $\nu_\phi E_\phi/(1-\nu_\phi\nu_\theta)$ |
| B22 | F10.0 | 21-30 | Isotropic - Blank<br>Orthotropic - $E_\theta/(1-\nu_\phi\nu_\theta)$ |
| B66 | F10.0 | 31-40 | Isotropic - Blank<br>Orthotropic - $G_{\phi\theta}$ |
| AL1 | F10.0 | 41-50 | Coefficient of thermal expansion, in meridional direction. Not used in AEP and NEP. |
| AL2 | F10.0 | 51-60 | Coefficient of thermal expansion in circumferential direction. Not used in AEP and NEP. |
| RHO | F10.0 | 61-70 | Mass density of material. For SP used only if spinning shell is analyzed. Not used for stability analysis. |
| IL1 | I5 | 71-75 | If IL1=0, then all elastic parameters are constant in this layer. For variable elastic parameters, see Comments. |

Comments:

1. An isotropic layer is identified by setting B22=0.0.

2. $E_\phi$ = Young's modulus in meridional direction.

$E_\theta$ = Young's modulus in circumferential direction.

$\nu_\phi$ = Poisson's ratio (contraction) in $\phi$ direction produced by normal stress in $\theta$ direction.

$\nu_\theta$ = Poisson's ratio (contraction) in $\theta$ direction produced by normal stress in $\phi$ direction.

$G_{\phi\theta}$ = shear modulus in $\phi,\theta$ plane.

Note that $\nu_\phi E_\phi = \nu_\theta E_\theta$ .

3. If IL1=+N, then N designates the code number of the first of six FGEN Card sets, by which variable B11, B12, B22, B66, AL1, AL2 are read, in that order, immediately after the Elastic Parameter card of an orthotropic layer.

4. If IL1=-N, then N designates the code number of one FGEN set, by which Young's modulus is read immediately after the Elastic Parameter card of an isotropic layer.

## Z-Coordinate Card

Application - SP, AEP, NEP.

Number of consecutive cards = one per Part.

| Variables | Format | Columns | Description |
|---|---|---|---|
| ZLY(I,1) | F10.0 | 1-10 | Z-coordinates of the bounding surfaces of layers, measured from the Reference Surface. See Figure 8. |
| ZLY(I,2) | F10.0 | 11-20 | |
| ZLY(I,3) | F10.0 | 21-30 | |
| ZLY(I,4) | F10.0 | 31-40 | |
| ZLY(I,5) | F10.0 | 41-50 | |
| IL2 | I5 | 51-55 | If IL2=0, all Z's are constants. For variable Z's, see Comments. |

Comments:

1. Note that the Z-coordinates in the negative Z direction must be negative numbers ($Z_1$ and $Z_2$ in Figure 8 are negative).

2. The number of Z-coordinates used equals the number of layers (MLY) plus one. Leave the unused ones blank.

3. If IL2=+N, then N designates the code number of the first of MLY+1 (number of layers plus one) FGEN sets by which the MLY+1 variable Z coordinates are read, immediately following the Z-coordinate card.

4. If Z's are variable, then the Z's entered on Z-coordinate card are used for reference only.

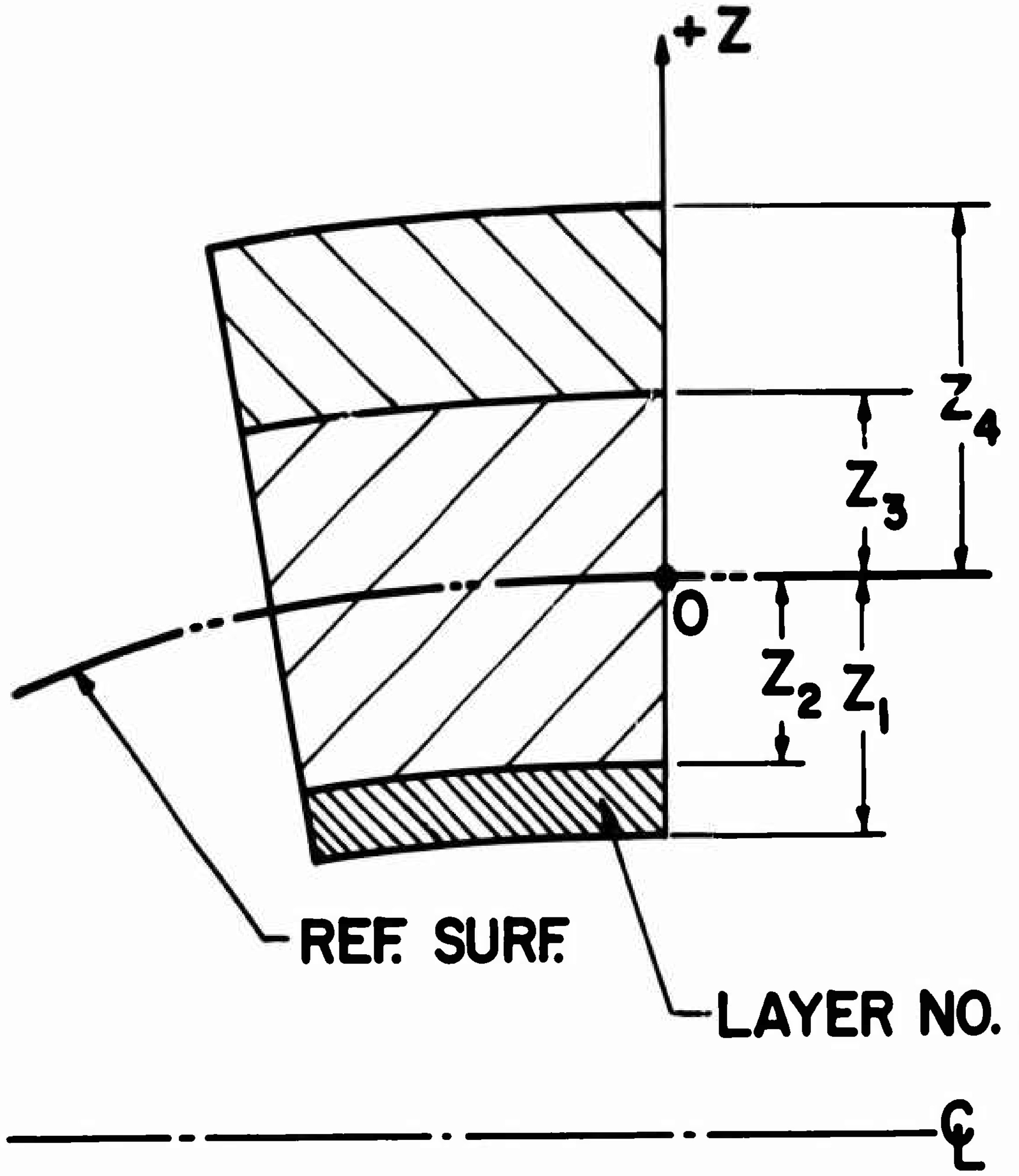



FIGURE 8. Z - COORDINATES FOR LAYERS

## Prestress ID Card

Application - AEP and NEP only.

Number of consecutive cards = one per Part.

| Variables | Format | Columns | Description |
|---|---|---|---|
| IBRT | I5 | 1-5 | Number of Part, for reference only. |
| NTR | I5 | 6-10 | Total number of points at which Prestress is given in this Part. |
| LCST | I5 | 11-15 | Indicates whether prestress is constant, zero, or variable. See Comments. |

Comments:

1. When counting the number of points for NTR, end points must be included.
2. If prestress is constant over this Part, set LCST=-1 and NTR=2. Then the constant prestress must be given at the beginning and end of this Part on Prestress Cards.
3. If prestress is zero in this Part, set LCST=0. No NTR need be given, so that Prestress ID Card can be a blank. Prestress cards must not be inserted.
4. If prestress is variable in this Part, set LCST=1.

## Prestress Card

Application - AEP and NEP only.

Number of consecutive cards = as many as NTR. One set per Part. Should not be inserted when LCST=0 on Prestress ID Card.

| Variables | Format | Columns | Description |
|---|---|---|---|
| S | F20.0 | 1-20 | Meridional coordinate |
| PAR(1) | F20.0 | 21-40 | Meridional membrane stress resultant, $N_\phi$, of prestressed state. |
| PAR(2) | F20.0 | 41-60 | Circumferential membrane stress resultant, $N_\theta$. |
| PAR(3) | F20.0 | 61-80 | Effective membrane shear stress resultant, N. Not used in AEP. |

Comments:

1. The meridional coordinate identifies the location on the meridian, at which the prestress variables apply.
2. S on the first card must equal SI and S on the last card must equal SX, as given on Shell Dimension Card.
3. Prestress ID and Prestress Cards can be punched directly by the Static Program.

## Wave Number Card

Application - SP only.

Number of consecutive cards = one per subcase.

| Variables | Format | Columns | Description |
|---|---|---|---|
| NX | I5 | 1-5 | Wave number for prescribed loads. |
| NPCH | I5 | 6-10 | If NPCH=0, cards with prestress variables of solution will not be punched. If NPCH=1, they will be punched. If prestress variables are to be punched, ISTK on Control Card must be 1 or 2. |

Comments:

1. The punched cards can be used directly as input for AEP and NEP.

2. If NX is input as a positive number, then the loads correspond to the upper trigonometric function as used in Item 8, Section 2, Chapter I, of Part II. If NX is input as a negative number, then the loads correspond to the lower trigonometric function.

## Eigenvalue Card

Application - AEP and NEP.

Number of consecutive cards = one per subcase.

| Variables | Format | Columns | Description |
|---|---|---|---|
| OMZER | F10.0 | 1-10 | Starting value of eigenvalue parameter. |
| DELOM | F10.0 | 11-20 | Increment in eigenvalue parameter. |
| OMFIN | F10.0 | 21-30 | Final value of eigenvalue parameter. |
| NFIN | I5 | 31-35 | Number of eigenvalues desired. |
| NX | I5 | 36-40 | Wave number for AEP. For NEP: lowest wave number which is selected to participate in the solution. |
| KT | I5 | 41-45 | For NEP only. Number of Fourier components used in solution. Wave numbers for these components will start with NX and will be multiples of NPRE given on Control Card. |

Comments:

1. OMZER and OMFIN represent the limits of the interval of eigenvalue parameter, within which eigenvalues will be searched.

2. Programs will calculate the first NFIN eigenvalues, if they actually occur within this interval.

## Boundary Condition Card - Starting Edge

Application - SP, AEP

Number of consecutive cards = one per subcase.

| Variables | Format | Columns | Description |
|---|---|---|---|
| IA(1) | I5 | 1-5 | Code number of first prescribed variable on Starting Edge. |
| GA(1) | F10.0 | 6-15 | Its prescribed value. |
| IA(2) | I5 | 16-20 | Second prescribed variable. |
| GA(2) | F10.0 | 21-30 | Its prescribed value. |
| IA(3) | I5 | 31-35 | Third prescribed variable. |
| GA(3) | F10.0 | 36-45 | Its prescribed value. |
| IA(4) | I5 | 46-50 | Fourth prescribed variable. |
| GA(4) | F10.0 | 51-60 | Its prescribed value. |

Comments:

1. For code numbers of prescribed variables see Item 6, Section 2, Chapter I, of Part II.
2. Remember that the prescribed edge loads are actually the Fourier coefficients for a given wave number NX.

## Boundary Condition Card - Final or Branch Edges

Application - SP, AEP

Number of consecutive cards = as many as Branches plus one. One set per subcase.

| Variables | Format | Columns | Description |
|---|---|---|---|
| IB(1,K) | I5 | 1-5 | Code number of first prescribed variable on Final or Branch Edge. |
| GB(1,K) | F10.0 | 6-15 | Its prescribed value. |
| IB(2,K) | I5 | 16-20 | Second prescribed variable. |
| GB(2,K) | F10.0 | 21-30 | Its prescribed value. |
| IB(3,K) | I5 | 31-35 | Third prescribed variable. |
| GB(3,K) | F10.0 | 36-45 | Its prescribed value. |
| IB(4,K) | I5 | 46-50 | Fourth prescribed variable. |
| GB(4,K) | F10.0 | 51-60 | Its prescribed value. |

Comments:

1. The first card of this set refers to the Final Edge of Main Shell. The second, third, etc., cards refer to Branch Edges in the order in which they are encountered, when going from the Starting to Final Edge of Main Shell.
2. For code numbers of prescribed variables, see Item 6, Section 2, Chapter I, of Part II.
3. Remember that the prescribed edge loads are actually the Fourier coefficients for a given wave number NX.

## Boundary Condition Card - Starting Edge

Application - NEP

Number of consecutive cards - one per subcase

| Variables | Format | Columns | Description |
|---|---|---|---|
| IA(1) | I5 | 1-5 | Code number of first prescribed variable on Starting Edge. |
| IA(2) | I5 | 6-10 | Second prescribed variable. |
| IA(3) | I5 | 11-15 | Third prescribed variable. |
| IA(4) | I5 | 16-20 | Fourth prescribed variable. |
| IA(5) | I5 | 21-25 | First unprescribed variable. |
| IA(6) | I5 | 26-30 | Second unprescribed variable. |
| IA(7) | I5 | 31-35 | Third unprescribed variable. |
| IA(8) | I5 | 36-40 | Fourth unprescribed variable. |

Comments:

1. For code numbers of variables, see Item 6, Section 2, Chapter I, of Part II.

## Boundary Condition Card - Final Edge

**Application - NEP**

**Number of consecutive cards - one per subcase**

| Variables | Format | Columns | Description |
|---|---|---|---|
| IB(1) | 15 | 1-5 | Code number of first unprescribed variable on Final Edge. |
| IB(2) | 15 | 6-10 | Second unprescribed variable. |
| IB(3) | 15 | 11-15 | Third unprescribed variable. |
| IB(4) | 15 | 16-20 | Fourth unprescribed variable. |
| IB(5) | 15 | 21-25 | First prescribed variable. |
| IB(6) | 15 | 26-30 | Second prescribed variable. |
| IB(7) | 15 | 31-35 | Third prescribed variable. |
| IB(8) | 15 | 36-40 | Fourth prescribed variable. |

## Ring Load Card

Application - SP only.

Number of consecutive cards = one per Part.

| Variables | Format | Columns | Description |
|---|---|---|---|
| DSC(I,2) | F10.0 | 1-10 | Value of discontinuity in effective transverse shear resultant, Q, at end of this Part. |
| DSC(I,4) | F10.0 | 11-20 | Value of discontinuity in membrane stress resultant $N_\phi$. |
| DSC(I,6) | F10.0 | 21-30 | Value of discontinuity in bending moment $M_\phi$. |
| DSC(I,8) | F10.0 | 31-40 | Value of discontinuity in effective membrane shear stress resultant N. |

Comments:

1. See Item 8, Section 2, Chapter I, of Part II for a discussion of Ring Loads.
2. The positive signs of the Ring Loads are the same as those of stress resultants on a leading edge (edge A in Figure 3) at the end of Part.
3. Remember that the prescribed values of the Ring Loads are actually the Fourier coefficients for a given wave number NX.

## Load Parameter Card

Application - SP only

Number of consecutive cards = one per Part.

| Variables | Format | Columns | Description |
|---|---|---|---|
| VK(I,1) | F10.0 | 1-10 | If INORM=0, then surface load along normal, p. If INORM=1, then surface load parallel to axis of symmetry, $p_1$. |
| VK(I,2) | F10.0 | 11-20 | If INORM=0, then surface load along meridian, $p_\phi$. If INORM=1, then surface load perpendicular to axis of symmetry, $p_2$. |
| VK(I,3) | F10.0 | 21-30 | Surface load along circumference, $p_\theta$. |
| VK(I,4) | F10.0 | 31-40 | Temperature on inside bounding surface of shell (in -Z direction), $T_L$. |
| VK(I,5) | F10.0 | 41-50 | Temperature on outside bounding surface of shell (in +Z direction), $T_U$. |
| IK2 | I5 | 51-55 | If IK2=0, then all loads are constant along meridian. Otherwise, IL2=number of variable loads |
| INORM | I5 | 56-60 | INORM=0 means that surface loads are along the normal and meridonal tangent of Reference Surface. INORM=1 means that they are parallel and perpendicular to axis of symmetry. |
| RPM | F10.0 | 61-70 | Revolutions per minute for a shell spinning about its axis of symmetry. |

Comments:

1. For positive directions of rotated loads, $p_1$ and $p_2$, see Figure 9.

2. The values of variable loads, entered on Load Parameter Card, are used for reference only.

3. Remember that the loads entered here are actually Fourier coefficients for a given wave number NX.

4. For a spinning shell, the same RPM must be entered on Load Parameter Card for every Part.

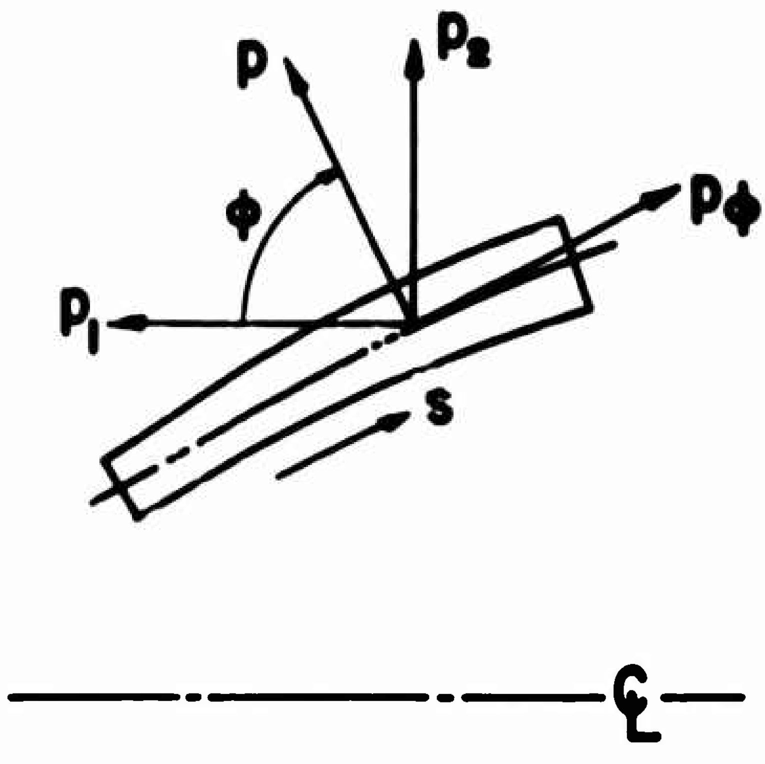



FIGURE 9. ROTATED SURFACE LOADS IN MERIDIONAL PLANE

## Variable Load Card

Application - SP only.

Number of consecutive cards = one per Part. This card should be inserted only if IK2>0 on Load Parameter Card.

| Variables | Format | Columns | Description |
|---|---|---|---|
| IKG(I,1) | I5 | 1-5 | Code number of first variable load. See Comments. |
| IKG(I,2) | I5 | 6-10 | Same of second variable load. |
| IKG(I,3) | I5 | 11-15 | Same of third variable load. |
| IKG(I,4) | I5 | 16-20 | Same of fourth variable load. |
| IKG(I,5) | I5 | 21-25 | Same of fifth variable load. |
| IK1 | I5 | 26-30 | Code number of FGEN set used for first variable load. |

Comments:

1. The code numbers of loads are as follows:

   1 for $p$ or $p_1$; 2 for $p_s$ or $p_2$

   3 for $p_\theta$; 4 for $T_L$; 5 for $T_U$

2. As many IKG's must be given as IK2 on load parameter card. Example: if surface temperatures are variable along meridian, then IK2=2 on Load Parameter Card, and IKG(I,1)=4, IKG(I,2)=5. Other IKG's are left blank.

3. IK1 designates the code number of the first of IK2 FGEN sets, by which the IK2 variable loads are read, immediately following the Variable Load Card.

## FGEN ID Card

Application - SP, AEP, NEP.

Number of consecutive cards = one per one FGEN set.

| Variables | Format | Columns | Description |
|---|---|---|---|
| NAMEG | A5 | 1-5 | Name of variable used in this FGEN. For reference only. |
| M | I5 | 6-10 | Total number of points used to describe this variable over one Part. |

Comments:

1. FGEN sets are used to handle parameters which vary along the meridian. Their values are read at discrete points, and in-between values are interpolated linearly.

2. Minimum M is 2 and maximum is 20.

3. One FGEN ID Card, followed by one set of FGEN Point Cards, constitutes one FGEN set, which can be used to describe any one variable parameter over one Part. At present, 30 FGEN sets are available for one case.

4. Each FGEN set is identified by a code number, from 1 to 30. Each variable parameter, defined over one Part, occupies one FGEN set. One given code number can be assigned only to one such FGEN set.

5. The code numbers are assigned either by IL1 on Variable Parameter Card, IL1 on Elastic Parameter Card, IL2 on Z-Coordinate Card, or IK1 on Variable Load Card. Where multiple FGEN sets are used, the code numbers are assigned consecutively upward and should not be assigned to another variable.

6. Example: If 3 loads are variable over one Part, and IK1=7 on Variable Load Card, then FGEN sets No. 7,8,9 cannot be assigned again to another variable parameter for this case.

## FGEN Point Card

Application - SP, AEP, NEP.

Number of consecutive cards = as many as needed to write M points at 4 points per card.

| Variables | Format | Columns | Description |
|---|---|---|---|
| XP(N,1) | F10.0 | 1-10 | Meridional coordinate S of point No. 1. |
| YP(N,1) | F10.0 | 11-20 | Value of variable at point No. 1. |
| XP(N,2) | F10.0 | 21-30 | Same at Point No. 2. |
| YP(N,2) | F10.0 | 31-40 | |
| XP(N,3) | F10.0 | 41-50 | As many pairs of XP and YP as required by M, as given on the FGEN ID Card. |
| YP(N,3) | F10.0 | 51-60 | |
| XP(N,4) | F10.0 | 61-70 | |
| YP(N,4) | F10.0 | 71-80 | |

Comments:

1. XP of point No. 1 must equal SI and XP of last point must equal SX, as given on Shell Dimension Card.

## Termination Card

Application - SP, AEP, MEP.

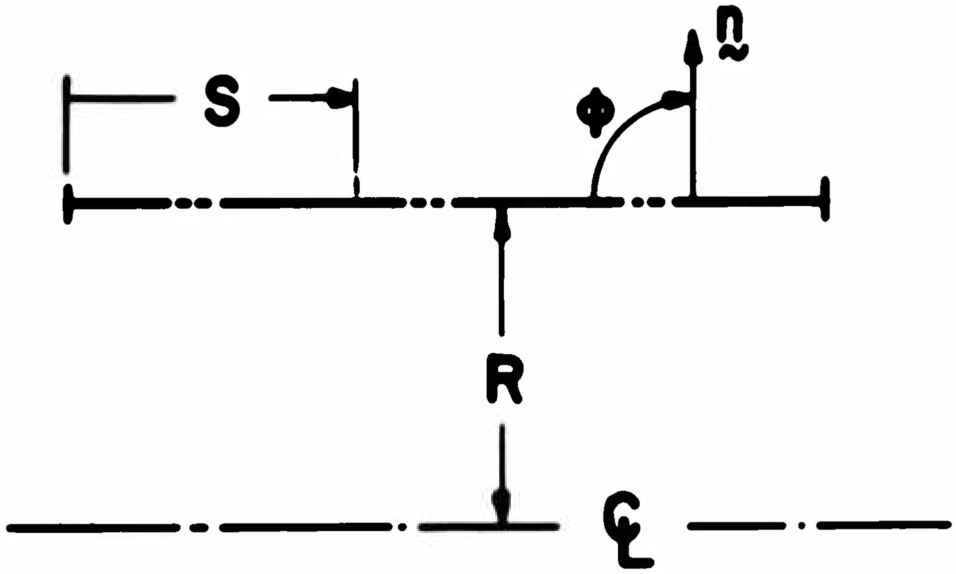
Number of consecutive cards - one blank card per job.

Comments:

1. Cases in all programs can be stacked by repeating the complete blocks of cards shown in Figures 5 and 6.

2. Program terminates execution by reading a Control Card with IBRM=0. Therefore, after all the data cards for each case have been inserted, a blank Termination Card at the end will terminate execution.

Cylindrical Reference Surface




Comments:

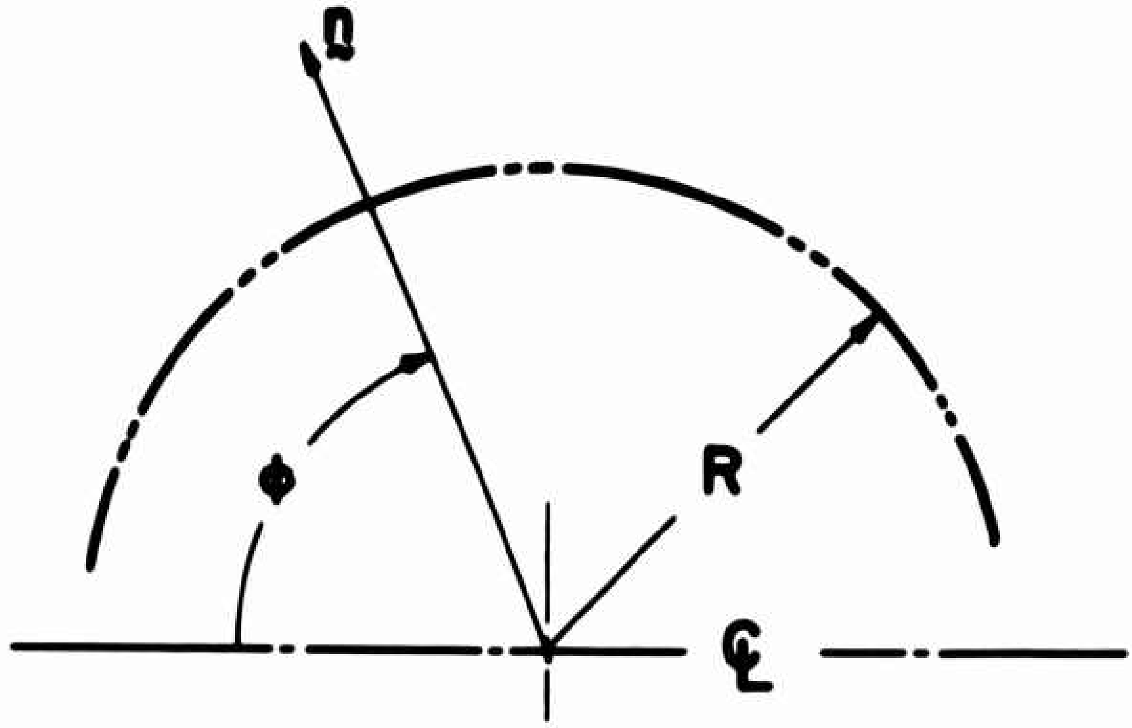
1. Radii of curvature: $1/R_\phi = 0$, $R_\theta = R$

2. Meridional coordinate = distance S along generator

3. $\phi$ can be either 90° or 270°, depending on whether normal points away from or toward axis of symmetry.

4. This shell can only be used if thickness and elastic properties are constant.

## Spherical Reference Surface



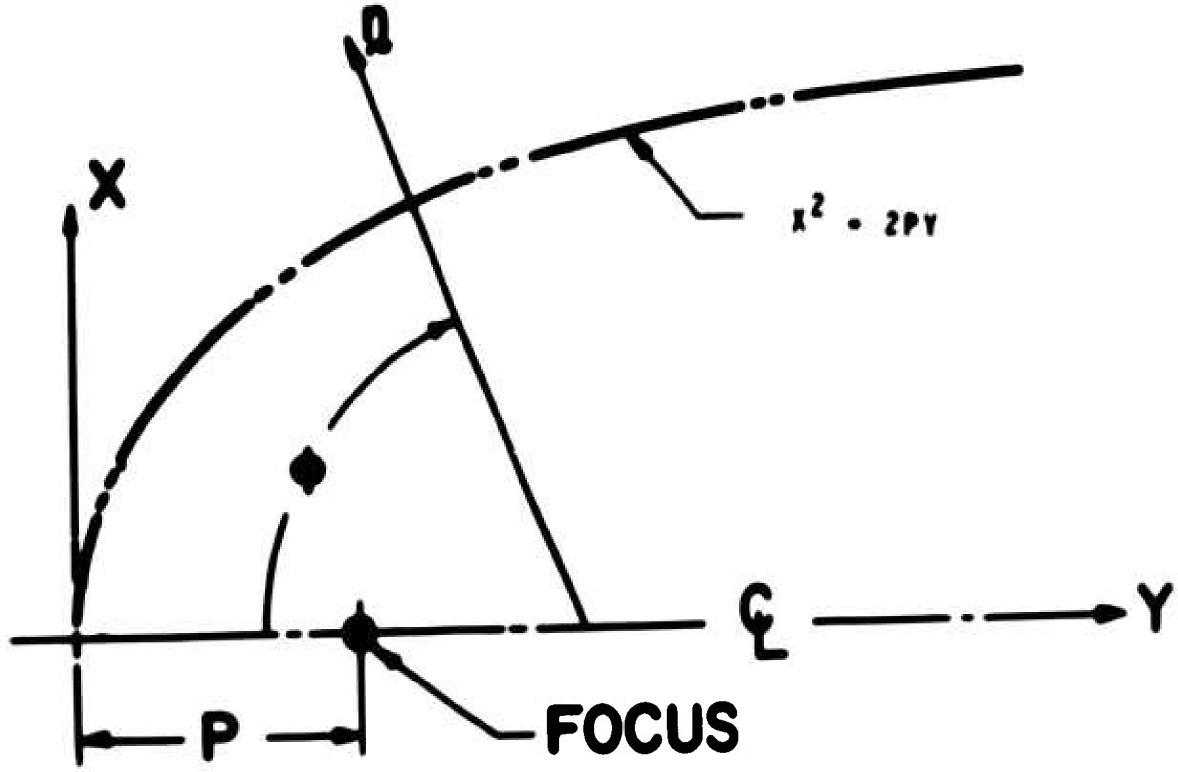

Comments:

1. Radii of curvature: $R_\phi = R_\theta = R$

2. Meridional coordinate = $\phi$ in radians

3. If normal points toward axis of symmetry, R must be inserted negative.

Paraboloidal Reference Surface




Comments:

1. Radii of curvature:

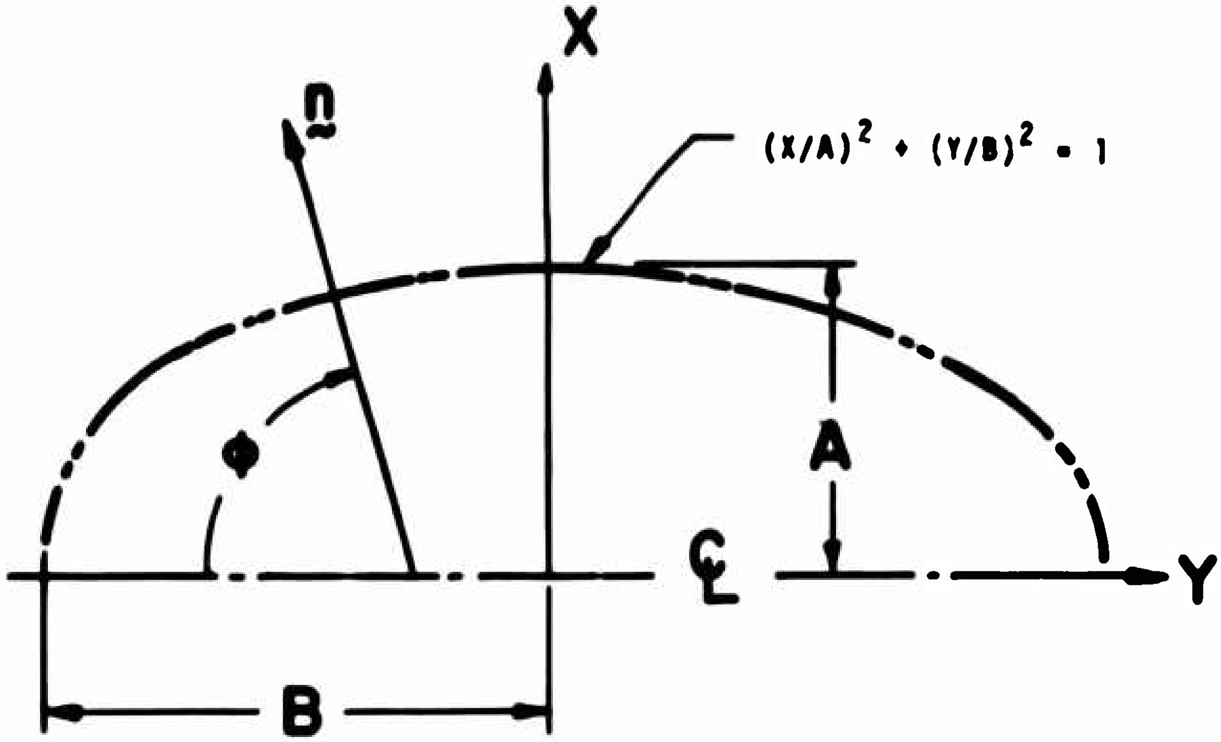
$$1/R_\phi = \cos^3\phi/2P$$

$$1/R_\theta = \cos\phi/2P$$

2. Meridional coordinate = $\phi$ in radians.

3. If normal points toward axis of symmetry, P must be inserted negative.

## Ellipsoidal Reference Surface




Comments:

1. Radii of curvature:

$$1/R_\phi = (A/B^2)[\sin^2\phi + (B/A)^2\cos^2\phi]^{3/2}$$

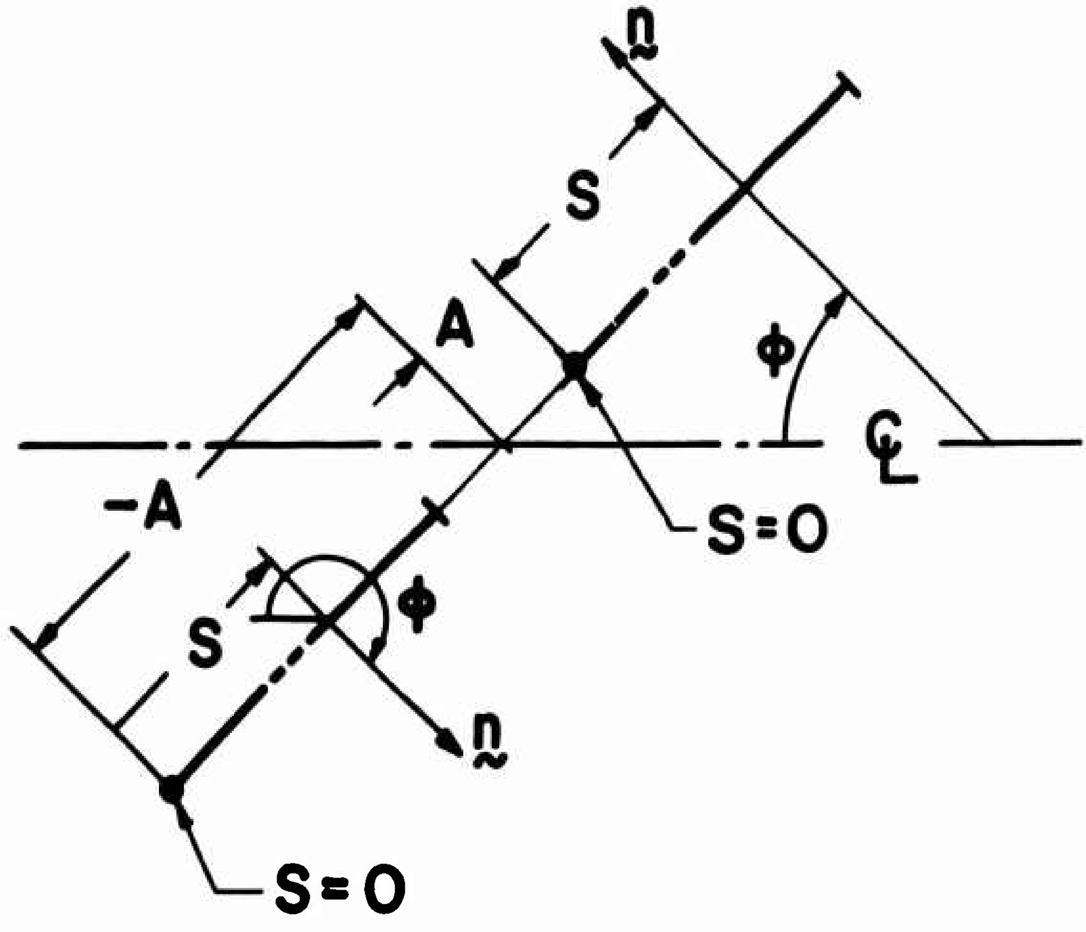
$$1/R_\theta = (1/A)[\sin^2\phi + (B/A)^2\cos^2\phi]^{1/2}$$

2. Meridional coordinate = $\phi$ in radians.

3. Note that A is the axis of the ellipse perpendicular and B parallel to the axis of symmetry.

4. If normal points toward axis of symmetry, A must be inserted negative.

Conical Reference Surface




Comments:

1. Radii of curvature:

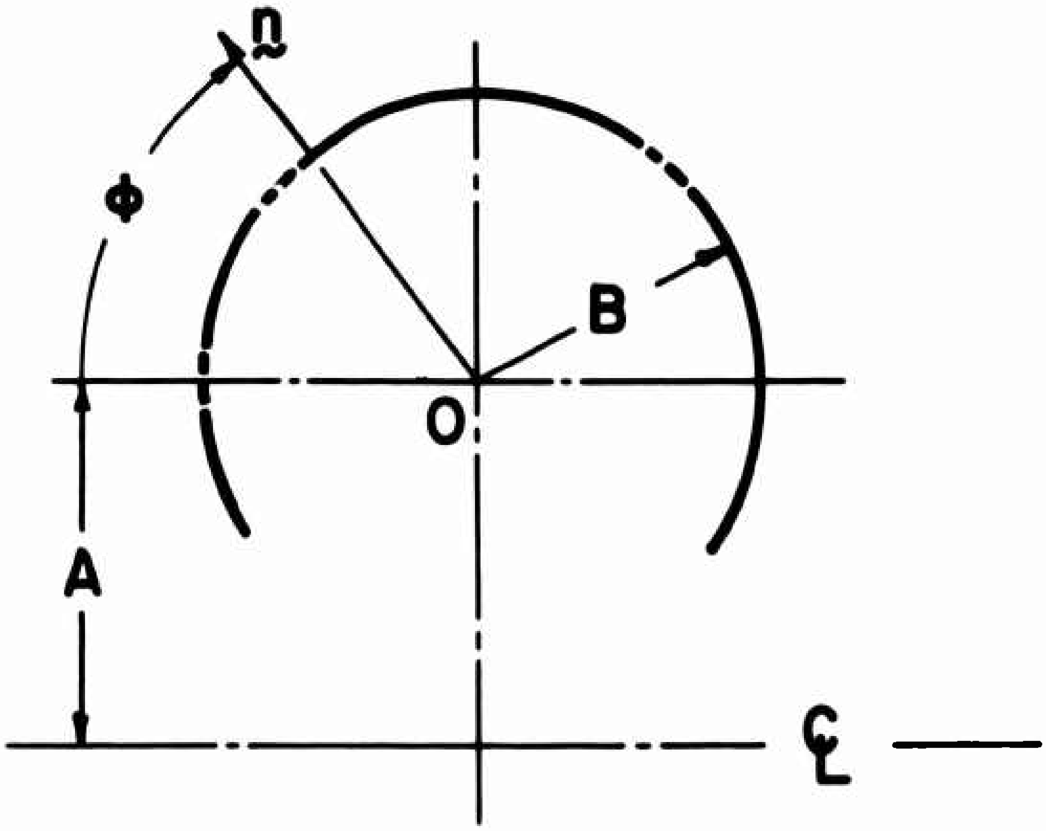
$$1/R_\phi = 0 \qquad 1/R_\theta = (A + S)\cos\phi$$

2. Meridional coordinate = distance S along generator

3. A is measured from apex, and it sets the origin of S (at which S = 0.0).

4. A must be inserted negative when integration is toward the apex.

Toroidal Reference Surface




Comments:

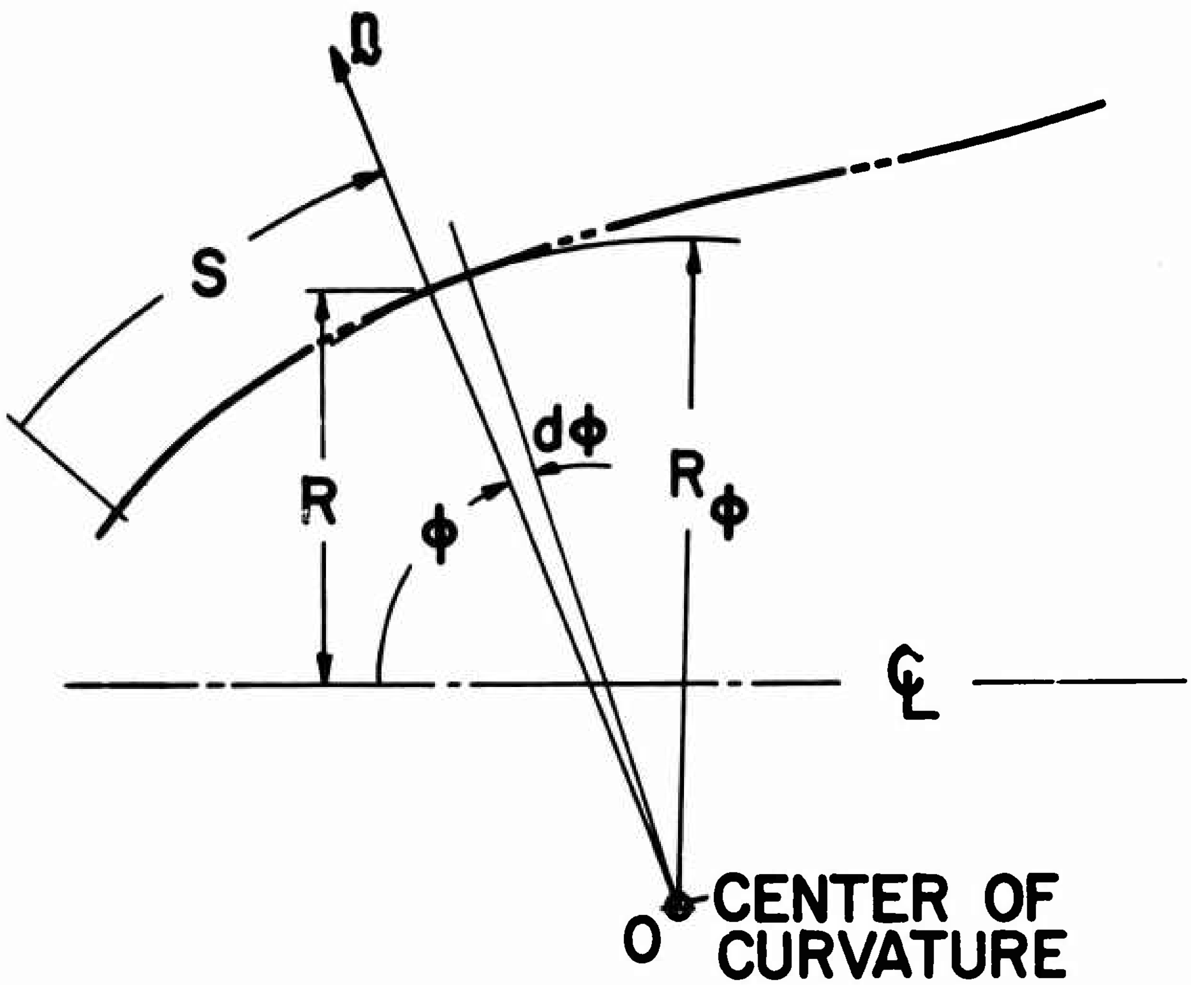
1. Radii of curvature: $R_\phi = B$

   $R_\theta = (A + B\sin\phi)/\sin\phi$

2. Meridional coordinate = $\phi$ in radians.

3. If normal points toward meridional center of curvature (point O), then B must be inserted negative.

## General Reference Surface



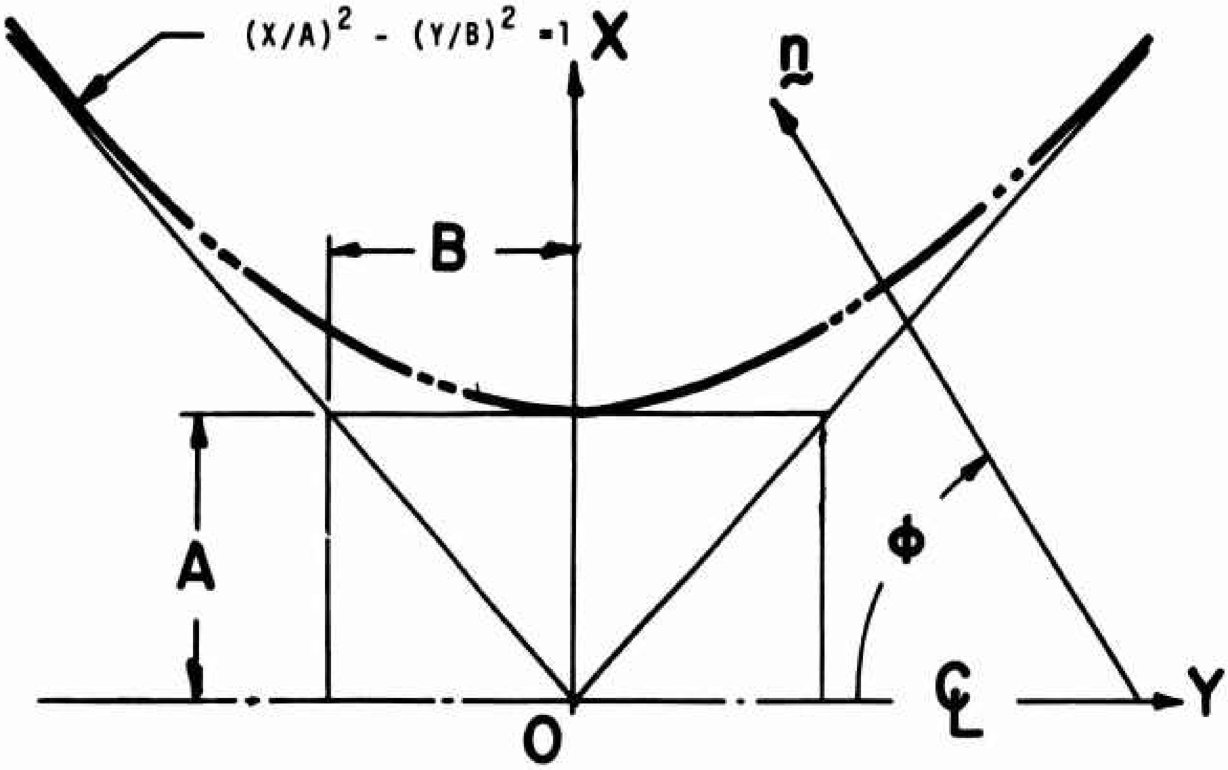

Comments:

1. Radii of curvature: $R_\phi$, $R_\theta$ [illegible] sin$\phi$

2. Meridional coordinate = arclength S along meridian.

3. If normal points toward meridional center of curvature (point O), $1/R_\phi$ must be inserted negative.

## Hyperboloidal Reference Surface




1. Radii of curvature:

$$1/R_\phi = -(A/B^2)[\sin^2\phi - (B/A)^2\cos^2\phi]^{3/2}$$

$$1/R_\theta = (1/A)[\sin^2\phi - (B/A)^2\cos^2\phi]^{1/2}$$

2. Meridional coordinate = $\phi$ in radians.

3. If normal points toward axis of symmetry, A must be inserted negative.

## 4. Interpretation of Results

### 1. Static Stress Analysis

Once the shell geometry is selected and the static external loads defined by their Fourier series coefficients, the solution, as obtained by the SP, predicts accurately the deflection of the shell and the resultant forces and couples which keep the RS in equilibrium. The only limitations are:

1. The yield limit of the material should not be exceeded anywhere in the shell.
2. The deflections should not be so large that a a linear theory is not applicable.

Both of these limitations can and should be checked after the results for a case are obtained. The proximity of the stress state to the yield point can be estimated with the use of some yield condition. Similarly, the normal deflection, w, should be smaller than, say, one thickness. If these two limitations are violated, the results are still useful as far as the trends of the stress distributions and deflections are concerned, but the possible errors in the results should be appreciated.

Caution should also be exercised when the stresses at certain points are interpreted. It should be borne in mind that shell theory ensures the equilibrium of the RS in terms of the stress resultants, but not in terms of the stresses. The stresses are calculated after the resultants are obtained by assuming a linear distribution through the thickness. Consequently, the stresses at points of discontinuous thickness or normal (points A, B, and C in Figure 1) cannot be accurately obtained by shell theory, and should be interpreted accordingly. Only a three-dimensional elasticity ( or perhaps plasticity) theory can provide the precise stress distribution in such cases.

In summary, the results given by the SP should be regarded as accurate in most cases. They have been verified experimentally for many shell configurations. In cases where their accuracy can be questioned, the simple fact is that better solutions are usually not available. If they are available, they should be used. If they are not, however, it is much better to have the results of the linear, elastic analysis of shell theory, than no results at all.

## 2. Free Vibration Analysis

Assuming that the object of the free vibration analysis, with or without prestress, is to obtain the resonant frequencies and mode shapes of the structure, the limitations with respect to the yield limit and the amplitude of the deflections are not significant in the free vibration analysis. For this reason, the free vibration results, as given by the AEP and NEP, should be regarded as accurate with even more confidence than those of the SP.

However, another limitation appears in free vibration problems, and that is with respect to the frequency. The classical theory of shells, which is used for the analysis in this report, neglects the transverse shear strains, and consequently the theory is only applicable in the low frequency range which is well below* the lowest antisymmetric thickness-shear mode of an infinite plate, given approximately by

$$\omega_s = c/10h$$

in cps, where c is the speed of sound of the material, defined by

$$c = \sqrt{E/\rho}$$

E is Young's modulus, $\rho$ is the mass density, and h is the thickness of the shell.

Our previous investigations** have shown that the classical theory predicts natural frequencies with great accuracy in the low frequency range, but the error reaches a maximum of about 5% at the frequency of $\omega_s/20$. If a 5% error is regarded as the maximum allowable error, then the free vibration results, given by the computer programs,

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*See A. Kalnins, "Dynamic Problems of Elastic Shells," Applied Mechanics Reviews, v. 18, 1965, pp. 867-871.

**See H. Kraus, Thin Elastic Shells, J. Wiley and Sons, New York, 1967, p. 349.

should be accurate in the frequency range

$$o < \omega < c/200h$$

For steel c=200,000 in/sec, so that the upper limit for a 0.5 in thick shell is 4000 cps, which is probably well above the frequency range of most applications. If higher frequencies are needed, a shear theory of shells must be employed, which takes the transverse shear strain effects into account. The governing equations of such a theory are well known, and there is no fundamental difficulty in applying the multisegment, direct numerical integration technique to the free vibration analysis by such a higher order theory. This was not done here simply because only in rare cases the frequency limit of

$$\omega = c/200h$$

would have to be exceeded.

When this frequency limit is observed, the free vibration results should be accepced with confidence. Natural frequencies, at least in the lower frequency range, have been experimentally verified on many occasions, including an experimental program conducted by the author at the Shell Vibration Laboratory of Lehigh University. One shell configuration which was tested is described under the test cases later in this Chapter. It was found that in the lower frequency range any natural frequency predicted by the computer program could be verified experimentally within about ±1 cps.

### 3. Stability Analysis

While the static and free vibration analyses of elastic shells have reached a definitive stage, where the meaning of the results is fully understood, the same cannot as yet be said about the stability analysis. It should be remembered that the stability analysis used in this report is based on the classical approach of solving a linear eigenvalue problem. The lowest eigenvalue, designated the classical buckling load, represents a prestressed state at which another infinitesimally different equilibrium state is possible. The computer programs for the stability analysis can find such a classical buckling load and the corresponding infinitesimal superimposed state, called the classical buckling mode, for an arbitrary shell of revolution. The purpose of this discussion is to give some indication on the meaning of such information to a shell analyst.

The uncertainty of the meaning of the classical buckling loads becomes evident when one begins to compare them with experimental results. It has been verified by many experiments that some shells of revolution, such as a cylindrical shell under axial load and a closed spherical shell under external pressure, can sustain only a fraction of the theoretically calculated classical buckling load. On the other hand, for other shells of revolution, such as ellipsoidal and cylindrical shells under external pressure, the classical buckling loads and the experimental pressures can agree very well.

It is clear from such experimental evidence that the mere knowledge of the classical buckling load and its corresponding mode is not sufficient to tell an analyst at what load will an arbitrary shell of revolution collapse. Further information is obviously necessary, which should provide an estimate on the expected degree of agreement between the classical buckling load and the actual collapse load of the structure.

The answer to our problem lies in the behavior of the shell after the classical buckling load has been reached. The various possibilities are shown in Figure 10. The branch OA represents symbolically the prestressed state where the abscissa is some deflection parameter. Whether OA is a straight line, which means that the prestressed state is linear, or a curve, is irrelevant for our discussion.

At point A the classical buckling load is reached, at which another load-deflection branch is attached. The deflection parameter of the new branch will be called the <u>buckling amplitude</u> and designated by $\delta$, and it need not be the same deflection parameter which was used to plot the prestressed state from O to A. We shall say that at A we

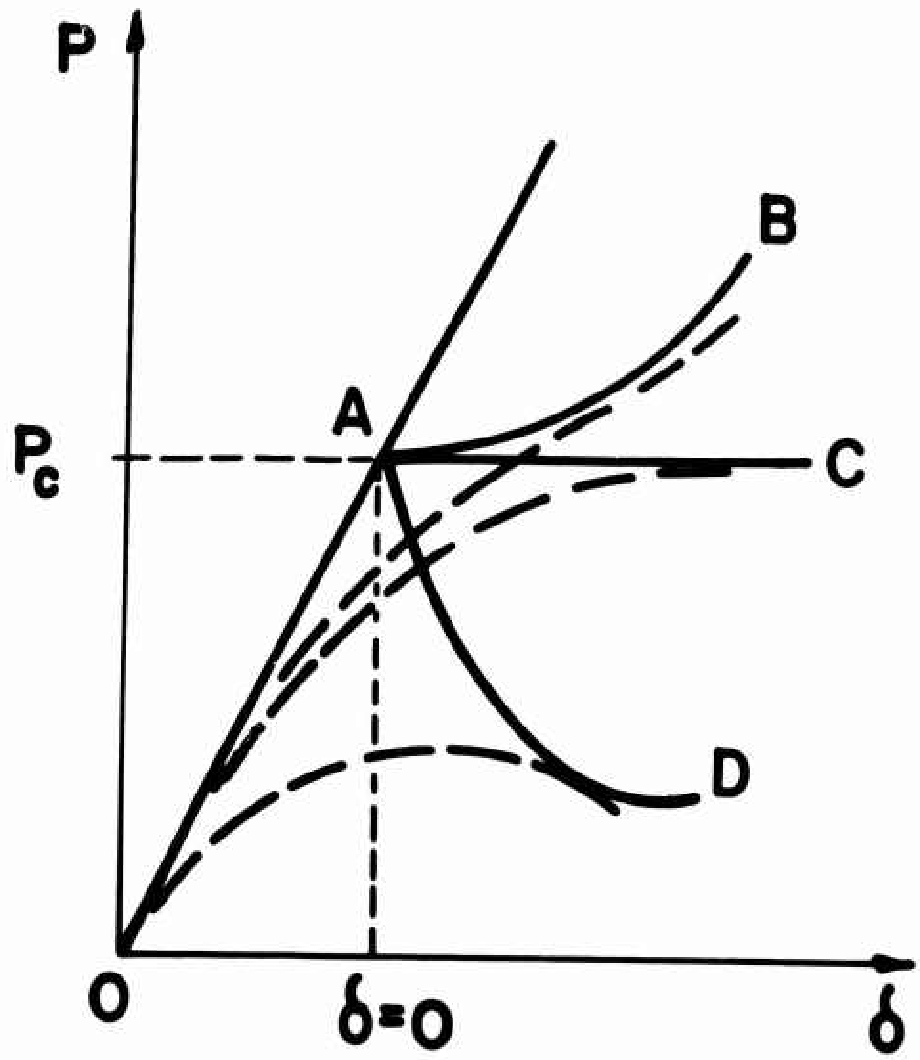


FIGURE 10. HYPOTHETICAL LOAD-DEFLECTION CURVES FOR A SHELL

have $\delta = 0$. Since our interest begins at A, the abscissa will be labeled $\delta$.

As shown in Figure 10, the load-buckling amplitude curve, after point A, can go either up, down, or stay horizontal. Because of imperfections in every manufactured shell, the actual P-$\delta$ curves will be more like the dashed curves in Figure 10, rather than the full curves.

If the postbuckling path is AD, then it is reasonable to expect that because of imperfections point A will never be reached, and that the collapse load of the actual shell will be well below the classical buckling load. For the path AC the collapse load may approach the classical buckling load, while for path AB it may surpass it. Such interpretation has been shown to be correct for an axially compressed cylindrical shell (path AD), column (AC), and a plate with restrained edges (AB). It seems reasonable to extend such reasoning to an arbitrary shell of revolution subjected to arbitrary loads.

If the above interpretation of the deflection of the shell is accepted, then, in addition to the classical buckling load and its mode, we must find some way to determine, at least in the initial stages, the character of the postbuckling behavior of the shell. Only then will the computer programs, described in this report, become meaningful tools for the stability analysis of a shell.

The additional information which could provide the analyst with a criterion on the accuracy of the classical buckling load could possibly come from three sources:

1. The classical buckling mode.
2. Initial postbuckling analysis.
3. Complete nonlinear postbuckling analysis.

The first source represents the simplest way to make an estimate on the accuracy of the classical buckling load, because the buckling modes, as printed out by the computer programs, contain all the fundamental variables and stresses of the superimposed state. Since theoretically buckling must begin at point A, it must be true that the very initial postbuckling state (i.e., prestressed plus superimposed state), must have some bearing on what happens to the shell next.

A procedure by which the classical buckling mode could be utilized for the desired accuracy estimate has been discussed by Gerard and Becker*. Their argument goes approximately as follows. Since an unstable state in a shell is caused by compressive membrane stresses, then if the superimposed state adds more compressive membrane stresses, the P-$\delta$ curve should be of the type AD in Figure 10, and poor agreement between the classical buckling load and the collapse load is expected. If, on the other hand, the

*G. Gerard and H. Becker, "Handbook of Structural Stability. Part III - Buckling of Curved Plates and Shells", NACA TN3783, August 1957, pp. 9-11.

superimposed state consists of tensile stresses, then the classical buckling load can be regarded as the collapse load or can be even exceeded.

While this argument seems very logical, it still requires further clarification. The superimposed state of the axially compressed cylindrical shell is a nonsymmetric one, where all variables are multiplied by cos$n\theta$ or sin$n\theta$, and n depends on the dimensions of the shell. This means that the positive and negative signs of the stresses of the superimposed state vary along the circumference with a period of $2\pi a/n$. It is not clear to the author how one can apply such an alternating stress field to Gerard and Becker's argument. It could be that this argument does not really apply to the stresses of the superimposed state of the classical buckling mode, but requires instead the knowledge of the stresses in the postbuckling range. If this is so, then their argument would require the information regarding the postbuckling range, which will be discussed next.

The second source from which an estimate on the accuracy of the classical buckling load could be obtained is concerned with the postbuckling region immediately following the bifurcation point (point A in Figure 10). Such initial postbuckling analysis has been proposed by Koiter* and

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*W. T. Koiter, "Elastic Stability and Postbuckling Behavior," in Nonlinear Problems, R. E. Langer, editor, University of Wisconsin Press, Madison, Wis., 1963, p. 257.

developed further by Budiansky and Hutchinson*. Koiter's theory is capable of predicting the slope of the P-$\delta$ curve in the postbuckling region at point A in Figure 10, as well as the imperfection sensitivity of the shell. When applied to an arbitrary shell of revolution, this approach seems to be the most promising one which could put the stability analysis on a much sounder foundation.

The third approach of dealing with our problem would require a complete nonlinear analysis of the deflection of the shell beyond the point A in Figure 10. While such an analysis would, of course, provide all the information that we are seeking, it can be carried out at this time only for a few simple configurations. Therefore, it would not be wise to recommend it for the stability analysis for an arbitrary shell of revolution, simply because it is too difficult, and because the computational difficulties could obscure the type of information which is sought.

In summary, the obtaining of the classical buckling load and mode by means of the computer programs should be regarded as the first step in the stability analysis of an arbitrary shell of revolution. Additional information, which is not included in this report, is needed for an extimate of the proximity of the classical buckling load

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*B. Budiansky and J. Hutchinson, Proc. 11th International Cong. Appl. Mech., Springer-Verlag, Berlin, 1965, pp. 636-651.

to the actual collapse load of the shell. Author believes that such information can be best obtained by following Koiter's theory of the initial postbuckling analysis of a shell. Until such time when a detailed procedure for the initial postbuckling analysis of an arbitrary shell of revolution is available, the user of the present computer programs must use them with the knowledge that the predicted buckling load may or may not represent the collapse load of the shell.

## 5. Test Cases Run by the Programs.

### 1. Static Program

#### A. Seven-Part Composite Shell.

The meridional profile of this shell is shown in Figure 11. The purpose of this case is to show the proper selection of $\phi$, normal, direction index, data for variable thickness and loads, layers, ring loads, and rotated boundary conditions. Two subcases are considered: for NX=0 and NX=2.

A recommended check for all static problems with NX=0 is the balance of the total applied loads with the edge reactions in the parallel direction to the axis of symmetry. The only edge load in this direction is at the Starting Edge, and a net force is produced by the Ring Load and pressure. Such a gross equilibrium check shows that the stress resultant at the Starting Edge should be $N_\phi = 50.219$, which is verified by the program.

The data sheets and the appropriate output for Case A follow.

TEST CASE FOR STATIC SHELL PROGRAM

SEVEN PART COMPOSITE SHELL (SEE FIG. 11)

```
   7      2      1      2
  0.0        177.617      210.0
   0.0      2.0            8      2      8      0      2
  0.075     0.0          2.0          90.0
  28.0      0.3
  14.0      0.4
  -0.025       0.0       0.025                                  7
Z1-P1     2
 0.0          -0.025    2.0        -0.025
Z2-P1     2
  0.0         0.0        2.0         0.0
Z3-P1     2
  0.0       0.05         2.0          0.025
   0.0     1.0             4      2      2      0      1
  0.05      2.0         90.0
  28.0      0.3
  -0.025      0.025
  1.5708   2.6180          6      2      7      1      1
 0.02       1.5         -0.5         -1.0
  28.0      0.3
 -0.01      0.01
  0.0       1.5            8      2      6      2      1
  0.02        210.0     -2.0207
  28.0      0.3
 -0.01      0.01
   0.0     1.0             4      2      2      0      1
  0.05      2.0         90.0
  28.0      0.3
  -0.025      0.025
  1.5708   2.0944          8      2      5      0      1
 0.05       2.0         1.0          1.0
  28.0      0.3
  -0.025      0.025
 2.0944       3.10         8      2      5      0      1
 0.05       2.0         1.0          1.0
  28.0      0.3
  -0.025      0.025
   0      0
   2                3                5                7
   2                3                5                7
   2                3                5                7
  0.0         50.0       0.0          0.0
  10.0                                                           1
   1                            3
PRESS     2
  0.0         20.0       2.0          0.0



 -5.0

 -5.0

  5.0

  5.0
```

```
 5.0
 2     0
 2                3                5                7
 2                3                5                7
 2                3                5                7



-20.0

-20.0

 20.0

 20.0

 20.0
 0
```

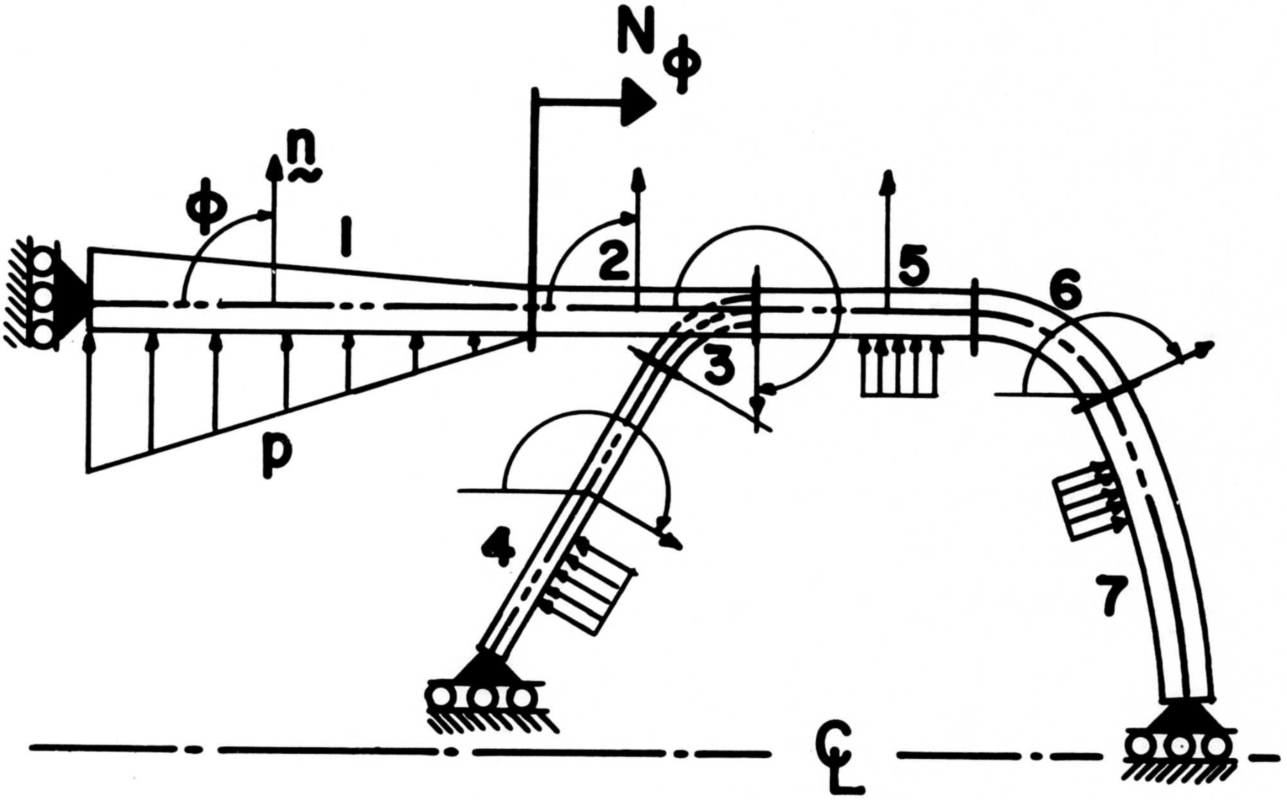


FIGURE 11. SEVEN-PART SHELL USED IN TEST CASE A

## B. Conical Shell

This is a simple conical shell with a 30° vertex angle, as shown in Figure 12. The direction of integration is toward the axis of symmetry, because the Starting Edge must be restrained from the motion as a rigid body along the axis of symmetry.

The purpose of this case is to illustrate the use of rotated surface loads (subcase No. 1) and a spinning shell (subcase No. 2). For each of these subcases solutions are available for a check of the results given by the program.

### a. Subcase No. 1

This is the solution for an aluminum, 0.01 in thick conical shell, subjected to its own weight and resting on a smooth table. The weight per unit volume of aluminum is $\gamma$=0.0975 lbs/in$^3$, and therefore the surface load, measured as weight per unit area of the reference surface, is $\gamma h$=0.000975 lbs/in$^2$. Since this surface load remains parallel to the axis of symmetry, we must have INORM=1 on the Load Parameter Card.

Except in the vicinity of the edges, the membrane solution for this case is verified by the program.

### b. Subcase No. 2

This subcase is intended to illustrate the analysis of a shell of revolution which is spinning about its axis of symmetry at a given RPM. The same aluminum conical shell is used. The main thing that must be remembered for a spinning shell is that INORM=1 and that now the mass density is given by

$$\rho = \gamma/g$$

where $\gamma$ is the weight density, and g is the acceleration imparted by gravity. Assuming that g=385.92 in/sec$^2$, the mass density $\rho$ on the Elastic Parameter Card must be input as $\rho$=0.0002526 lb sec$^2$/in$^4$.

If the shell is spinning with 100 RPM, the membrane theory of shells predicts $N_\theta$=0.00062 at the Starting Edge and $N_\theta$=0.000069 at the Final Edge. These values are verified by the program.

Data sheets and the appropriate output for case B follow.

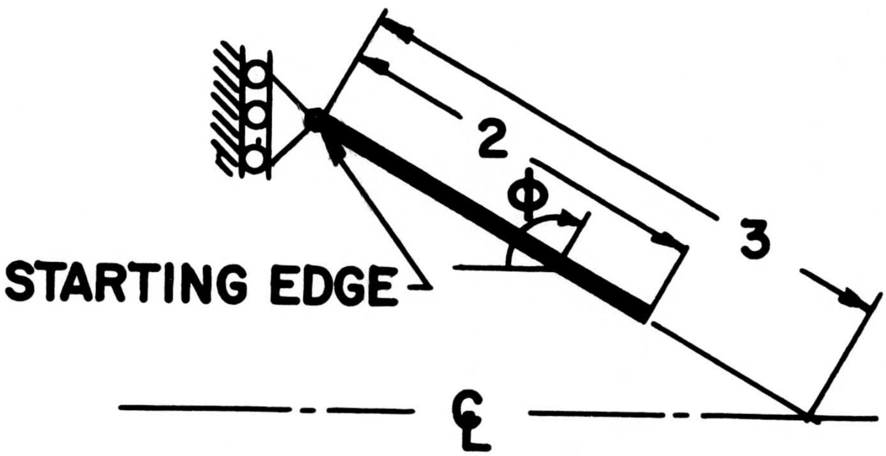


FIGURE 12. CONICAL SHELL USED IN TEST CASE B

```
     CONICAL SHELL                  (SEE FIG. 12)

          SUBCASE (1) - ROTATED SURFACE LOADS
          SUBCASE (2) - SPINNING SHELL


    1     2     0     3
   30.0
    0.0      2.0        10     3     6     0     1
  0.01      120.0      -3.0
10300000.     0.3                           0.000012440.000012440.00025264
  -0.005    0.005
    0     0
    2               3              6              7
    2               4              6              8

 0.000975                                                   1
    0     0
    2 -100.0        3              6              7
    2               4              6              8


    0     0
    2               3              6              7
    2               4              6              8

     0.0                                                    1    100.0
```

```
STRESS ANALYSIS PROGRAM OF SHELLS OF REVOLUTION UNDER STATIC LOADS, BY A. KALNINS, LEHIGH UNIV, BETHLEHEM, PA
         WRIGHT PATTERSON AIR FORCE BASE FLIGHT DYNAMICS LABORATORY VERSION, 22 JULY 1968

          STATIC ANALYSIS     PARTS=  7  BRANCHES=  1  NUMBER OF SUBCASES=  2

ANGLES OF ROTATION OF BOUNDARY CONDITIONS ARE ALFL= 0.          ALFRS= 0.17762E 03 0.21000E 03

          PART NO  1

SI= 0.              SX= 0.20000E 01   IPAR=  8   ING=  2    SHELL TYPE 8   NTP= 0   LAYERS MLY= 2

GENERAL SHELL NO  8  H= 0.75000E-01 1/RFI= 0.            R= 0.20000E 01 FI=    90.000 DEG

          Z1-P1  LINEAR FUNCTION GENERATOR NO.   7 FROM   2 POINTS

Y COORDINATES       -0.02500  -0.02500

X COORDINATES        0.        2.00000

          Z2-P1  LINEAR FUNCTION GENERATOR NO.   8 FROM   2 POINTS

Y COORDINATES        0.        0.

X COORDINATES        0.        2.00000

          Z3-P1  LINEAR FUNCTION GENERATOR NO.   9 FROM   2 POINTS

Y COORDINATES        0.05000   0.02500

X COORDINATES        0.        2.00000

LAYER NO  1  FROM Z=-0.25000E-01  TO Z= 0.
     CONSISTS OF ISOTROPIC MATERIAL, YOUNGS MODULUS E= 0.28000E 02  POISSONS RATIO NU= 0.30000E-00
     COEFFICIENTS OF THERMAL EXPANSION  AFI=-0.           ATHETA=-0.           MASS DENSITY RHO=-0.

LAYER NO  2  FROM Z= 0.          TO Z= 0.25000E-01
     CONSISTS OF ISOTROPIC MATERIAL, YOUNGS MODULUS E= 0.14000E 02  POISSONS RATIO NU= 0.40000E-00
     COEFFICIENTS OF THERMAL EXPANSION  AFI=-0.           ATHETA=-0.           MASS DENSITY RHO=-0.


          PART NO  2

SI= 0.              SX= 0.10000E 01   IPAR=  4   ING=  2    SHELL TYPE 2   NTP= 0   LAYERS MLY= 1

CYLINDRICAL SHELL NO  2    H= 0.50000E-01    R= 0.20000E 01  PHI=    90.000  DEGREES

LAYER NO  1  FROM Z=-0.25000E-01  TO Z= 0.25000E-01
     CONSISTS OF ISOTROPIC MATERIAL, YOUNGS MODULUS E= 0.28000E 02  POISSONS RATIO NU= 0.30000E-00
     COEFFICIENTS OF THERMAL EXPANSION  AFI=-0.           ATHETA=-0.           MASS DENSITY RHO=-0.


          PART NO  3

SI= 0.15708E 01   SX= 0.26180E 01   IPAR=  6   ING=  2    SHELL TYPE 7   NTP= 1   LAYERS MLY= 1

TOROIDAL SHELL NO  7     H= 0.20000E-01    A= 0.15000E 01    B=-0.50000E 00  DIRECTN=-1.

LAYER NO  1  FROM Z=-1.00000E-02  TO Z= 1.00000E-02
     CONSISTS OF ISOTROPIC MATERIAL, YOUNGS MODULUS E= 0.28000E 02  POISSONS RATIO NU= 0.30000E-00
     COEFFICIENTS OF THERMAL EXPANSION  AFI=-0.           ATHETA=-0.           MASS DENSITY RHO=-0.


          PART NO  4
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SI= 0.          SX= 0.15000E 01   IPAR=  5   ING=  2    SHELL TYPE 6   NTP= 2   LAYERS MLY= 1

CONICAL SHELL NO  6    H=  0.020000  PHI=   210.000 DEGREES    A=-0.20207E 01

LAYER NO  1  FROM Z=-1.00000E-02  TO Z= 1.00000E-02
     CONSISTS OF ISOTROPIC MATERIAL, YOUNGS MODULUS E= 0.28000E 02  POISSONS RATIO NU= 0.30000E-00
     COEFFICIENTS OF THERMAL EXPANSION  AFI=-0.          ATHETA=-0.          MASS DENSITY RHO=-0.

          PART NO  5

SI= 0.          SX= 0.10000E 01   IPAR=  4   ING=  2    SHELL TYPE 2   NTP= 0   LAYERS MLY= 1

CYLINDRICAL SHELL NO  2    H= 0.50000E-01    R= 0.20000E 01  PHI=    90.000  DEGREES

LAYER NO  1  FROM Z=-0.25000E-01  TO Z= 0.25000E-01
     CONSISTS OF ISOTROPIC MATERIAL, YOUNGS MODULUS E= 0.28000E 02  POISSONS RATIO NU= 0.30000E-00
     COEFFICIENTS OF THERMAL EXPANSION  AFI=-0.          ATHETA=-0.          MASS DENSITY RHO=-0.

          PART NO  6

SI= 0.15708E 01   SX= 0.20944E 01   IPAR=  8   ING=  2    SHELL TYPE 5   NTP= 0   LAYERS MLY= 1

ELLIPSOIDAL SHELL NO  5     H= 0.50000E-01    A= 0.20000E 01    B= 0.10000E 01  DIRECTN= 1.

LAYER NO  1  FROM Z=-0.25000E-01  TO Z= 0.25000E-01
     CONSISTS OF ISOTROPIC MATERIAL, YOUNGS MODULUS E= 0.28000E 02  POISSONS RATIO NU= 0.30000E-00
     COEFFICIENTS OF THERMAL EXPANSION  AFI=-0.          ATHETA=-0.          MASS DENSITY RHO=-0.

          PART NO  7

SI= 0.20944E 01   SX= 0.31000E 01   IPAR=  8   ING=  2    SHELL TYPE 5   NTP= 0   LAYERS MLY= 1

ELLIPSOIDAL SHELL NO  5     H= 0.50000E-01    A= 0.20000E 01    B= 0.10000E 01  DIRECTN= 1.

LAYER NO  1  FROM Z=-0.25000E-01  TO Z= 0.25000E-01
     CONSISTS OF ISOTROPIC MATERIAL, YOUNGS MODULUS E= 0.28000E 02  POISSONS RATIO NU= 0.30000E-00
     COEFFICIENTS OF THERMAL EXPANSION  AFI=-0.          ATHETA=-0.          MASS DENSITY RHO=-0.
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STRESS ANALYSIS PROGRAM OF SHELLS OF REVOLUTION UNDER STATIC LOADS, BY A. KALNINS, LEHIGH UNIV, BETHLEHEM, PA
WRIGHT PATTERSON AIR FORCE BASE FLIGHT DYNAMICS LABORATORY VERSION, 22 JULY 1968

                    SUBCASE NO  1   FOR FOURIER HARMONIC COS 0 THETA

BOUNDARY CONDITIONS AT STARTING EDGE        2-0.         3-0.         5-0.         7-0.
BOUNDARY CONDITIONS AT FINAL    EDGE        2-0.         3-0.         5-0.         7-0.
BOUNDARY CONDITION AT BRANCH EDGE NO  1     2-0.         3-0.         5-0.         7-0.

                LOADS FOR PART NO  1  SUBCASE NO  1

RING LOADS AT END OF THIS PART ARE   Q= 0.          NPHI= 0.50000E 02  MPHI= 0.          N= 0.
SURFACE AND TEMP LOADS ARE  P= 0.10000E 02  PFI=-0.          PTHETA=-0.          TL=-0.          TU=-0.
VARIABLE LOAD PARAMETERS ARE    1   -0   -0   -0   -0
      PRESS  LINEAR FUNCTION GENERATOR NO.   3 FROM   2 POINTS
Y COORDINATES     20.00000   0.
X COORDINATES      0.         2.00000

                LOADS FOR PART NO  2  SUBCASE NO  1

RING LOADS AT END OF THIS PART ARE   Q=-0.          NPHI=-0.          MPHI=-0.          N=-0.
SURFACE AND TEMP LOADS ARE  P=-0.          PFI=-0.          PTHETA=-0.          TL=-0.          TU=-0.

                LOADS FOR PART NO  3  SUBCASE NO  1

RING LOADS AT END OF THIS PART ARE   Q=-0.          NPHI=-0.          MPHI=-0.          N=-0.
SURFACE AND TEMP LOADS ARE  P=-0.50000E 01  PFI=-0.          PTHETA=-0.          TL=-0.          TU=-0.

                LOADS FOR PART NO  4  SUBCASE NO  1

RING LOADS AT END OF THIS PART ARE   Q=-0.          NPHI=-0.          MPHI=-0.          N=-0.
SURFACE AND TEMP LOADS ARE  P=-0.50000E 01  PFI=-0.          PTHETA=-0.          TL=-0.          TU=-0.

                LOADS FOR PART NO  5  SUBCASE NO  1

RING LOADS AT END OF THIS PART ARE   Q=-0.          NPHI=-0.          MPHI=-0.          N=-0.
SURFACE AND TEMP LOADS ARE  P= 0.50000E 01  PFI=-0.          PTHETA=-0.          TL=-0.          TU=-0.

                LOADS FOR PART NO  6  SUBCASE NO  1

RING LOADS AT END OF THIS PART ARE   Q=-0.          NPHI=-0.          MPHI=-0.          N=-0.
SURFACE AND TEMP LOADS ARE  P= 0.50000E 01  PFI=-0.          PTHETA=-0.          TL=-0.          TU=-0.

                LOADS FOR PART NO  7  SUBCASE NO  1
```

```
RING LOADS AT END OF THIS PART ARE   Q=-0.          MPHI=-0.          NPHI=-0.          N=-0.
SURFACE AND TEMP LOADS ARE  P= 0.50000E 01  PFI=-0.          PTHETA=-0.          TL=-0.          TU=-0.
```

STRESS ANALYSIS PROGRAM OF SHELLS OF REVOLUTION UNDER STATIC LOADS, BY A. KALNINS, LEHIGH UNIV, BETHLEHEM, PA
WRIGHT PATTERSON AIR FORCE BASE FLIGHT DYNAMICS LABORATORY VERSION, 22 JULY 1968

STATIC SOLUTION AT POINTS ALONG MERIDIAN FOR WAVE NUMBER NX= 0

| S | W | Q | UPHI | NPHI | BPHI | MPHI | UTHETA | N | NTHETA | MTHETA |
|---|---|---|---|---|---|---|---|---|---|---|
| MAIN SHELL PART NO 1 | | | | | | | | | | |
| 0. | 0.25207E 02 | 0. | 0. | 0.50220E 02 | 0. | 0.38776E-00 | 0. | 0. | 0.35406E 02 | 0.29438E-00 |
| 0.12500 | 0.24933E 02 | -0.22093E-00 | 0.33642E 01 | 0.50220E 02 | 0.49977E 01 | 0.37262E-00 | 0. | 0. | 0.34943E 02 | 0.27357E-00 |
| 0.25000 | 0.23876E 02 | -0.33165E-00 | 0.68009E 01 | 0.50220E 02 | 0.12007E 02 | 0.33713E-00 | 0. | 0. | 0.33913E 02 | 0.24336E-00 |
| 0.25000 | 0.23878E 02 | -0.33137E-00 | 0.68008E 01 | 0.50220E 02 | 0.12000E 02 | 0.33718E-00 | 0. | 0. | 0.33915E 02 | 0.24338E-00 |
| 0.37500 | 0.21950E 02 | -0.36858E-00 | 0.10346E 02 | 0.50220E 02 | 0.18687E 02 | 0.29265E-00 | 0. | 0. | 0.32306E 02 | 0.20899E-00 |
| 0.50000 | 0.19264E 02 | -0.36531E-00 | 0.14024E 02 | 0.50220E 02 | 0.24040E 02 | 0.24670E-00 | 0. | 0. | 0.30235E 02 | 0.17449E-00 |
| 0.50000 | 0.19266E 02 | -0.36502E-00 | 0.14024E 02 | 0.50220E 02 | 0.24032E 02 | 0.24674E-00 | 0. | 0. | 0.30236E 02 | 0.17451E-00 |
| 0.62500 | 0.16004E 02 | -0.34534E-00 | 0.17850E 02 | 0.50220E 02 | 0.27923E 02 | 0.20227E-00 | 0. | 0. | 0.27853E 02 | 0.14251E-00 |
| 0.75000 | 0.12334E 02 | -0.32470E-00 | 0.21835E 02 | 0.50220E 02 | 0.30644E 02 | 0.16044E-00 | 0. | 0. | 0.25285E 02 | 0.11423E-00 |
| 0.75000 | 0.12335E 02 | -0.32441E-00 | 0.21835E 02 | 0.50220E 02 | 0.30638E 02 | 0.16048E-00 | 0. | 0. | 0.25286E 02 | 0.11425E-00 |
| 0.87500 | 0.83766E 01 | -0.31138E-00 | 0.25985E 02 | 0.50220E 02 | 0.32598E 02 | 0.12086E-00 | 0. | 0. | 0.22620E 02 | 0.90002E-01 |
| 1.00000 | 0.42076E 01 | -0.31028E-00 | 0.30307E 02 | 0.50220E 02 | 0.34019E 02 | 0.82140E-01 | 0. | 0. | 0.19911E 02 | 0.69541E-01 |
| 1.00000 | 0.42067E 01 | -0.31031E-00 | 0.30307E 02 | 0.50220E 02 | 0.34014E 02 | 0.82128E-01 | 0. | 0. | 0.19911E 02 | 0.69536E-01 |
| 1.12500 | -0.10010E-00 | -0.32246E-00 | 0.34807E 02 | 0.50220E 02 | 0.34740E 02 | 0.42714E-01 | 0. | 0. | 0.17206E 02 | 0.52340E-01 |
| 1.25000 | -0.44209E 01 | -0.34555E-00 | 0.39488E 02 | 0.50220E 02 | 0.34067E 02 | 0.10481E-02 | 0. | 0. | 0.14577E 02 | 0.37775E-01 |
| 1.25000 | -0.44220E 01 | -0.34561E-00 | 0.39488E 02 | 0.50220E 02 | 0.34061E 02 | 0.10322E-02 | 0. | 0. | 0.14577E 02 | 0.37770E-01 |
| 1.37500 | -0.85032E 01 | -0.37084E-00 | 0.44350E 02 | 0.50220E 02 | 0.30638E 02 | -0.43804E-01 | 0. | 0. | 0.12170E 02 | 0.25167E-01 |
| 1.50000 | -0.11893E 02 | -0.37708E-00 | 0.49380E 02 | 0.50220E 02 | 0.22698E 02 | -0.90913E-01 | 0. | 0. | 0.10236E 02 | 0.13961E-01 |
| 1.50000 | -0.11894E 02 | -0.37721E-00 | 0.49380E 02 | 0.50220E 02 | 0.22695E 02 | -0.90928E-01 | 0. | 0. | 0.10235E 02 | 0.13958E-01 |
| 1.62500 | -0.13935E 02 | -0.32429E-00 | 0.54552E 02 | 0.50220E 02 | 0.89826E 01 | -0.13568E-00 | 0. | 0. | 0.91294E 01 | 0.40570E-02 |
| 1.75000 | -0.13961E 02 | -0.14876E-00 | 0.59832E 02 | 0.50220E 02 | -0.87709E 01 | -0.16692E-00 | 0. | 0. | 0.92033E 01 | -0.31563E-02 |
| 1.75000 | -0.13961E 02 | -0.14876E-00 | 0.59832E 02 | 0.50220E 02 | -0.87710E 01 | -0.16692E-00 | 0. | 0. | 0.92033E 01 | -0.31561E-02 |
| 1.87500 | -0.11953E 02 | 0.22651E-00 | 0.65200E 02 | 0.50220E 02 | -0.20928E 02 | -0.16451E-00 | 0. | 0. | 0.10462E 02 | -0.26961E-02 |
| 2.00000 | -0.99893E 01 | 0.65049E 00 | 0.70729E 02 | 0.50220E 02 | -0.19624E 01 | -0.99756E-01 | 0. | 0. | 0.11837E 02 | 0.17965E-01 |
| MAIN SHELL PART NO 2 | | | | | | | | | | |
| 0. | -0.99893E 01 | 0.85049E 00 | 0.70729E 02 | 0.21981E-00 | -0.19833E 01 | -0.99757E-01 | 0. | 0. | -0.69266E 01 | -0.29927E-01 |
| 0.12500 | -0.80657E 01 | 0.44996E-00 | 0.70421E 02 | 0.21981E-00 | -0.23581E 02 | -0.19385E-01 | 0. | 0. | -0.55800E 01 | -0.58154E-02 |
| 0.25000 | -0.50207E 01 | 0.16830E-00 | 0.71061E 02 | 0.21981E-00 | -0.22736E 02 | 0.17856E-01 | 0. | 0. | -0.34485E 01 | 0.53567E-02 |
| 0.25000 | -0.50207E 01 | 0.16830E-00 | 0.71061E 02 | 0.21981E-00 | -0.22736E 02 | 0.17856E-01 | 0. | 0. | -0.34485E 01 | 0.53567E-02 |
| 0.37500 | -0.27360E 01 | 0.71510E-02 | 0.71150E 02 | 0.21981E-00 | -0.13186E 02 | 0.27777E-01 | 0. | 0. | -0.18493E 01 | 0.83330E-02 |
| 0.50000 | -0.17459E 01 | -0.82094E-01 | 0.71207E 02 | 0.21981E-00 | -0.29955E 01 | 0.22643E-01 | 0. | 0. | -0.11562E 01 | 0.67930E-02 |
| 0.50000 | -0.17459E 01 | -0.82094E-01 | 0.71207E 02 | 0.21982E-00 | -0.29955E 01 | 0.22643E-01 | 0. | 0. | -0.11562E 01 | 0.67930E-02 |
| 0.62500 | -0.18221E 01 | -0.15314E-00 | 0.71257E 02 | 0.21982E-00 | 0.32627E 01 | 0.79811E-02 | 0. | 0. | -0.12095E 01 | 0.23943E-02 |
| 0.75000 | -0.22488E 01 | -0.23862E-00 | 0.71314E 02 | 0.21982E-00 | 0.19880E 01 | -0.16301E-01 | 0. | 0. | -0.15082E 01 | -0.48904E-02 |
| 0.75000 | -0.22488E 01 | -0.23862E-00 | 0.71314E 02 | 0.21982E-00 | 0.19880E 01 | -0.16301E-01 | 0. | 0. | -0.15082E 01 | -0.48904E-02 |
| 0.87500 | -0.18334E 01 | -0.32966E-00 | 0.71372E 02 | 0.21982E-00 | -0.10955E 02 | -0.51996E-01 | 0. | 0. | -0.12175E 01 | -0.15599E 01 |
| 1.00000 | 0.11535E 01 | -0.35351E-00 | 0.71402E 02 | 0.21982E-00 | -0.39715E 02 | -0.96040E-01 | 0. | 0. | 0.87340E 00 | -0.288[illegible]-01 |
| BRANCH SHELL PART NO 3 | | | | | | | | | | |
| 1.57080 | -0.11532E 01 | 0.90621E 00 | -0.71402E 02 | 0.47459E 01 | -0.39715E 02 | -0.75945E-01 | 0. | 0. | 0.17467E 01 | -0.22784E-01 |
| 1.65807 | 0.97395E 01 | 0.68295E 00 | -0.70722E 02 | 0.46136E 01 | -0.16275E 03 | -0.41352E-01 | 0. | 0. | 0.45392E-00 | -0.12273E-01 |

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1.74533    0.24437E 02 0.49407E-00-0.68854E 02 0.48858E 01-0.22235E 03-0.15896E-01 0.        0.        -0.19377E 01-0.44071E-02

1.74533    0.24437E 02 0.49407E-00-0.68854E 02 0.48858E 01-0.22235E 03-0.15894E-01 0.        0.        -0.19378E 01-0.44066E-02
1.83260    0.40452E 02 0.35596E-00-0.65564E 02 0.49622E 01-0.23593E 03 0.24369E-02 0.        0.        -0.47537E 01 0.13059E-02
1.91987    0.55944E 02 0.26834E-00-0.60814E 02 0.50637E 01-0.21619E 03 0.15929E-01 0.        0.        -0.75128E 01 0.54793E-02

1.91987    0.55945E 02 0.26635E-00-0.60814E 02 0.50637E 01-0.21619E 03 0.15930E-01 0.        0.        -0.75129E 01 0.54796E-02
2.00713    0.69527E 02 0.22029E-00-0.54717E 02 0.52032E 01-0.17110E 03 0.26598E-01 0.        0.        -0.98755E 01 0.66706E-02
2.09440    0.80088E 02 0.19366E-00-0.47499E 02 0.53882E 01-0.10497E 03 0.35812E-01 0.        0.        -0.11597E 02 0.11250E-01

2.09440    0.80089E 02 0.19367E-00-0.47499E 02 0.53882E 01-0.10497E 03 0.35813E-01 0.        0.        -0.11597E 02 0.11251E-01
2.18167    0.86681E 02 0.16605E-00-0.39478E 02 0.56191E 01-0.20075E 02 0.44046E-01 0.        0.        -0.12497E 02 0.13326E-01
2.26893    0.88479E 02 0.11346E-00-0.31056E 02 0.58880E 01 0.81253E 02 0.50748E-01 0.        0.        -0.12454E 02 0.14707E-01

2.26893    0.88479E 02 0.11346E-00-0.31056E 02 0.58880E 01 0.81257E 02 0.50749E-01 0.        0.        -0.12454E 02 0.14707E-01
2.35620    0.84638E 02 0.13208E-01-0.22701E 02 0.61786E 01 0.19441E 03 0.54322E-01 0.        0.        -0.11421E 02 0.14912E-01
2.44347    0.75448E 02-0.15295E-00-0.14927E 02 0.64674E 01 0.31034E 03 0.52231E-01 0.        0.        -0.94548E 01 0.13233E-01

2.44347    0.75448E 02-0.15294E-00-0.14927E 02 0.64674E 01 0.31035E 03 0.52232E-01 0.        0.        -0.94547E 01 0.13233E-01
2.53073    0.60570E 02-0.39570E-00-0.82500E 01 0.67267E 01 0.41375E 03 0.41245E-01 0.        0.        -0.67523E 01 0.88327E-02
2.61800    0.41335E 02-0.71576E 00-0.31240E 01 0.69292E 01 0.48190E 03 0.17796E-01 0.        0.        -0.36690E 01 0.88710E-03

BRANCH SHELL PART NO  4
0.         0.41335E 02-0.71572E 00-0.31242E 01 0.69292E 01 0.48190E 03 0.17796E-01 0.        0.        -0.36690E 01 0.88702E-03
0.09375   -0.40517E 01-0.26653E-00-0.19242E 01 0.72730E 01 0.45243E 03-0.26930E-01 0.        0.         0.34209E 01-0.12462E-01
0.18750   -0.39674E 02 0.50574E-02-0.10322E 01 0.73063E 01 0.29824E 03-0.38879E-01 0.        0.         0.95043E 01-0.14701E-01

0.18750   -0.39695E 02 0.49901E-02-0.10323E 01 0.73062E 01 0.29830E 03-0.38876E-01 0.        0.         0.95080E 01-0.14700E-01
0.28125   -0.59719E 02 0.12472E-00-0.40965E-00 0.70731E 01 0.13306E 03-0.33031E-01 0.        0.         0.13354E 02-0.11337E-01
0.37500   -0.66011E 02 0.14264E-00 0.22405E-01 0.66601E 01 0.10146E 02-0.20914E-01 0.        0.         0.14959E 02-0.63893E-02

0.37500   -0.66010E 02 0.14282E-00 0.22378E-01 0.66601E 01 0.10138E 02-0.20916E-01 0.        0.         0.14959E 02-0.63899E-02
0.46875   -0.63330E 02 0.10966E-00 0.34016E-00 0.61536E 01-0.58779E 02-0.94828E-02 0.        0.         0.14917E 02-0.21378E-02
0.56250   -0.56358E 02 0.62929E-01 0.59820E 00 0.56181E 01-0.84108E 02-0.16773E-02 0.        0.         0.13952E 02 0.57350E-03

0.56250   -0.56358E 02 0.62915E-01 0.59617E 00 0.56181E 01-0.84120E 02-0.16801E-02 0.        0.         0.13951E 02 0.57278E-03
0.65625   -0.48375E 02 0.24076E-01 0.82573E 00 0.50895E 01-0.83208E 02 0.22423E-02 0.        0.         0.12651E 02 0.18110E-02
0.75000   -0.41094E 02 0.66562E-03 0.10321E 01 0.45795E 01-0.71278E 02 0.33487E-02 0.        0.         0.11375E 02 0.20517E-02

0.75000   -0.41094E 02 0.65022E-03 0.10320E 01 0.45795E 01-0.71290E 02 0.33457E-02 0.        0.         0.11375E 02 0.20510E-02
0.84375   -0.35046E 02-0.89267E-02 0.12142E 01 0.40634E 01-0.58004E 02 0.29675E-02 0.        0.         0.10275E 02 0.18102E-02
0.93750   -0.30122E 02-0.10582E-01 0.13641E 01 0.35882E 01-0.47675E 02 0.20937E-02 0.        0.         0.93622E 01 0.14497E-02

0.93750   -0.30122E 02-0.10601E-01 0.13640E 01 0.35882E 01-0.47689E 02 0.20902E-02 0.        0.         0.93620E 01 0.14489E-02
1.03125   -0.25974E 02-0.10949E-01 0.14723E 01 0.30789E 01-0.41517E 02 0.11222E-02 0.        0.         0.85778E 01 0.11199E-02
1.12500   -0.22188E 02-0.14373E-01 0.15314E 01 0.25429E 01-0.40178E 02-0.77896E-04 0.        0.         0.78144E 01 0.81395E-03

1.12500   -0.22187E 02-0.14402E-01 0.15314E 01 0.25428E 01-0.40196E 02-0.83308E-04 0.        0.         0.78142E 01 0.81271E-03
1.21875   -0.18224E 02-0.17261E-01 0.15401E 01 0.19787E 01-0.45724E 02-0.18022E-02 0.        0.         0.68654E 01 0.52365E-03
1.31250   -0.13344E 02 0.89404E-02 0.15145E 01 0.14243E 01-0.59530E 02-0.30372E-02 0.        0.         0.53215E 01 0.65793E-03

1.31250   -0.13343E 02 0.88998E-02 0.15144E 01 0.14243E 01-0.59556E 02-0.30451E-02 0.        0.         0.53211E 01 0.65624E-03
1.40625   -0.71569E 01 0.15334E-00 0.15090E 01 0.10155E 01-0.68418E 02 0.26796E-02 0.        0.         0.26953E 01 0.28824E-02
1.50000   -0.27823E 01 0.56021E 00 0.16059E 01 0.97030E 00 0.48797E-01 0.35382E-01 0.        0.         0.29157E-00 0.10613E-01

MAIN SHELL PART NO  5
0.         0.11535E 01 0.55471E 00 0.71402E 02 0.49656E 01-0.39715E 02-0.20095E-01 0.        0.         0.22971E 01-0.60284E-02
0.12500    0.61554E 01 0.18384E-00 0.71737E 02 0.49656E 01-0.37493E 02 0.23770E-01 0.        0.         0.57985E 01 0.71311E-02
0.25000    0.10133E 02 0.13782E-01 0.71985E 02 0.49656E 01-0.25481E 02 0.34306E-01 0.        0.         0.85826E 01 0.10292E-01

0.25000    0.10133E 02 0.13793E-01 0.71985E 02 0.49656E 01-0.25481E 02 0.34308E-01 0.        0.         0.85825E 01 0.10292E-01
0.37500    0.12484E 02-0.17326E-01 0.72174E 02 0.49656E 01-0.12231E 02 0.33016E-01 0.        0.         0.10228E 02 0.99049E-02
```

| | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| 0.50000 | 0.13219E 02 | 0.18780E-01 | 0.72334E 02 | 0.49656E 01 | 0.44688E-00 | 0.32772E-01 | 0. | 0. | 0.10743E 02 | 0.98317E-02 |
| 0.50000 | 0.13219E 02 | 0.18790E-01 | 0.72334E 02 | 0.49656E 01 | 0.44764E-00 | 0.32775E-01 | 0. | 0. | 0.10743E 02 | 0.98324E-02 |
| 0.62500 | 0.12334E 02 | 0.52032E-01 | 0.72495E 02 | 0.49656E 01 | 0.14031E 02 | 0.37602E-01 | 0. | 0. | 0.10123E 02 | 0.11201E-01 |
| 0.75000 | 0.96130E 01 | 0.74392E-02 | 0.72690E 02 | 0.49656E 01 | 0.29838E 02 | 0.42557E-01 | 0. | 0. | 0.82188E 01 | 0.12767E-01 |
| 0.75000 | 0.96129E 01 | 0.74491E-02 | 0.72690E 02 | 0.49656E 01 | 0.29839E 02 | 0.42560E-01 | 0. | 0. | 0.82187E 01 | 0.12768E-01 |
| 0.87500 | 0.48753E 01 | -0.20037E-00 | 0.72954E 02 | 0.49656E 01 | 0.45346E 02 | 0.32664E-01 | 0. | 0. | 0.49024E 01 | 0.97993E-02 |
| 1.00000 | -0.12917E 01 | -0.65156E 00 | 0.73323E 02 | 0.49656E 01 | 0.50085E 02 | -0.17753E-01 | 0. | 0. | 0.58547E 00 | -0.53259E-02 |
| MAIN SHELL PART NO 6 | | | | | | | | | | |
| 1.57080 | -0.12915E 01 | -0.65153E 00 | 0.73323E 02 | 0.49656E 01 | 0.50086E 02 | -0.17751E-01 | 0. | 0. | 0.58545E 00 | -0.53252E-02 |
| 1.60352 | 0.29833E-00 | -0.56813E 00 | 0.73397E 02 | 0.49862E 01 | 0.48921E 02 | -0.27729E-01 | 0. | 0. | 0.23161E-01 | -0.85523E-02 |
| 1.63625 | 0.19109E 01 | -0.48896E-00 | 0.73420E 02 | 0.50056E 01 | 0.47279E 02 | -0.36405E-01 | 0. | 0. | -0.52616E 00 | -0.11373E-01 |
| 1.63625 | 0.19109E 01 | -0.48896E-00 | 0.73420E 02 | 0.50056E 01 | 0.47279E 02 | -0.36405E-01 | 0. | 0. | -0.52616E 00 | -0.11373E-01 |
| 1.66897 | 0.35502E 01 | -0.41412E-00 | 0.73393E 02 | 0.50243E 01 | 0.45219E 02 | -0.43862E-01 | 0. | 0. | -0.10585E 01 | -0.13806E-01 |
| 1.70170 | 0.52196E 01 | -0.34367E-00 | 0.73314E 02 | 0.50427E 01 | 0.42790E 02 | -0.50178E-01 | 0. | 0. | -0.15701E 01 | -0.15870E-01 |
| 1.70170 | 0.52196E 01 | -0.34367E-00 | 0.73314E 02 | 0.50427E 01 | 0.42790E 02 | -0.50178E-01 | 0. | 0. | -0.15701E 01 | -0.15870E-01 |
| 1.73442 | 0.69219E 01 | -0.27768E-00 | 0.73182E 02 | 0.50614E 01 | 0.40038E 02 | -0.55425E-01 | 0. | 0. | -0.20580E 01 | -0.17582E-01 |
| 1.76715 | 0.86598E 01 | -0.21617E-00 | 0.72997E 02 | 0.50807E 01 | 0.37004E 02 | -0.59669E-01 | 0. | 0. | -0.25192E 01 | -0.18959E-01 |
| 1.76715 | 0.86598E 01 | -0.21617E-00 | 0.72997E 02 | 0.50807E 01 | 0.37004E 02 | -0.59669E-01 | 0. | 0. | -0.25192E 01 | -0.18959E-01 |
| 1.79987 | 0.10436E 02 | -0.15920E-00 | 0.72757E 02 | 0.51011E 01 | 0.33724E 02 | -0.62970E-01 | 0. | 0. | -0.29510E 01 | -0.20015E-01 |
| 1.83260 | 0.12252E 02 | -0.10677E-00 | 0.72462E 02 | 0.51232E 01 | 0.30234E 02 | -0.65385E-01 | 0. | 0. | -0.33508E 01 | -0.20767E-01 |
| 1.83260 | 0.12252E 02 | -0.10677E-00 | 0.72462E 02 | 0.51232E 01 | 0.30234E 02 | -0.65385E-01 | 0. | 0. | -0.33508E 01 | -0.20767E-01 |
| 1.86532 | 0.14110E 02 | -0.58884E-01 | 0.72110E 02 | 0.51474E 01 | 0.26564E 02 | -0.66968E-01 | 0. | 0. | -0.37164E 01 | -0.21228E-01 |
| 1.89805 | 0.16013E 02 | -0.15548E-01 | 0.71700E 02 | 0.51742E 01 | 0.22745E 02 | -0.67769E-01 | 0. | 0. | -0.40454E 01 | -0.21412E-01 |
| 1.89805 | 0.16013E 02 | -0.15548E-01 | 0.71700E 02 | 0.51742E 01 | 0.22745E 02 | -0.67769E-01 | 0. | 0. | -0.40454E 01 | -0.21412E-01 |
| 1.93077 | 0.17964E 02 | 0.23258E-01 | 0.71230E 02 | 0.52041E 01 | 0.18805E 02 | -0.67837E-01 | 0. | 0. | -0.43357E 01 | -0.21334E-01 |
| 1.96350 | 0.19964E 02 | 0.57564E-01 | 0.70699E 02 | 0.52376E 01 | 0.14773E 02 | -0.67219E-01 | 0. | 0. | -0.45849E 01 | -0.21008E-01 |
| 1.96350 | 0.19964E 02 | 0.57564E-01 | 0.70699E 02 | 0.52376E 01 | 0.14773E 02 | -0.67219E-01 | 0. | 0. | -0.45849E 01 | -0.21008E-01 |
| 1.99622 | 0.22016E 02 | 0.87413E-01 | 0.70107E 02 | 0.52753E 01 | 0.10676E 02 | -0.65961E-01 | 0. | 0. | -0.47910E 01 | -0.20447E-01 |
| 2.02895 | 0.24124E 02 | 0.11287E-00 | 0.69451E 02 | 0.53177E 01 | 0.65421E 01 | -0.64110E-01 | 0. | 0. | -0.49519E 01 | -0.19668E-01 |
| 2.02895 | 0.24124E 02 | 0.11287E-00 | 0.69451E 02 | 0.53177E 01 | 0.65421E 01 | -0.64110E-01 | 0. | 0. | -0.49519E 01 | -0.19668E-01 |
| 2.06167 | 0.26291E 02 | 0.13400E-00 | 0.68729E 02 | 0.53652E 01 | 0.23994E 01 | -0.61711E-01 | 0. | 0. | -0.50653E 01 | -0.18684E-01 |
| 2.09440 | 0.28521E 02 | 0.15092E-00 | 0.67941E 02 | 0.54185E 01 | -0.17232E 01 | -0.58813E-01 | 0. | 0. | -0.51292E 01 | -0.17513E-01 |
| MAIN SHELL PART NO 7 | | | | | | | | | | |
| 2.09440 | 0.28521E 02 | 0.15092E-00 | 0.67941E 02 | 0.54185E 01 | -0.17232E 01 | -0.58813E-01 | 0. | 0. | -0.51292E 01 | -0.17513E-01 |
| 2.15725 | 0.32992E 02 | 0.17203E-00 | 0.66233E 02 | 0.55391E 01 | -0.94736E 01 | -0.52029E-01 | 0. | 0. | -0.51058E 01 | -0.14803E-01 |
| 2.22010 | 0.37737E 02 | 0.17917E-00 | 0.64259E 02 | 0.56667E 01 | -0.16813E 02 | -0.43961E-01 | 0. | 0. | -0.48790E 01 | -0.11603E-01 |
| 2.22010 | 0.37737E 02 | 0.17917E-00 | 0.64259E 02 | 0.56667E 01 | -0.16813E 02 | -0.43961E-01 | 0. | 0. | -0.48790E 01 | -0.11603E-01 |
| 2.28295 | 0.42788E 02 | 0.17348E-00 | 0.62003E 02 | 0.58050E 01 | -0.23496E 02 | -0.35029E-01 | 0. | 0. | -0.44376E 01 | -0.80700E-02 |
| 2.34580 | 0.48175E 02 | 0.15820E-00 | 0.59451E 02 | 0.60769E 01 | -0.29262E 02 | -0.25701E-01 | 0. | 0. | -0.37746E 01 | -0.43855E-02 |
| 2.34580 | 0.48175E 02 | 0.15820E-00 | 0.59451E 02 | 0.60769E 01 | -0.29263E 02 | -0.25701E-01 | 0. | 0. | -0.37745E 01 | -0.43855E-02 |
| 2.40865 | 0.53921E 02 | 0.13465E-00 | 0.56585E 02 | 0.63247E 01 | -0.33848E 02 | -0.16492E-01 | 0. | 0. | -0.28894E 01 | -0.75115E-03 |
| 2.47150 | 0.60033E 02 | 0.10629E-00 | 0.53388E 02 | 0.66090E 01 | -0.37006E 02 | -0.79405E-02 | 0. | 0. | -0.17913E 01 | 0.26193E-02 |
| 2.47150 | 0.60033E 02 | 0.10629E-00 | 0.53388E 02 | 0.66090E 01 | -0.37007E 02 | -0.79408E-02 | 0. | 0. | -0.17913E 01 | 0.26192E-02 |
| 2.53435 | 0.66493E 02 | 0.76515E-01 | 0.49844E 02 | 0.69281E 01 | -0.38546E 02 | -0.56152E-03 | 0. | 0. | -0.50173E 00 | 0.55189E-02 |
| 2.59720 | 0.73245E 02 | 0.48357E-01 | 0.45941E 02 | 0.72771E 01 | -0.38371E 02 | 0.52368E-02 | 0. | 0. | 0.94256E 00 | 0.77793E-02 |
| 2.59720 | 0.73245E 02 | 0.48861E-01 | 0.45941E 02 | 0.72771E 01 | -0.38371E 02 | 0.52374E-02 | 0. | 0. | 0.94272E 00 | 0.77795E-02 |
| 2.66005 | 0.80180E 02 | 0.26502E-01 | 0.41672E 02 | 0.76468E 01 | -0.36517E 02 | 0.92653E-02 | 0. | 0. | 0.24896E 01 | 0.93118E-02 |

| | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| 2.72290 | 0.87130E 02 | 0.11722E-01 | 0.37041E 02 | 0.80226E 01 | -0.33143E 02 | 0.11674E-01 | 0. | 0. | 0.40729E 01 | 0.10144E-01 |
| | | | | | | | | | | |
| 2.72290 | 0.87131E 02 | 0.11758E-01 | 0.37041E 02 | 0.80225E 01 | -0.33144E 02 | 0.11677E-01 | 0. | 0. | 0.40734E 01 | 0.10145E-01 |
| 2.78575 | 0.93351E 02 | 0.52267E-02 | 0.32062E 02 | 0.83824E 01 | -0.28427E 02 | 0.13051E-01 | 0. | 0. | 0.56148E 01 | 0.10429E-01 |
| 2.84860 | 0.99993E 02 | 0.48750E-02 | 0.26767E 02 | 0.86967E 01 | -0.22328E 02 | 0.14320E-01 | 0. | 0. | 0.70172E 01 | 0.10330E-01 |
| | | | | | | | | | | |
| 2.84860 | 0.99993E 02 | 0.48610E-02 | 0.26767E 02 | 0.86967E 01 | -0.22330E 02 | 0.14316E-01 | 0. | 0. | 0.70171E 01 | 0.10330E-01 |
| 2.91145 | 0.10503E 03 | 0.34418E-02 | 0.21209E 02 | 0.89303E 01 | -0.14323E 02 | 0.16254E-01 | 0. | 0. | 0.81365E 01 | 0.96690E-02 |
| 2.97430 | 0.10812E 03 | -0.18720E-01 | 0.15481E 02 | 0.90737E 01 | -0.36427E 01 | 0.17726E-01 | 0. | 0. | 0.87145E 01 | 0.69546E-02 |
| | | | | | | | | | | |
| 2.97430 | 0.10812E 03 | -0.18540E-01 | 0.15480E 02 | 0.90725E 01 | -0.36238E 01 | 0.17777E-01 | 0. | 0. | 0.87164E 01 | 0.69615E-02 |
| 3.03715 | 0.10835E 03 | -0.11614E-00 | 0.97516E 01 | 0.93405E 01 | 0.79518E 01 | 0.10058E-01 | 0. | 0. | 0.82502E 01 | -0.26033E-02 |
| 3.10000 | 0.10681E 03 | -0.54218E 00 | 0.44475E 01 | 0.13027E 02 | -0.98109E-01 | -0.60923E-01 | 0. | 0. | 0.38888E 01 | -0.18105E-01 |

STRESS ANALYSIS PROGRAM OF SHELLS OF REVOLUTION UNDER STATIC LOADS, BY A. KALNINS, LEHIGH UNIV, BETHLEHEM, PA
WRIGHT PATTERSON AIR FORCE BASE FLIGHT DYNAMICS LABORATORY VERSION, 22 JULY 1968

STATIC SOLUTION AT POINTS ALONG MERIDIAN FOR WAVE NUMBER NX= 0

MAIN SHELL PART NO 1

| S | W | UPHI | UTHETA | BPHI | SPHI IN | SPHI OUT | STHETA IN | STHETA OUT | SFITH IN | SFITH OUT |
|---|---|---|---|---|---|---|---|---|---|---|
| 0. | 0.25207E 02 | 0. | 0. | 0. | 0.92209E 03 | 0.93925E 03 | 0.62952E 03 | 0.63467E 03 | -0. | 0. |
| 0. | 0.25207E 02 | 0. | 0. | 0. | 0.52977E 03 | 0.54536E 03 | 0.38836E 03 | 0.39579E 03 | 0. | 0. |
| 0.12500 | 0.24933E 02 | 0.33642E 01 | 0. | 0.49977E 01 | 0.91020E 03 | 0.95038E 03 | 0.62213E 03 | 0.63418E 03 | -0. | 0. |
| 0.12500 | 0.24933E 02 | 0.33642E 01 | 0. | 0.49977E 01 | 0.53557E 03 | 0.57773E 03 | 0.38876E 03 | 0.40563E 03 | 0. | 0. |
| 0.25000 | 0.23876E 02 | 0.68009E 01 | 0. | 0.12007E 02 | 0.92457E 03 | 0.96829E 03 | 0.61164E 03 | 0.62475E 03 | -0. | 0. |
| 0.25000 | 0.23876E 02 | 0.68009E 01 | 0. | 0.12007E 02 | 0.54439E 03 | 0.58880E 03 | 0.38489E 03 | 0.40265E 03 | 0. | 0. |
| 0.25000 | 0.23878E 02 | 0.68008E 01 | 0. | 0.12000E 02 | 0.92450E 03 | 0.96828E 03 | 0.61165E 03 | 0.62478E 03 | -0. | 0. |
| 0.25000 | 0.23878E 02 | 0.68008E 01 | 0. | 0.12000E 02 | 0.54438E 03 | 0.58885E 03 | 0.38490E 03 | 0.40269E 03 | 0. | 0. |
| 0.37500 | 0.21950E 02 | 0.10346E 02 | 0. | 0.18687E 02 | 0.95199E 03 | 0.98952E 03 | 0.59289E 03 | 0.60415E 03 | -0. | 0. |
| 0.37500 | 0.21950E 02 | 0.10346E 02 | 0. | 0.18687E 02 | 0.55428E 03 | 0.59114E 03 | 0.37536E 03 | 0.39010E 03 | 0. | 0. |
| 0.50000 | 0.19264E 02 | 0.14024E 02 | 0. | 0.24040E 02 | 0.98381E 03 | 0.10121E 04 | 0.56483E 03 | 0.57331E 03 | -0. | 0. |
| 0.50000 | 0.19264E 02 | 0.14024E 02 | 0. | 0.24040E 02 | 0.56425E 03 | 0.59102E 03 | 0.36054E 03 | 0.37125E 03 | 0. | 0. |
| 0.50000 | 0.19266E 02 | 0.14024E 02 | 0. | 0.24032E 02 | 0.98375E 03 | 0.10120E 04 | 0.56484E 03 | 0.57333E 03 | -0. | 0. |
| 0.50000 | 0.19266E 02 | 0.14024E 02 | 0. | 0.24032E 02 | 0.56424E 03 | 0.59107E 03 | 0.36056E 03 | 0.37129E 03 | 0. | 0. |
| 0.62500 | 0.16004E 02 | 0.17850E 02 | 0. | 0.27923E 02 | 0.10150E 04 | 0.10349E 04 | 0.52855E 03 | 0.53453E 03 | -0. | 0. |
| 0.62500 | 0.16004E 02 | 0.17850E 02 | 0. | 0.27923E 02 | 0.57391E 03 | 0.59213E 03 | 0.34159E 03 | 0.34888E 03 | 0. | 0. |
| 0.75000 | 0.12334E 02 | 0.21835E 02 | 0. | 0.30644E 02 | 0.10439E 04 | 0.10579E 04 | 0.48585E 03 | 0.49004E 03 | -0. | 0. |
| 0.75000 | 0.12334E 02 | 0.21835E 02 | 0. | 0.30644E 02 | 0.58331E 03 | 0.59563E 03 | 0.31966E 03 | 0.32459E 03 | 0. | 0. |
| 0.75000 | 0.12335E 02 | 0.21835E 02 | 0. | 0.30638E 02 | 0.10438E 04 | 0.10579E 04 | 0.48584E 03 | 0.49006E 03 | -0. | 0. |
| 0.75000 | 0.12335E 02 | 0.21835E 02 | 0. | 0.30638E 02 | 0.56331E 03 | 0.59568E 03 | 0.31967E 03 | 0.32462E 03 | 0. | 0. |
| 0.87500 | 0.83766E 01 | 0.25985E 02 | 0. | 0.32598E 02 | 0.10708E 04 | 0.10812E 04 | 0.43853E 03 | 0.44162E 03 | -0. | 0. |
| 0.87500 | 0.83766E 01 | 0.25965E 02 | 0. | 0.32598E 02 | 0.59261E 03 | 0.60135E 03 | 0.29568E 03 | 0.29918E 03 | 0. | 0. |
| 1.00000 | 0.42076E 01 | 0.30307E 02 | 0. | 0.34019E 02 | 0.10979E 04 | 0.11050E 04 | 0.38929E 03 | 0.39040E 03 | -0. | 0. |
| 1.00000 | 0.42076E 01 | 0.30307E 02 | 0. | 0.34019E 02 | 0.60203E 03 | 0.60774E 03 | 0.27027E 03 | 0.27255E 03 | 0. | 0. |
| 1.00000 | 0.42067E 01 | 0.30307E 02 | 0. | 0.34014E 02 | 0.10980E 04 | 0.11050E 04 | 0.38929E 03 | 0.39039E 03 | -0. | 0. |
| 1.00000 | 0.42067E 01 | 0.30307E 02 | 0. | 0.34014E 02 | 0.60203E 03 | 0.60773E 03 | 0.27026E 03 | 0.27254E 03 | 0. | 0. |
| 1.12500 | -0.10010E-00 | 0.34807E 02 | 0. | 0.34740E 02 | 0.11282E 04 | 0.11294E 04 | 0.33706E 03 | 0.33743E 03 | -0. | 0. |
| 1.12500 | -0.10010E-00 | 0.34807E 02 | 0. | 0.34740E 02 | 0.61169E 03 | 0.61263E 03 | 0.24397E 03 | 0.24435E 03 | 0. | 0. |
| 1.25000 | -0.44209E 01 | 0.39488E 02 | 0. | 0.34067E 02 | 0.11652E 04 | 0.11543E 04 | 0.28766E 03 | 0.28439E 03 | -0. | 0. |
| 1.25000 | -0.44209E 01 | 0.39488E 02 | 0. | 0.34067E 02 | 0.62155E 03 | 0.61344E 03 | 0.21768E 03 | 0.21443E 03 | 0. | 0. |
| 1.25000 | -0.44220E 01 | 0.39488E 02 | 0. | 0.34061E 02 | 0.11652E 04 | 0.11543E 04 | 0.28766E 03 | 0.28436E 03 | -0. | 0. |
| 1.25000 | -0.44220E 01 | 0.39488E 02 | 0. | 0.34061E 02 | 0.62155E 03 | 0.61342E 03 | 0.21767E 03 | 0.21441E 03 | 0. | 0. |
| 1.37500 | -0.85032E 01 | 0.44350E 02 | 0. | 0.30638E 02 | 0.12120E 04 | 0.11789E 04 | 0.24455E 03 | 0.23462E 03 | -0. | 0. |
| 1.37500 | -0.85032E 01 | 0.44350E 02 | 0. | 0.30636E 02 | 0.63147E 03 | 0.60793E 03 | 0.19307E 03 | 0.18365E 03 | 0. | 0. |
| 1.50000 | -0.11893E 02 | 0.49380E 02 | 0. | 0.22698E 02 | 0.12683E 04 | 0.12021E 04 | 0.21398E 03 | 0.19414E 03 | -0. | 0. |
| 1.50000 | -0.11893E 02 | 0.49380E 02 | 0. | 0.22698E 02 | 0.64125E 03 | 0.59647E 03 | 0.17325E 03 | 0.15534E 03 | 0. | 0. |
| 1.50000 | -0.11894E 02 | 0.49380E 02 | 0. | 0.22695E 02 | 0.12683E 04 | 0.12021E 04 | 0.21397E 03 | 0.19412E 03 | -0. | 0. |
| 1.50000 | -0.11894E 02 | 0.49380E 02 | 0. | 0.22695E 02 | 0.64125E 03 | 0.59645E 03 | 0.17324E 03 | 0.15532E 03 | 0. | 0. |
| 1.62500 | -0.13935E 02 | 0.54552E 02 | 0. | 0.89826E 01 | 0.13249E 04 | 0.12236E 04 | 0.20239E 03 | 0.17199E 03 | -0. | 0. |
| 1.62500 | -0.13935E 02 | 0.54552E 02 | 0. | 0.89826E 01 | 0.65116E 03 | 0.58598E 03 | 0.16292E 03 | 0.13685E 03 | 0. | 0. |
| 1.75000 | -0.13961E 02 | 0.59832E 02 | 0. | -0.87709E 01 | 0.13539E 04 | 0.12458E 04 | 0.21071E 03 | 0.17827E 03 | -0. | 0. |
| 1.75000 | -0.13961E 02 | 0.59832E 02 | 0. | -0.87709E 01 | 0.66315E 03 | 0.59726E 03 | 0.16753E 03 | 0.14117E 03 | 0. | 0. |
| 1.75000 | -0.13961E 02 | 0.59832E 02 | 0. | -0.87710E 01 | 0.13539E 04 | 0.12458E 04 | 0.21070E 03 | 0.17827E 03 | -0. | 0. |
| 1.75000 | -0.13961E 02 | 0.59832E 02 | 0. | -0.87710E 01 | 0.66315E 03 | 0.59726E 03 | 0.16753E 03 | 0.14117E 03 | 0. | 0. |
| 1.87500 | -0.11953E 02 | 0.65200E 02 | 0. | -0.20928E 02 | 0.12961E 04 | 0.12797E 04 | 0.22148E 03 | 0.21656E 03 | -0. | 0. |
| 1.87500 | -0.11953E 02 | 0.65200E 02 | 0. | -0.20926E 02 | 0.68321E 03 | 0.67378E 03 | 0.18961E 03 | 0.18584E 03 | 0. | 0. |
| 2.00000 | -0.99893E 01 | 0.70729E 02 | 0. | -0.19624E 01 | 0.10522E 04 | 0.13531E 04 | 0.17581E 03 | 0.26609E 03 | -0. | 0. |

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2.00000  -0.99893E 01 0.70729E 02 0.        -0.19824E 01 0.72462E 03 0.88763E 03 0.21992E 03 0.28513E 03 0.          0.

MAIN SHELL PART NO  2
0.       -0.99893E 01 0.70729E 02 0.        -0.19833E 01 0.24381E 03-0.23502E 03-0.66706E 02-0.21036E 03-0.          0.
0.12500  -0.80657E 01 0.70921E 02 0.        -0.23581E 02 0.50919E 02-0.42127E 02-0.97644E 02-0.12556E 03-0.          0.
0.25000  -0.50207E 01 0.71061E 02 0.        -0.22736E 02-0.38458E 02 0.47250E 02-0.81827E 02-0.56114E 02-0.          0.

0.25000  -0.50207E 01 0.71061E 02 0.        -0.22736E 02-0.38457E 02 0.47250E 02-0.81827E 02-0.56114E 02-0.          0.
0.37500  -0.27360E 01 0.71150E 02 0.        -0.13186E 02-0.62268E 02 0.71061E 02-0.56985E 02-0.16986E 02-0.          0.
0.50000  -0.17459E 01 0.71207E 02 0.        -0.29955E 01-0.49948E 02 0.58741E 02-0.39427E 02-0.68199E 01-0.          0.

0.50000  -0.17459E 01 0.71207E 02 0.        -0.29955E 01-0.49948E 02 0.58741E 02-0.39427E 02-0.68199E 01-0.          0.
0.62500  -0.18221E 01 0.71257E 02 0.         0.32627E 01-0.14758E 02 0.23551E 02-0.29437E 02-0.18444E 02-0.          0.
0.75000  -0.22488E 01 0.71314E 02 0.         0.19880E 01 0.43519E 02-0.34727E 02-0.18427E 02-0.41901E 02-0.          0.

0.75000  -0.22488E 01 0.71314E 02 0.         0.19880E 01 0.43519E 02-0.34727E 02-0.18427E 02-0.41901E 02-0.          0.
0.87500  -0.18334E 01 0.71372E 02 0.        -0.10955E 02 0.12919E 03-0.12039E 03 0.13088E 02-0.61786E 02-0.          0.
1.00000   0.11535E 01 0.71402E 02 0.        -0.39715E 02 0.23489E 03-0.22610E 03 0.86617E 02-0.51681E 02-0.          0.

BRANCH SHELL PART NO  3
1.57080  -0.11532E 01-0.71402E 02 0.        -0.39715E 02 0.13765E 04-0.90189E 03 0.42909E 03-0.25442E 03-0.          0.
1.65807   0.97395E 01-0.70722E 02 0.        -0.16275E 03 0.86121E 03-0.37934E 03 0.20679E 03-0.16140E 03-0.          0.
1.74533   0.24437E 02-0.68854E 02 0.        -0.22235E 03 0.48274E 03 0.58487E 01-0.30778E 02-0.16299E 03-0.          0.

1.74533   0.24437E 02-0.68854E 02 0.        -0.22235E 03 0.48271E 03 0.58754E 01-0.30789E 02-0.16299E 03-0.          0.
1.83260   0.40452E 02-0.65564E 02 0.        -0.23593E 03 0.21156E 03 0.28466E 03-0.25727E 03-0.21810E 03-0.          0.
1.91987   0.55944E 02-0.60614E 02 0.        -0.21619E 03 0.14254E 02 0.49211E 03-0.45783E 03-0.29345E 03-0.          0.

279
1.91987   0.55945E 02-0.60814E 02 0.        -0.21619E 03 0.14236E 02 0.49213E 03-0.45784E 03-0.29345E 03-0.          0.
2.00713   0.69527E 02-0.54717E 02 0.        -0.17110E 03-0.13882E 03 0.65913E 03-0.62383E 03-0.36372E 03-0.          0.
2.09440   0.80088E 02-0.47499E 02 0.        -0.10497E 03-0.26777E 03 0.80659E 03-0.74859E 03-0.41107E 03-0.          0.

2.09440   0.80089E 02-0.47499E 02 0.        -0.10497E 03-0.26778E 03 0.80660E 03-0.74859E 03-0.41107E 03-0.          0.
2.18167   0.86681E 02-0.39478E 02 0.        -0.20075E 02-0.37974E 03 0.94165E 03-0.82472E 03-0.42493E 03-0.          0.
2.26893   0.88479E 02-0.31056E 02 0.         0.81253E 02-0.46682E 03 0.10556E 04-0.84329E 03-0.40209E 03-0.          0.

2.26893   0.88479E 02-0.31056E 02 0.         0.61257E 02-0.46683E 03 0.10556E 04-0.84329E 03-0.40209E 03-0.          0.
2.35620   0.84838E 02-0.22701E 02 0.         0.19441E 03-0.50590E 03 0.11238E 04-0.79472E 03-0.34735E 03-0.          0.
2.44347   0.75448E 02-0.14927E 02 0.         0.31034E 03-0.46010E 03 0.11068E 04-0.67123E 03-0.27424E 03-0.          0.

2.44347   0.75448E 02-0.14927E 02 0.         0.31035E 03-0.46010E 03 0.11068E 04-0.67123E 03-0.27424E 03-0.          0.
2.53073   0.60570E 02-0.82500E 01 0.         0.41375E 03-0.28234E 03 0.95501E 03-0.47010E 03-0.20512E 03-0.          0.
2.61800   0.41335E 02-0.31240E 01 0.         0.48190E 03 0.79527E 02 0.61340E 03-0.19676E 03-0.17015E 03-0.          0.

BRANCH SHELL PART NO  4
0.        0.41335E 02-0.31242E 01 0.         0.48190E 03 0.79527E 02 0.61340E 03-0.19676E 03-0.17015E 03-0.          0.
0.09375  -0.40517E 01-0.19242E 01 0.         0.45243E 03 0.76761E 03-0.40304E 02 0.35797E 03-0.15883E 02-0.          0.
0.18750  -0.39674E 02-0.10322E 01 0.         0.29824E 03 0.94851E 03-0.21787E 03 0.69573E 03 0.25471E 03-0.          0.

0.18750  -0.39695E 02-0.10323E 01 0.         0.29830E 03 0.94846E 03-0.21784E 03 0.69591E 03 0.25489E 03-0.          0.
0.28125  -0.59719E 02-0.40965E-00 0.         0.13306E 03 0.84912E 03-0.14181E 03 0.83775E 03 0.49764E 03-0.          0.
0.37500  -0.66011E 02 0.22405E-01 0.         0.10146E 02 0.64672E 03 0.19295E 02 0.84378E 03 0.65211E 03 0.         -0.

0.37500  -0.66010E 02 0.22378E-01 0.         0.10138E 02 0.64675E 03 0.19260E 02 0.84379E 03 0.65210E 03 0.         -0.
0.46875  -0.63330E 02 0.34016E-00 0.        -0.58779E 02 0.44992E 03 0.16544E 03 0.77791E 03 0.71377E 03 0.         -0.
0.56250  -0.56358E 02 0.59820E 00 0.        -0.64108E 02 0.30606E 03 0.25575E 03 0.68897E 03 0.70618E 03 0.         -0.

0.56250  -0.56358E 02 0.59817E 00 0.        -0.84120E 02 0.30611E 03 0.25570E 03 0.68898E 03 0.70616E 03 0.         -0.
0.65625  -0.48375E 02 0.82573E 00 0.        -0.83208E 02 0.22084E 03 0.28811E 03 0.60538E 03 0.65971E 03 0.         -0.
0.75000  -0.41094E 02 0.10321E 01 0.        -0.71278E 02 0.17875E 03 0.27921E 03 0.53797E 03 0.59952E 03 0.         -0.
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0.75000  -0.41094E 02 0.10320E 01 0.      -0.71290E 02 0.17879E 03 0.27916E 03 0.53798E 03 0.59951E 03 0.     -0.
0.84375  -0.35046E 02 0.12142E 01 0.      -0.58004E 02 0.15966E 03 0.24868E 03 0.48657E 03 0.54088E 03 0.     -0.
0.93750  -0.30122E 02 0.13641E 01 0.      -0.47675E 02 0.14801E 03 0.21082E 03 0.44636E 03 0.48985E 03 0.     -0.

0.93750  -0.30122E 02 0.13640E 01 0.      -0.47689E 02 0.14806E 03 0.21076E 03 0.44637E 03 0.48983E 03 0.     -0.
1.03125  -0.25974E 02 0.14723E 01 0.      -0.41517E 02 0.13711E 03 0.17078E 03 0.41209E 03 0.44569E 03 0.     -0.
1.12500  -0.22188E 02 0.15314E 01 0.      -0.40178E 02 0.12831E 03 0.12598E 03 0.37851E 03 0.40293E 03 0.     -0.

1.12500  -0.22187E 02 0.15314E 01 0.      -0.40196E 02 0.12839E 03 0.12589E 03 0.37852E 03 0.40290E 03 0.     -0.
1.21875  -0.18224E 02 0.15401E 01 0.      -0.45724E 02 0.12597E 03 0.71902E 02 0.33542E 03 0.35113E 03 0.     -0.
1.31250  -0.13344E 02 0.15145E 01 0.      -0.59530E 02 0.11677E 03 0.25659E 02 0.25621E 03 0.27594E 03 0.     -0.

1.31250  -0.13343E 02 0.15144E 01 0.      -0.59556E 02 0.11689E 03 0.25536E 02 0.25621E 03 0.27590E 03 0.     -0.
1.40625  -0.71569E 01 0.15090E 01 0.      -0.68418E 02 0.10584E 02 0.90971E 02 0.91529E 02 0.17800E 03 0.     -0.
1.50000  -0.27823E 01 0.16059E 01 0.       0.48797E-01-0.48221E 03 0.57924E 03-0.14461E 03 0.17377E 03 0.     -0.


MAIN SHELL PART NO  5
0.        0.11535E 01 0.71402E 02 0.      -0.39715E 02 0.14754E 03 0.51085E 02 0.60411E 02 0.31474E 02-0.      0.
0.12500   0.61554E 01 0.71737E 02 0.      -0.37493E 02 0.42263E 02 0.15636E 03 0.98855E 02 0.13308E 03-0.      0.
0.25000   0.10133E 02 0.71985E 02 0.      -0.25481E 02 0.16978E 02 0.18165E 03 0.14695E 03 0.19635E 03-0.      0.

0.25000   0.10133E 02 0.71985E 02 0.      -0.25481E 02 0.16973E 02 0.18165E 03 0.14695E 03 0.19635E 03-0.      0.
0.37500   0.12484E 02 0.72174E 02 0.      -0.12231E 02 0.20073E 02 0.17855E 03 0.18079E 03 0.22833E 03-0.      0.
0.50000   0.13217E 02 0.72334E 02 0.       0.44688E-00 0.20659E 02 0.17797E 03 0.19126E 03 0.23845E 03-0.      0.

0.50000   0.13219E 02 0.72334E 02 0.       0.44764E-00 0.20653E 02 0.17797E 03 0.19126E 03 0.23845E 03-0.      0.
0.62500   0.12334E 02 0.72495E 02 0.       0.14031E 02 0.90660E 01 0.18956E 03 0.17539E 03 0.22954E 03-0.      0.
0.75000   0.96130E 01 0.72690E 02 0.       0.29638E 02-0.28260E 01 0.20145E 03 0.13373E 03 0.19502E 03-0.      0.

0.75000   0.96129E 01 0.72690E 02 0.       0.29839E 02-0.28315E 01 0.20146E 03 0.13373E 03 0.19502E 03-0.      0.
0.87500   0.48753E 01 0.72954E 02 0.       0.45346E 02 0.20917E 02 0.17771E 03 0.74530E 02 0.12157E 03-0.      0.
1.00000  -0.12917E 01 0.73323E 02 0.       0.50085E 02 0.14192E 03 0.56705E 02 0.24492E 02-0.10727E 01-0.      0.

MAIN SHELL PART NO  6
1.57080  -0.12915E 01 0.73323E 02 0.       0.50086E 02 0.14191E 03 0.56711E 02 0.24489E 02-0.10715E 01-0.      0.
1.60352   0.29833E-00 0.73397E 02 0.       0.48921E 02 0.16627E 03 0.33173E 02 0.20989E 02-0.20062E 02-0.      0.
1.63625   0.19109E 01 0.73420E 02 0.       0.47279E 02 0.18748E 03 0.12740E 02 0.16771E 02-0.37818E 02-0.      0.

1.63625   0.19109E 01 0.73420E 02 0.       0.47279E 02 0.18748E 03 0.12740E 02 0.16771E 02-0.37818E 02-0.      0.
1.66897   0.35502E 01 0.73393E 02 0.       0.45219E 02 0.20575E 03-0.47829E 01 0.11965E 02-0.54303E 02-0.      0.
1.70170   0.52196E 01 0.73314E 02 0.       0.42790E 02 0.22128E 03-0.19573E 02 0.66845E 01-0.69491E 02-0.      0.

1.73170   0.52196E 01 0.73314E 02 0.       0.42790E 02 0.22128E 03-0.19573E 02 0.66845E 01-0.69491E 02-0.      0.
1.73442   0.69219E 01 0.73162E 02 0.       0.40638E 02 0.23425E 03-0.31793E 02 0.10362E 01-0.83357E 02-0.      0.
1.76715   0.86598E 01 0.72997E 02 0.       0.37004E 02 0.24482E 03-0.41592E 02-0.48830E 01-0.95885E 02-0.      0.

1.76715   0.86598E 01 0.72997E 02 0.       0.37004E 02 0.24482E 03-0.41592E 02-0.48830E 01-0.95885E 02-0.      0.
1.79987   0.10436E 02 0.72757E 02 0.       0.33724E 02 0.25315E 03-0.49105E 02-0.10982E 02-0.10706E 03-0.      0.
1.83260   0.12252E 02 0.72462E 02 0.       0.30234E 02 0.25939E 03-0.54460E 02-0.17176E 02-0.11686E 03-0.      0.

1.83260   0.12252E 02 0.72462E 02 0.       0.30234E 02 0.25939E 03-0.54460E 02-0.17176E 02-0.11686E 03-0.      0.
1.86532   0.14116E 02 0.72110E 02 0.       0.26564E 02 0.26367E 03-0.57775E 02-0.23382E 02-0.12527E 03-0.      0.
1.89805   0.16013E 02 0.71700E 02 0.       0.22745E 02 0.26613E 03-0.59162E 02-0.29520E 02-0.13230E 03-0.      0.

1.89805   0.16013E 02 0.71700E 02 0.       0.22745E 02 0.26613E 03-0.59162E 02-0.29520E 02-0.13230E 03-0.      0.
1.93077   0.17964E 02 0.71230E 02 0.       0.18605E 02 0.26649E 03-0.58727E 02-0.35512E 02-0.13792E 03-0.      0.
1.96350   0.19964E 02 0.70699E 02 0.       0.14773E 02 0.26608E 03-0.56573E 02-0.41280E 02-0.14212E 03-0.      0.

1.96350   0.19964E 02 0.70699E 02 0.       0.14773E 02 0.26608E 03-0.56573E 02-0.41280E 02-0.14212E 03-0.      0.
1.99622   0.22015E 02 0.70107E 02 0.       0.10676E 02 0.26381E 03-0.52601E 02-0.46747E 02-0.14489E 03-0.      0.
2.02895   0.24124E 02 0.69451E 02 0.       0.65421E 01 0.26022E 03-0.47511E 02-0.51835E 02-0.14624E 03-0.      0.
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2.02895    0.24124E 02 0.69451E 02 0.          0.65421E 01 0.26022E 03-0.47511E 02-0.51835E 02-0.14624E 03-0.        0.
2.06167    0.26291E 02 0.68729E 02 0.          0.23994E 01 0.25541E 03-0.40803E 02-0.56464E 02-0.14615E 03-0.        0.
2.09440    0.28521E 02 0.67941E 02 0.         -0.17232E 01 0.24952E 03-0.32781E 02-0.60553E 02-0.14462E 03-0.        0.

MAIN SHELL PART NO  7
2.09440    0.28521E 02 0.67941E 02 0.         -0.17232E 01 0.24952E 03-0.32781E 02-0.60553E 02-0.14462E 03-0.        0.
2.15725    0.32992E 02 0.66233E 02 0.         -0.94736E 01 0.23565E 03-0.14087E 02-0.66590E 02-0.13764E 03-0.        0.
2.22010    0.37737E 02 0.64259E 02 0.         -0.16813E 02 0.21924E 03 0.82270E 01-0.69733E 02-0.12543E 03-0.        0.

2.22010    0.37737E 02 0.64259E 02 0.         -0.16813E 02 0.21924E 03 0.82269E 01-0.69733E 02-0.12543E 03-0.        0.
2.28295    0.42788E 02 0.62003E 02 0.         -0.23496E 02 0.20137E 03 0.33229E 02-0.69383E 02-0.10812E 03-0.        0.
2.34580    0.48175E 02 0.59451E 02 0.         -0.29262E 02 0.18322E 03 0.59656E 02-0.64966E 02-0.86016E 02-0.        0.

2.34560    0.48175E 02 0.59451E 02 0.         -0.29263E 02 0.18322E 03 0.59855E 02-0.64966E 02-0.86016E 02-0.        0.
2.40865    0.53921E 02 0.56585E 02 0.         -0.33848E 02 0.16608E 03 0.86914E 02-0.55986E 02-0.59591E 02-0.        0.
2.47150    0.60033E 02 0.53368E 02 0.         -0.37006E 02 0.15124E 03 0.11312E 03-0.42112E 02-0.29540E 02-0.        0.

2.47150    0.60033E 02 0.53368E 02 0.         -0.37007E 02 0.15124E 03 0.11312E 03-0.42112E 02-0.29539E 02-0.        0.
2.53435    0.66493E 02 0.49844E 02 0.         -0.38546E 02 0.13991E 03 0.13721E 03-0.23280E 02 0.32107E 01-0.        0.
2.59720    0.73245E 02 0.45941E 02 0.         -0.38371E 02 0.13297E 03 0.15811E 03 0.18104E-00 0.37522E 02-0.        0.

2.59720    0.73245E 02 0.45941E 02 0.         -0.38371E 02 0.13297E 03 0.15811E 03 0.18365E-00 0.37525E 02-0.        0.
2.66005    0.80180E 02 0.41672E 02 0.         -0.36517E 02 0.13070E 03 0.17517E 03 0.27444E 02 0.72141E 02-0.        0.
2.72290    0.87130E 02 0.37041E 02 0.         -0.33143E 02 0.13243E 03 0.18847E 03 0.57112E 02 0.10580E 03-0.        0.

2.72290    0.87131E 02 0.37041E 02 0.         -0.33144E 02 0.13243E 03 0.18847E 03 0.57120E 02 0.10582E 03-0.        0.
2.78575    0.93851E 02 0.32062E 02 0.         -0.28427E 02 0.13633E 03 0.19897E 03 0.87266E 02 0.13733E 03-0.        0.
2.84860    0.99993E 02 0.26767E 02 0.         -0.22328E 02 0.13957E 03 0.20830E 03 0.11555E 03 0.16514E 03-0.        0.

2.84860    0.99993E 02 0.26767E 02 0.         -0.22330E 02 0.13958E 03 0.20829E 03 0.11555E 03 0.16513E 03-0.        0.
2.91145    0.10503E 03 0.21209E 02 0.         -0.14323E 02 0.13960E 03 0.21762E 03 0.13952E 03 0.18594E 03-0.        0.
2.97430    0.10812E 03 0.15461E 02 0.         -0.36427E 01 0.13893E 03 0.22401E 03 0.15760E 03 0.19098E 03-0.        0.

2.97430    0.10812E 03 0.15480E 02 0.         -0.36238E 01 0.13879E 03 0.22411E 03 0.15762E 03 0.19103E 03-0.        0.
3.03715    0.10835E 03 0.97516E 01 0.          0.79518E 01 0.16267E 03 0.21095E 03 0.17125E 03 0.15876E 03-0.        0.
3.10000    0.10681E 03 0.44475E 01 0.         -0.98109E-01 0.40676E 03 0.11432E 03 0.12123E 03 0.34324E 02-0.        0.
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                          SUBCASE NO  2   FOR FOURIER HARMONIC COS 2 THETA

BOUNDARY CONDITIONS AT STARTING EDGE      2-0.          3-0.          5-0.          7-0.

BOUNDARY CONDITIONS AT FINAL    EDGE      2-0.          3-0.          5-0.          7-0.

BOUNDARY CONDITION AT BRANCH EDGE NO  1   2-0.          3-0.          5-0.          7-0.


                   LOADS FOR PART NO  1  SUBCASE NO  2

RING LOADS AT END OF THIS PART ARE   Q=-0.          NPHI=-0.          MPHI=-0.          N=-0.

SURFACE AND TEMP LOADS ARE  P=-0.            PFI=-0.           PTHETA=-0.          TL=-0.          TU=-0.


                   LOADS FOR PART NO  2  SUBCASE NO  2

RING LOADS AT END OF THIS PART ARE   Q=-0.          NPHI=-0.          MPHI=-0.          N=-0.

SURFACE AND TEMP LOADS ARE  P=-0.            PFI=-0.           PTHETA=-0.          TL=-0.          TU=-0.


                   LOADS FOR PART NO  3  SUBCASE NO  2

RING LOADS AT END OF THIS PART ARE   Q=-0.          NPHI=-0.          MPHI=-0.          N=-0.

SURFACE AND TEMP LOADS ARE  P=-0.20000E 02  PFI=-0.           PTHETA=-0.          TL=-0.          TU=-0.


                   LOADS FOR PART NO  4  SUBCASE NO  2

RING LOADS AT END OF THIS PART ARE   Q=-0.          NPHI=-0.          MPHI=-0.          N=-0.

SURFACE AND TEMP LOADS ARE  P=-0.20000E 02  PFI=-0.           PTHETA=-0.          TL=-0.          TU=-0.


                   LOADS FOR PART NO  5  SUBCASE NO  2

RING LOADS AT END OF THIS PART ARE   Q=-0.          NPHI=-0.          MPHI=-0.          N=-0.

SURFACE AND TEMP LOADS ARE  P= 0.20000E 02  PFI=-0.           PTHETA=-0.          TL=-0.          TU=-0.


                   LOADS FOR PART NO  6  SUBCASE NO  2

RING LOADS AT END OF THIS PART ARE   Q=-0.          NPHI=-0.          MPHI=-0.          N=-0.

SURFACE AND TEMP LOADS ARE  P= 0.20000E 02  PFI=-0.           PTHETA=-0.          TL=-0.          TU=-0.


                   LOADS FOR PART NO  7  SUBCASE NO  2

RING LOADS AT END OF THIS PART ARE   Q=-0.          NPHI=-0.          MPHI=-0.          N=-0.

SURFACE AND TEMP LOADS ARE  P= 0.20000E 02  PFI=-0.           PTHETA=-0.          TL=-0.          TU=-0.
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STRESS ANALYSIS PROGRAM OF SHELLS OF REVOLUTION UNDER STATIC LOADS, BY A. KALNINS, LEHIGH UNIV, BETHLEHEM, PA
WRIGHT PATTERSON AIR FORCE BASE FLIGHT DYNAMICS LABORATORY VERSION, 22 JULY 1968

STATIC SOLUTION AT POINTS ALONG MERIDIAN FOR WAVE NUMBER NX= 2

```
   S          W           V          UPHI         NPHI        BPHI         MPHI       UTHETA        N          NTHETA      MTHETA

MAIN SHELL PART NO  1
0.        0.16961E 02-0.          -0.          -0.17639E 02-0.          -0.23399E-00-0.          -0.14160E 02 0.56076E 01-0.14414E-01
0.12500   0.18245E 02 0.23025E-00-0.14971E 01-0.15907E 02-0.20263E 02-0.19962E-00-0.34607E 01-0.13695E 02 0.21498E 01-0.31966E-01
0.25000   0.21899E 02 0.29337E-00-0.28092E 01-0.14209E 02-0.37556E 02-0.15264E-00-0.71017E 01-0.13562E 02 0.21655E-00-0.30589E-01

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0.50000   0.16489E 03-0.11741E-00 0.94761E-01 0.17186E 02-0.38236E 02 0.18088E-01-0.87643E 02-0.14452E 02-0.21237E 01 0.40737E-01

0.50000   0.16489E 03-0.11730E-00 0.94709E-01 0.17186E 02-0.38230E 02 0.18110E-01-0.87642E 02-0.14452E 02-0.21236E 01 0.40743E-01
0.62500   0.16968E 03-0.26851E-00 0.17772E 01 0.19011E 02-0.40106E 02-0.73237E-02-0.90917E 02-0.14761E 02-0.28066E 01 0.34033E-01
0.75000   0.17550E 03-0.44100E-00 0.36331E 01 0.20879E 02-0.56070E 02-0.54056E-01-0.94050E 02-0.15112E 02-0.25581E 01 0.21254E-01

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1.00000   0.20028E 03-0.30394E-00 0.76431E 01 0.24679E 02-0.16002E 03-0.18410E-00-0.99700E 02-0.14839E 02 0.80195E 01-0.11355E-01

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1.91987  -0.90305E 02 0.95989E 00-0.55037E 02 0.17601E 02-0.62016E 03 0.50041E-01-0.11157E 03-0.10645E 02-0.28679E 02 0.16293E-01

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MAIN SHELL PART NO  5
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0.37500   0.23367E 03 0.40310E-01 0.15077E 02 0.40297E 02 0.11576E 02 0.20468E-00-0.96290E 02 0.48665E 01 0.40855E 02 0.11552E-00
0.50000   0.22758E 03 0.16094E-00 0.18521E 02 0.39363E 02 0.66737E 02 0.21895E-00-0.92392E 02 0.10088E 02 0.41768E 02 0.11859E-00
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| 0.62500 | 0.21158E 03 | 0.18717E-00 | 0.20394E 02 | 0.37708E 02 | 0.17134E 03 | 0.24996E-00 | -0.86989E 02 | 0.15118E 02 | 0.37659E 02 | 0.12402E-00 |
| 0.75000 | 0.18424E 03 | -0.22496E-00 | 0.23288E 02 | 0.35615E 02 | 0.26745E 03 | 0.26552E-00 | -0.80208E 02 | 0.19260E 02 | 0.27359E 02 | 0.12169E-00 |
| 0.75000 | 0.18424E 03 | -0.22490E-00 | 0.23283E 02 | 0.35615E 02 | 0.26747E 03 | 0.26555E-00 | -0.80206E 02 | 0.19260E 02 | 0.27359E 02 | 0.12170E-00 |
| 0.87500 | 0.14449E 03 | -0.15101E 01 | 0.25838E 02 | 0.33030E 02 | 0.35553E 03 | 0.18482E-00 | -0.72338E 02 | 0.21679E 02 | 0.10131E 02 | 0.87186E-01 |
| 1.00000 | 0.98181E 02 | -0.40433E 01 | 0.28681E 02 | 0.30291E 02 | 0.37360E 03 | -0.12784E-00 | -0.63848E 02 | 0.21605E 02 | -0.11573E 02 | -0.19028E-01 |

MAIN SHELL PART NO 6

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| 1.60352 | 0.93023E 02 | -0.34698E 01 | 0.25951E 02 | 0.30065E 02 | 0.36523E 03 | -0.18765E-00 | -0.62738E 02 | 0.21399E 02 | -0.14330E 02 | -0.39867E-01 |
| 1.63625 | 0.87925E 02 | -0.29686E 01 | 0.23396E 02 | 0.29837E 02 | 0.35408E 03 | -0.23773E-00 | -0.61658E 02 | 0.21158E 02 | -0.17040E 02 | -0.58009E-01 |
| 1.63625 | 0.87925E 02 | -0.29686E 01 | 0.23396E 02 | 0.29837E 02 | 0.35408E 03 | -0.23773E-00 | -0.61658E 02 | 0.21156E 02 | -0.17040E 02 | -0.58009E-01 |
| 1.66897 | 0.82922E 02 | -0.24797E 01 | 0.21016E 02 | 0.29609E 02 | 0.34051E 03 | -0.28047E-00 | -0.60606E 02 | 0.20883E 02 | -0.19677E 02 | -0.73588E-01 |
| 1.70170 | 0.78045E 02 | -0.20234E 01 | 0.18307E 02 | 0.29383E 02 | 0.32487E 03 | -0.31584E-00 | -0.59578E 02 | 0.20573E 02 | -0.22215E 02 | -0.86740E-01 |
| 1.70170 | 0.78045E 02 | -0.20234E 01 | 0.18307E 02 | 0.29383E 02 | 0.32487E 03 | -0.31584E-00 | -0.59578E 02 | 0.20573E 02 | -0.22215E 02 | -0.86740E-01 |
| 1.73442 | 0.73323E 02 | -0.15997E 01 | 0.16766E 02 | 0.29161E 02 | 0.30745E 03 | -0.34437E-00 | -0.58570E 02 | 0.20229E 02 | -0.24634E 02 | -0.97593E-01 |
| 1.76715 | 0.68781E 02 | -0.12086E 01 | 0.14887E 02 | 0.28947E 02 | 0.28852E 03 | -0.36654E-00 | -0.57579E 02 | 0.19850E 02 | -0.26913E 02 | -0.10627E-00 |
| 1.76715 | 0.68781E 02 | -0.12086E 01 | 0.14887E 02 | 0.28947E 02 | 0.28852E 03 | -0.36654E-00 | -0.57579E 02 | 0.19850E 02 | -0.26913E 02 | -0.10627E-00 |
| 1.79987 | 0.64444E 02 | -0.84999E 00 | 0.13166E 02 | 0.28742E 02 | 0.26831E 03 | -0.38281E-00 | -0.56603E 02 | 0.19436E 02 | -0.29035E 02 | -0.11289E-00 |
| 1.83260 | 0.60335E 02 | -0.52369E 00 | 0.11596E 02 | 0.28549E 02 | 0.24705E 03 | -0.39360E-00 | -0.55638E 02 | 0.18987E 02 | -0.30985E 02 | -0.11755E-00 |
| 1.83260 | 0.60335E 02 | -0.52369E 00 | 0.11596E 02 | 0.28549E 02 | 0.24705E 03 | -0.39360E-00 | -0.55638E 02 | 0.18987E 02 | -0.30985E 02 | -0.11755E-00 |
| 1.86532 | 0.56476E 02 | -0.22936E-00 | 0.10170E 02 | 0.28371E 02 | 0.22494E 03 | -0.39930E-00 | -0.54683E 02 | 0.18504E 02 | -0.32748E 02 | -0.12038E-00 |
| 1.89805 | 0.52890E 02 | 0.33381E-01 | 0.88815E 01 | 0.28210E 02 | 0.20217E 03 | -0.40031E-00 | -0.53735E 02 | 0.17986E 02 | -0.34310E 02 | -0.12147E-00 |
| 1.89805 | 0.52890E 02 | 0.33379E-01 | 0.88815E 01 | 0.28210E 02 | 0.20217E 03 | -0.40031E-00 | -0.53735E 02 | 0.17986E 02 | -0.34310E 02 | -0.12147E-00 |
| 1.93077 | 0.49598E 02 | 0.26563E-00 | 0.77215E 01 | 0.28069E 02 | 0.17892E 03 | -0.39697E-00 | -0.52791E 02 | 0.17433E 02 | -0.35658E 02 | -0.12093E-00 |
| 1.96350 | 0.46623E 02 | 0.46618E-00 | 0.66814E 01 | 0.27951E 02 | 0.15537E 03 | -0.38965E-00 | -0.51850E 02 | 0.16846E 02 | -0.36782E 02 | -0.11885E-00 |
| 1.96350 | 0.46623E 02 | 0.46618E-00 | 0.66814E 01 | 0.27951E 02 | 0.15537E 03 | -0.38965E-00 | -0.51850E 02 | 0.16846E 02 | -0.36782E 02 | -0.11885E-00 |
| 1.99622 | 0.43989E 02 | 0.63755E 00 | 0.57518E 01 | 0.27859E 02 | 0.13169E 03 | -0.37669E-00 | -0.50910E 02 | 0.16225E 02 | -0.37668E 02 | -0.11535E-00 |
| 2.02895 | 0.41717E 02 | 0.77997E 00 | 0.49225E 01 | 0.27797E 02 | 0.10604E 03 | -0.36443E-00 | -0.49969E 02 | 0.15569E 02 | -0.38307E 02 | -0.11053E-00 |
| 2.02895 | 0.41717E 02 | 0.77997E 00 | 0.49225E 01 | 0.27797E 02 | 0.10604E 03 | -0.36443E-00 | -0.49969E 02 | 0.15569E 02 | -0.38307E 02 | -0.11053E-00 |
| 2.06167 | 0.39833E 02 | 0.89441E 00 | 0.41828E 01 | 0.27767E 02 | 0.84587E 02 | -0.34722E-00 | -0.49026E 02 | 0.14879E 02 | -0.38669E 02 | -0.10449E-00 |
| 2.09440 | 0.38361E 02 | 0.98199E 00 | 0.35214E 01 | 0.27773E 02 | 0.61505E 02 | -0.32741E-00 | -0.48079E 02 | 0.14157E 02 | -0.38805E 02 | -0.97348E-01 |

MAIN SHELL PART NO 7

| | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| 2.09440 | 0.38361E 02 | 0.98199E 00 | 0.35214E 01 | 0.27773E 02 | 0.61505E 02 | -0.32741E-00 | -0.48079E 02 | 0.14157E 02 | -0.38805E 02 | -0.97348E-01 |
| 2.15725 | 0.36776E 02 | 0.10797E 01 | 0.24255E 01 | 0.27893E 02 | 0.18824E 02 | -0.28336E-00 | -0.46248E 02 | 0.12680E 02 | -0.38252E 02 | -0.80937E-01 |
| 2.22010 | 0.36973E 02 | 0.10942E 01 | 0.14813E 01 | 0.28198E 02 | -0.20665E 02 | -0.23364E-00 | -0.44399E 02 | 0.11098E 02 | -0.36647E 02 | -0.61706E-01 |
| 2.22010 | 0.36972E 02 | 0.10942E 01 | 0.14813E 01 | 0.28198E 02 | -0.20665E 02 | -0.23364E-00 | -0.44398E 02 | 0.11098E 02 | -0.36647E 02 | -0.61706E-01 |
| 2.28295 | 0.39114E 02 | 0.10394E 01 | 0.58924E 00 | 0.28698E 02 | -0.55737E 02 | -0.18100E-00 | -0.42536E 02 | 0.94314E 01 | -0.33971E 02 | -0.40552E-01 |
| 2.34580 | 0.43336E 02 | 0.93201E 00 | -0.35551E-00 | 0.29431E 02 | -0.85200E 02 | -0.12833E-00 | -0.40672E 02 | 0.77130E 01 | -0.30248E 02 | -0.18400E-01 |
| 2.34580 | 0.43336E 02 | 0.93201E 00 | -0.35552E-00 | 0.29431E 02 | -0.85200E 02 | -0.12833E-00 | -0.40672E 02 | 0.77130E 01 | -0.30248E 02 | -0.18400E-01 |
| 2.40865 | 0.49723E 02 | 0.79215E 00 | -0.14592E 01 | 0.30428E 02 | -0.10798E 03 | -0.78458E-01 | -0.38831E 02 | 0.59963E 01 | -0.25550E 02 | 0.38690E-02 |
| 2.47150 | 0.58274E 02 | 0.64244E 00 | -0.28227E 01 | 0.31724E 02 | -0.12325E 03 | -0.33933E-01 | -0.37048E 02 | 0.43616E 01 | -0.20026E 02 | 0.25549E-01 |
| 2.47150 | 0.58274E 02 | 0.64244E 00 | -0.28227E 01 | 0.31724E 02 | -0.12325E 03 | -0.33933E-01 | -0.37048E 02 | 0.43616E 01 | -0.20026E 02 | 0.25549E-01 |
| 2.53435 | 0.68860E 02 | 0.50703E 00 | -0.45315E 01 | 0.33355E 02 | -0.13052E 03 | 0.35127E-02 | -0.35373E 02 | 0.29278E 01 | -0.13910E 02 | 0.46329E-01 |
| 2.59720 | 0.81182E 02 | 0.40927E-00 | -0.66404E 01 | 0.35356E 02 | -0.12970E 03 | 0.33640E-01 | -0.33862E 02 | 0.18677E 01 | -0.75496E 01 | 0.66604E-01 |
| 2.59720 | 0.81183E 02 | 0.40930E-00 | -0.66405E 01 | 0.35356E 02 | -0.12970E 03 | 0.33641E-01 | -0.33862E 02 | 0.18678E 01 | -0.75492E 01 | 0.66605E-01 |
| 2.66005 | 0.94692E 02 | 0.36792E-00 | -0.91530E 01 | 0.37761E 02 | -0.12075E 03 | 0.58508E-01 | -0.32563E 02 | 0.14291E 01 | -0.14190E 01 | 0.87843E-01 |
| 2.72290 | 0.10844E 03 | 0.39072E-00 | -0.11990E 02 | 0.40593E 02 | -0.10285E 03 | 0.82992E-01 | -0.31483E 02 | 0.19627E 01 | 0.38335E 01 | 0.11284E-00 |

| | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| 2.72290 | 0.10844E 03 | 0.39073E-00 | -0.11990E 02 | 0.40593E 02 | -0.10285E 03 | 0.82991E-01 | -0.31483E 02 | 0.19628E 01 | 0.38337E 01 | 0.11284E-00 |
| 2.78575 | 0.12067E 03 | 0.46359E-00 | -0.14941E 02 | 0.43855E 02 | -0.72809E 02 | 0.11409E-00 | -0.30521E 02 | 0.39547E 01 | 0.72937E 01 | 0.14545E-00 |
| 2.84860 | 0.12803E 03 | 0.52933E 00 | -0.17570E 02 | 0.47512E 02 | -0.23634E 02 | 0.15693E-00 | -0.29357E 02 | 0.80275E 01 | 0.75904E 01 | 0.18875E-00 |
| | | | | | | | | | | |
| 2.84860 | 0.12803E 03 | 0.52927E 00 | -0.17570E 02 | 0.47512E 02 | -0.23638E 02 | 0.15692E-00 | -0.29357E 02 | 0.80273E 01 | 0.75904E 01 | 0.18875E-00 |
| 2.91145 | 0.12439E 03 | 0.44791E-00 | -0.19051E 02 | 0.51543E 02 | 0.53060E 02 | 0.20365E-00 | -0.27322E 02 | 0.14783E 02 | 0.27264E 01 | 0.23980E-00 |
| 2.97430 | 0.10134E 03 | -0.65539E-01 | -0.17949E 02 | 0.56368E 02 | 0.15212E 03 | 0.21217E-00 | -0.23300E 02 | 0.24118E 02 | -0.93975E 01 | 0.27845E-00 |
| | | | | | | | | | | |
| 2.97430 | 0.10135E 03 | -0.65388E-01 | -0.17950E 02 | 0.56363E 02 | 0.15209E 03 | 0.21220E-00 | -0.23298E 02 | 0.24122E 02 | -0.93871E 01 | 0.27849E-00 |
| 3.03715 | 0.55753E 02 | -0.15590E 01 | -0.12104E 02 | 0.65441E 02 | 0.21684E 03 | 0.80070E-01 | -0.15811E 02 | 0.33209E 02 | -0.27347E 02 | 0.25127E-00 |
| 3.10000 | 0.15244E 02 | -0.34895E 01 | 0.61152E 00 | 0.81283E 02 | 0.14416E 01 | -0.88505E 00 | 0.33914E-01 | 0.69407E 02 | 0.25150E 02 | 0.37822E-00 |

STRESS ANALYSIS PROGRAM OF SHELLS OF REVOLUTION UNDER STATIC LOADS, BY A. KALNINS, LEHIGH UNIV, BETHLEHEM, PA
WRIGHT PATTERSON AIR FORCE BASE FLIGHT DYNAMICS LABORATORY VERSION, 22 JULY 1968

STATIC SOLUTION AT POINTS ALONG MERIDIAN FOR WAVE NUMBER NX= 2

| S | W | UPHI | UTHETA | BPHI | SPHI IN | SPHI OUT | STHETA IN | STHETA OUT | SFITH IN | SFITH OUT |
|---|---|---|---|---|---|---|---|---|---|---|
| MAIN SHELL PART NO 1 | | | | | | | | | | |
| 0. | 0.16961E 02 | -0. | -0. | -0. | -0.19540E 03 | -0.31846E 03 | 0.16696E 03 | 0.14191E 03 | -0.28837E 03 | -0.29202E 03 |
| 0. | 0.16961E 02 | -0. | -0. | -0. | -0.15837E 03 | -0.29026E 03 | 0.55378E 02 | 0.14492E 02 | -0.13558E 03 | -0.13897E 03 |
| 0.12500 | 0.18243E 02 | -0.14971E 01 | -0.34607E 01 | -0.20263E 02 | -0.17680E 03 | -0.29118E 03 | 0.93898E 02 | 0.71143E 02 | -0.29622E 03 | -0.28912E 03 |
| 0.12500 | 0.18243E 02 | -0.14971E 01 | -0.34607E 01 | -0.20263E 02 | -0.14829E 03 | -0.26699E 03 | 0.19933E 02 | -0.16351E 02 | -0.13424E 03 | -0.12785E 03 |
| 0.25000 | 0.21899E 02 | -0.28092E 01 | -0.71017E 01 | -0.37556E 02 | -0.17955E 03 | -0.26863E 03 | 0.41027E 02 | 0.27145E 02 | -0.30847E 03 | -0.29222E 03 |
| 0.25000 | 0.21899E 02 | -0.28092E 01 | -0.71017E 01 | -0.37556E 02 | -0.13910E 03 | -0.22814E 03 | -0.17713E 01 | -0.25348E 02 | -0.13570E 03 | -0.12161E 03 |
| 0.25000 | 0.21698E 02 | -0.28092E 01 | -0.71017E 01 | -0.37551E 02 | -0.17953E 03 | -0.26863E 03 | 0.41017E 02 | 0.27131E 02 | -0.30847E 03 | -0.29229E 03 |
| 0.25000 | 0.21698E 02 | -0.28092E 01 | -0.71017E 01 | -0.37551E 02 | -0.13910E 03 | -0.22815E 03 | -0.17786E 01 | -0.25361E 02 | -0.13570E 03 | -0.12161E 03 |
| 0.37500 | 0.27445E 02 | -0.39767E 01 | -0.10953E 02 | -0.50460E 02 | -0.18486E 03 | -0.24563E 03 | 0.67022E 01 | 0.38482E 01 | -0.32124E 03 | -0.29834E 03 |
| 0.37500 | 0.27445E 02 | -0.39767E 01 | -0.10953E 02 | -0.50460E 02 | -0.12844E 03 | -0.18644E 03 | -0.12605E 02 | -0.21872E 02 | -0.13851E 03 | -0.11924E 03 |
| 0.50000 | 0.34337E 02 | -0.50146E 01 | -0.15020E 02 | -0.59186E 02 | -0.18358E 03 | -0.21994E 03 | -0.13680E 02 | -0.58106E 01 | -0.33270E 03 | -0.30532E 03 |
| 0.50000 | 0.34337E 02 | -0.50146E 01 | -0.15020E 02 | -0.59186E 02 | -0.11555E 03 | -0.14807E 03 | -0.16135E 02 | -0.12710E 02 | -0.14176E 03 | -0.11951E 03 |
| 0.50000 | 0.34337E 02 | -0.50146E 01 | -0.15020E 02 | -0.59186E 02 | -0.18358E 03 | -0.21994E 03 | -0.13680E 02 | -0.58111E 01 | -0.33270E 03 | -0.30532E 03 |
| 0.50000 | 0.34337E 02 | -0.50146E 01 | -0.15020E 02 | -0.59186E 02 | -0.11555E 03 | -0.14807E 03 | -0.16136E 02 | -0.12710E 02 | -0.14176E 03 | -0.11951E 03 |
| 0.62500 | 0.42105E 02 | -0.59234E 01 | -0.19289E 02 | -0.64648E 02 | -0.17271E 03 | -0.19101E 03 | -0.25157E 02 | -0.79266E 01 | -0.34236E 03 | -0.31225E 03 |
| 0.62500 | 0.42105E 02 | -0.59234E 01 | -0.19289E 02 | -0.64648E 02 | -0.10053E 03 | -0.11498E 03 | -0.15522E 02 | -0.21302E 01 | -0.14498E 03 | -0.12138E 03 |
| 0.75000 | 0.50408E 02 | -0.66959E 01 | -0.23742E 02 | -0.67911E 02 | -0.15269E 03 | -0.15911E 03 | -0.31846E 02 | -0.67932E 01 | -0.35044E 03 | -0.31876E 03 |
| 0.75000 | 0.50408E 02 | -0.66959E 01 | -0.23742E 02 | -0.67911E 02 | -0.83745E 02 | -0.86779E 02 | -0.13029E 02 | 0.76754E 01 | -0.14799E 03 | -0.12409E 03 |
| 0.75000 | 0.50408E 02 | -0.66959E 01 | -0.23742E 02 | -0.67911E 02 | -0.15269E 03 | -0.15910E 03 | -0.31844E 02 | -0.67922E 01 | -0.35044E 03 | -0.31876E 03 |
| 0.75000 | 0.50408E 02 | -0.66959E 01 | -0.23742E 02 | -0.67911E 02 | -0.83745E 02 | -0.86780E 02 | -0.13028E 02 | 0.76756E 01 | -0.14799E 03 | -0.12409E 03 |
| 0.87500 | 0.59030E 02 | -0.73223E 01 | -0.28355E 02 | -0.69887E 02 | -0.12535E 03 | -0.12469E 03 | -0.36524E 02 | -0.49291E 01 | -0.35740E 03 | -0.32481E 03 |
| 0.87500 | 0.59030E 02 | -0.73223E 01 | -0.28355E 02 | -0.69887E 02 | -0.65609E 02 | -0.62129E 02 | -0.10004E 02 | 0.15916E 02 | -0.15081E 03 | -0.12717E 03 |
| 1.00000 | 0.67854E 02 | -0.77927E 01 | -0.33105E 02 | -0.71237E 02 | -0.92688E 02 | -0.88203E 02 | -0.40702E 02 | -0.34452E 01 | -0.36373E 03 | -0.33056E 03 |
| 1.00000 | 0.67854E 02 | -0.77927E 01 | -0.33105E 02 | -0.71237E 02 | -0.46407E 02 | -0.39557E 02 | -0.70548E 01 | 0.22618E 02 | -0.15347E 03 | -0.13037E 03 |
| 1.00000 | 0.67854E 02 | -0.77927E 01 | -0.33105E 02 | -0.71237E 02 | -0.92688E 02 | -0.88203E 02 | -0.40701E 02 | -0.34443E 01 | -0.36373E 03 | -0.33056E 03 |
| 1.00000 | 0.67854E 02 | -0.77927E 01 | -0.33105E 02 | -0.71237E 02 | -0.46407E 02 | -0.39557E 02 | -0.70544E 01 | 0.22619E 02 | -0.15347E 03 | -0.13037E 03 |
| 1.12500 | 0.76831E 02 | -0.80977E 01 | -0.37969E 02 | -0.72381E 02 | -0.56323E 02 | -0.49914E 02 | -0.44896E 02 | -0.24810E 01 | -0.36983E 03 | -0.33614E 03 |
| 1.12500 | 0.76831E 02 | -0.80977E 01 | -0.37969E 02 | -0.72381E 02 | -0.26293E 02 | -0.17839E 02 | -0.42706E 01 | 0.28215E 02 | -0.15607E 03 | -0.13359E 03 |
| 1.25000 | 0.85950E 02 | -0.82292E 01 | -0.42925E 02 | -0.73539E 02 | -0.17480E 02 | -0.99654E 01 | -0.48993E 02 | -0.15973E 01 | -0.37589E 03 | -0.34167E 03 |
| 1.25000 | 0.85950E 02 | -0.82292E 01 | -0.42925E 02 | -0.73539E 02 | -0.53151E 01 | 0.39763E 01 | -0.14299E 01 | 0.33321E 02 | -0.15863E 03 | -0.13679E 03 |
| 1.25000 | 0.85950E 02 | -0.82292E 01 | -0.42925E 02 | -0.73539E 02 | -0.17483E 02 | -0.99656E 01 | -0.48993E 02 | -0.15960E 01 | -0.37589E 03 | -0.34167E 03 |
| 1.25000 | 0.85950E 02 | -0.82292E 01 | -0.42925E 02 | -0.73539E 02 | -0.53151E 01 | 0.39784E 01 | -0.14292E 01 | 0.33323E 02 | -0.15863E 03 | -0.13679E 03 |
| 1.37500 | 0.95219E 02 | -0.81803E 01 | -0.47951E 02 | -0.74761E 02 | 0.22702E 02 | 0.31623E 02 | -0.52615E 02 | -0.68984E-01 | -0.38197E 03 | -0.34716E 03 |
| 1.37500 | 0.95219E 02 | -0.81803E 01 | -0.47951E 02 | -0.74761E 02 | 0.16560E 02 | 0.26798E 02 | 0.18462E 01 | 0.38669E 02 | -0.16118E 03 | -0.13996E 03 |
| 1.50000 | 0.10464E 03 | -0.79447E 01 | -0.53021E 02 | -0.75876E 02 | 0.62610E 02 | 0.74983E 02 | -0.55585E 02 | 0.28151E 01 | -0.38788E 03 | -0.35250E 03 |
| 1.50000 | 0.10464E 03 | -0.79447E 01 | -0.53021E 02 | -0.75876E 02 | 0.39444E 02 | 0.51891E 02 | 0.59379E 01 | 0.45097E 02 | -0.16366E 03 | -0.14313E 03 |
| 1.50000 | 0.10464E 03 | -0.79447E 01 | -0.53021E 02 | -0.75876E 02 | 0.62609E 02 | 0.74983E 02 | -0.55583E 02 | 0.28169E 01 | -0.38788E 03 | -0.35250E 03 |
| 1.50000 | 0.10464E 03 | -0.79447E 01 | -0.53021E 02 | -0.75876E 02 | 0.39444E 02 | 0.51891E 02 | 0.59388E 01 | 0.45098E 02 | -0.16366E 03 | -0.14313E 03 |
| 1.62500 | 0.11416E 03 | -0.75158E 01 | -0.58111E 02 | -0.76317E 02 | 0.99428E 02 | 0.12056E 03 | -0.58599E 02 | 0.73129E 01 | -0.39321E 03 | -0.35759E 03 |
| 1.62500 | 0.11416E 03 | -0.75158E 01 | -0.58111E 02 | -0.76317E 02 | 0.63584E 02 | 0.81385E 02 | 0.11007E 02 | 0.53499E 02 | -0.16603E 03 | -0.14639E 03 |
| 1.75000 | 0.12364E 03 | -0.68834E 01 | -0.63192E 02 | -0.74810E 02 | 0.12869E 03 | 0.16939E 03 | -0.64285E 02 | 0.12354E 02 | -0.39726E 03 | -0.36243E 03 |
| 1.75000 | 0.12364E 03 | -0.68834E 01 | -0.63192E 02 | -0.74810E 02 | 0.89464E 02 | 0.11858E 03 | 0.16554E 02 | 0.64442E 02 | -0.16827E 03 | -0.15008E 03 |
| 1.75000 | 0.12364E 03 | -0.68834E 01 | -0.63192E 02 | -0.74809E 02 | 0.12868E 03 | 0.16939E 03 | -0.64284E 02 | 0.12358E 02 | -0.39726E 03 | -0.36243E 03 |
| 1.75000 | 0.12364E 03 | -0.68834E 01 | -0.63192E 02 | -0.74809E 02 | 0.89465E 02 | 0.11859E 03 | 0.16556E 02 | 0.64446E 02 | -0.16827E 03 | -0.15008E 03 |
| 1.87500 | 0.13269E 03 | -0.60273E 01 | -0.68237E 02 | -0.68990E 02 | 0.14485E 03 | 0.22347E 03 | -0.78576E 02 | 0.14005E 02 | -0.39927E 03 | -0.36752E 03 |
| 1.87500 | 0.13269E 03 | -0.60273E 01 | -0.68237E 02 | -0.68990E 02 | 0.11789E 03 | 0.16750E 03 | 0.20638E 02 | 0.77136E 02 | -0.17064E 03 | -0.15498E 03 |
| 2.00000 | 0.14056E 03 | -0.49083E 01 | -0.73227E 02 | -0.55312E 02 | 0.14492E 03 | 0.28552E 03 | -0.11183E 03 | 0.31118E 01 | -0.39894E 03 | -0.37450E 03 |
| 2.00000 | 0.14056E 03 | -0.49083E 01 | -0.73227E 02 | -0.55312E 02 | 0.14974E 03 | 0.23023E 03 | 0.18625E 02 | 0.87200E 02 | -0.17387E 03 | -0.16253E 03 |

```
MAIN SHELL PART NO  2
0.          0.14056E 03-0.49083E 01-0.73227E 02-0.55312E 02 0.19389E 03 0.21131E 03-0.97135E 02 0.53614E 02-0.30253E 03-0.25120E 03
0.12500     0.14749E 03-0.38961E 01-0.77007E 02-0.54664E 02 0.19204E 03 0.28271E 03-0.11002E 03 0.69767E 02-0.30487E 03-0.25404E 03
0.25000     0.15404E 03-0.27334E 01-0.80681E 02-0.49679E 02 0.20778E 03 0.33713E 03-0.11973E 03 0.78259E 02-0.30475E 03-0.25903E 03

0.25000     0.15404E 03-0.27335E 01-0.80681E 02-0.49674E 02 0.20773E 03 0.33718E 03-0.11974E 03 0.78275E 02-0.30474E 03-0.25903E 03
0.37500     0.15984E 03-0.14063E 01-0.84230E 02-0.43093E 02 0.24145E 03 0.37428E 03-0.13061E 03 0.74058E 02-0.30426E 03-0.26535E 03
0.50000     0.16489E 03 0.94761F-01-0.87643E 02-0.38236E 02 0.30030E 03 0.38713E 03-0.14024E 03 0.55295E 02-0.30610E 03-0.27213E 03

0.50000     0.16489E 03 0.94709E-01-0.87642E 02-0.38230E 02 0.30025E 03 0.38718E 03-0.14026E 03 0.55311E 02-0.30609E 03-0.27213E 03
0.62500     0.16968E 03 0.17772E 01-0.90917E 02-0.40106E 02 0.39780E 03 0.36265E 03-0.13781E 03 0.25549E 02-0.31345E 03-0.27716E 03
0.75000     0.17550E 03 0.36331E 01-0.94050E 02-0.56070E 02 0.54731E 03 0.28785E 03-0.10217E 03-0.15174E-00-0.32925E 03-0.27545E 03

0.75000     0.17550E 03 0.36330E 01-0.94050E 02-0.56063E 02 0.54726E 03 0.28790E 03-0.10219E 03-0.13474E-00-0.32924E 03-0.27544E 03
0.87500     0.18464E 03 0.56209E 01-0.97010E 02-0.94550E 02 0.74432E 03 0.16703E 03-0.33543E 01 0.14044E 02-0.35388E 03-0.25819E 03
1.00000     0.20028E 03 0.76431E 01-0.99700E 02-0.16002E 03 0.93542E 03 0.51728E 02 0.18764E 03 0.13314E 03-0.38061E 03-0.21365E 03

BRANCH SHELL PART NO  3
1.57080    -0.20028E 03-0.76438E 01-0.99699E 02-0.16001E 03 0.37173E 04-0.22152E 04 0.11696E 04-0.69436E 03-0.46940E 03-0.39204E 03
1.65807    -0.19423E 03-0.23399E 02-0.10202E 03-0.48227E 03 0.24406E 04-0.88242E 03 0.50495E 03-0.55504E 03-0.53590E 03-0.31601E 03
1.74533    -0.15651E 03-0.36964E 02-0.10480E 03-0.64324E 03 0.14768E 04 0.13835E 03-0.23216E 03-0.66150E 03-0.58503E 03-0.29147E 03

1.74533    -0.15651E 03-0.36984E 02-0.10479E 03-0.64296E 03 0.14759E 04 0.13923E 03-0.23228E 03-0.66111E 03-0.58495E 03-0.29151E 03
1.83260    -0.12352E 03-0.47616E 02-0.10799E 03-0.68089E 03 0.73812E 03 0.94178E 03-0.97410E 03-0.90389E 03-0.63011E 03-0.31555E 03
1.91987    -0.90305E 02-0.55037E 02-0.11157E 03-0.62016E 03 0.12946E 03 0.16307E 04-0.16783E 04-0.11895E 04-0.67847E 03-0.38555E 03

1.91987    -0.90304E 02-0.55037E 02-0.11157E 03-0.62016E 03 0.12943E 03 0.16307E 04-0.16783E 04-0.11895E 04-0.67847E 03-0.38555E 03
2.00713    -0.61163E 02-0.59415E 02-0.11547E 03-0.47128E 03-0.43274E 03 0.22958E 04-0.23120E 04-0.14415E 04-0.73167E 03-0.49989E 03
2.09440    -0.40154E 02-0.61299F 02-0.11968E 03-0.23462E 03-0.99718E 03 0.29912E 04-0.28391E 04-0.16016F 04-0.78697E 03-0.65747E 03

2.09440    -0.40153E 02-0.61299E 02-0.11968E 03-0.23462E 03-0.99718E 03 0.29912E 04-0.28391E 04-0.16016E 04-0.78697E 03-0.65747E 03
2.18167    -0.31367E 02-0.61609E 02-0.12416E 03 0.93545E 02-0.15631E 04 0.37190E 04-0.32132E 04-0.16324E 04-0.83856E 03-0.85660E 03
2.26893    -0.38905E 02-0.61643E 02-0.12893E 03 0.51290E 03-0.20686E 04 0.44166E 04-0.33769E 04-0.15227E 04-0.87936E 03-0.10932E 04

2.26893    -0.38904E 02-0.61643E 02-0.12893E 03 0.51290E 03-0.20686E 04 0.44166E 04-0.33769E 04-0.15228E 04-0.87936E 03-0.10932E 04
2.35620    -0.66464E 02-0.63057E 02-0.13405E 03 0.10096E 04-0.23850E 04 0.49509E 04-0.32679E 04-0.12953E 04-0.90351E 03-0.13589E 04
2.44347    -0.11651E 03-0.67793E 02-0.13962E 03 0.15466E 04-0.23179E 04 0.51188E 04-0.28321E 04-0.10137E 04-0.90965E 03-0.16385E 04

2.44347    -0.11651F 03-0.67793E 02-0.13962F 03 0.15466E 04-0.23179E 04 0.51188E 04-0.28321E 04-0.10137E 04-0.90965E 03-0.16385E 04
2.53073    -0.18903E 03-0.77909E 02-0.14582E 03 0.20541E 04-0.16171E 04 0.46595E 04-0.20408E 04-0.78836E 03-0.90503E 03-0.19088E 04
2.61800    -0.27995E 03-0.95275F 02-0.15289E 03 0.24225E 04 0.43711E 01 0.32744E 04-0.90747E 03-0.77463E 03-0.91047E 03-0.21369E 04

BRANCH SHELL PART NO  4
0.         -0.27995E 03-0.95274E 02-0.15289E 03 0.24225E 04 0.43865E 01 0.32744E 04-0.90747E 03-0.77465E 03-0.90332E 03-0.21441E 04
0.09375    -0.51485E 03-0.89146E 02-0.16945E 03 0.24339E 04 0.33739E 04 0.44602E 03 0.14840E 04-0.48170E 03-0.10109E 04-0.23827E 04
0.18750    -0.71788E 03-0.83470E 02-0.18557E 03 0.18459E 04 0.46588E 04-0.35099E 03 0.30376E 04 0.37400E 03-0.11434E 04-0.23327E 04

0.18750    -0.71798E 03-0.83472E 02-0.18557E 03 0.18461E 04 0.46591E 04-0.35142E 03 0.30387E 04 0.37471E 03-0.11432E 04-0.23326E 04
0.28125    -0.85762E 03-0.78046E 02-0.20008E 03 0.11392E 04 0.47598E 04-0.25973E 02 0.37988E 04 0.11991E 04-0.12032E 04-0.21101E 04
0.37500    -0.93542E 03-0.72524E 02-0.21200E 03 0.54955E 03 0.43840E 04 0.72535E 03 0.39879E 04 0.17301E 04-0.11597E 04-0.18077E 04

0.37500    -0.93543E 03-0.72524E 02-0.21200E 03 0.54953E 03 0.43842E 04 0.72511E 03 0.39880E 04 0.17301E 04-0.11597E 04-0.18077E 04
0.46875    -0.96646E 03-0.66592E 02-0.22066E 03 0.14237E 03 0.39728E 04 0.14778E 04 0.38572E 04 0.19273E 04-0.10208E 04-0.14826E 04
0.56250    -0.96699E 03-0.60033E 02-0.22560E 03-0.10990F 03 0.37273E 04 0.20433E 04 0.36055E 04 0.18634E 04-0.81042E 03-0.11588E 04

0.56250    -0.96700E 03-0.60033E 02-0.22560E 03-0.10995E 03 0.37276E 04 0.20429E 04 0.36056E 04 0.18634E 04-0.81041E 03-0.11587E 04
0.65625    -0.94879E 03-0.52748E 02-0.22653E 03-0.26600E 03 0.36861E 04 0.23679E 04 0.33543E 04 0.16436E 04-0.55285E 03-0.83530E 03
0.75000    -0.91787E 03-0.44727E 02-0.22321E 03-0.38966E 03 0.38020E 04 0.25532E 04 0.31560F 04 0.13585E 04-0.26390E 03-0.49575E 03

0.75000    -0.91787E 03-0.44726E 02-0.22321E 03-0.38987E 03 0.38033E 04 0.25517E 04 0.31566E 04 0.13583E 04-0.26383F 03-0.49553E 03
0.84375    -0.87551E 03-0.36036E 02-0.21534E 03-0.51731E 03 0.40090E 04 0.25871E 04 0.30170E 04 0.10650E 04 0.50875E 02-0.11352E 03
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0.93750  -0.81986E 03-0.26742E 02-0.20256E 03-0.67722E 03 0.42489E 04 0.25161E 04 0.29146E 04 0.78685E 03 0.39377E 03 0.34469E 03

0.93750  -0.81967E 03-0.26743E 02-0.20256E 03-0.67750E 03 0.42506E 04 0.25141E 04 0.29153E 04 0.78657E 03 0.39364E 03 0.34501E 03
1.03125  -0.74606E 03-0.17174E 02-0.18437E 03-0.89227E 03 0.44943E 04 0.23156E 04 0.28026E 04 0.51635E 03 0.76808E 03 0.92490E 03
1.12500  -0.65009E 03-0.74145E 01-0.16017E 03-0.11860E 04 0.47001E 04 0.19536E 04 0.25750E 04 0.22212E 03 0.11659E 04 0.16861E 04

1.12500  -0.65010E 03-0.74160E 01-0.16017E 03-0.11864E 04 0.47022E 04 0.19532E 04 0.25759E 04 0.22182E 03 0.11659E 04 0.16866E 04
1.21875  -0.52132E 03 0.22020E 01-0.12942E 03-0.15756E 04 0.46785E 04 0.15041E 04 0.19912E 04-0.13771E 03 0.15506E 04 0.26922E 04
1.31250  -0.35375E 03 0.11406E 02-0.91841E 02-0.19533E 04 0.38031E 04 0.15496E 04 0.57998E 03-0.49764E 03 0.18692E 04 0.39225E 04

1.31250  -0.35371E 03 0.11408E 02-0.91839E 02-0.19837E 04 0.38049E 04 0.15477E 04 0.57950E 03-0.49664E 03 0.18688E 04 0.39228E 04
1.40625  -0.16187E 03 0.19894E 02-0.48206E 02-0.19577E 04-0.80351E 02 0.41114E 04-0.20447E 04-0.18808E 03 0.23035E 04 0.49789E 04
1.50000  -0.44968E 02 0.25920E 02-0.18935E-02 0.10015E 01-0.11716E 05 0.13439E 05-0.32637E 04 0.37861E 04 0.47738E 04 0.44768E 04


MAIN SHELL PART NO  5
0.          0.20028E 03 0.76430E 01-0.99699E 02-0.16001E 03 0.76118E 03 0.82682E 03 0.13537E 03 0.36566E 03-0.21045E 03-0.39191E 02
0.12500     0.21878E 03 0.10704E 02-0.99751E 02-0.12659E 03 0.41819E 03 0.11957E 04 0.27712E 03 0.74683E 03-0.14527E 03-0.79804E 01
0.25000     0.23067E 03 0.13495E 02-0.98700E 02-0.60433E 02 0.33220E 03 0.12917E 04 0.43780E 03 0.97941E 03-0.33580E 02 0.35210E 02

0.25000     0.23061E 03 0.13493E 02-0.98697E 02-0.60491E 02 0.33214E 03 0.12917E 04 0.43777E 03 0.97940E 03-0.33601E 02 0.35195E 02
0.37500     0.23367E 03 0.16077E 02-0.96230E 02 0.11676E 02 0.31471E 03 0.12972E 04 0.53985E 03 0.10943E 04 0.10029E 03 0.94477E 02
0.50000     0.22758E 03 0.18521E 02-0.92392E 02 0.86737E 02 0.26176E 03 0.13127E 04 0.55075E 03 0.11200E 04 0.24362E 03 0.16024E 03

0.50000     0.22758E 03 0.18521E 02-0.92391E 02 0.86747E 02 0.26168E 03 0.13128E 04 0.55072E 03 0.11200E 04 0.24363E 03 0.16025E 03
0.62500     0.21156E 03 0.20694E 02-0.86989E 02 0.17134E 03 0.15565E 03 0.13556E 04 0.45552E 03 0.10508E 04 0.38837E 03 0.21705E 03
0.75000     0.19424E 03 0.23288E 02-0.60206E 02 0.26746E 03 0.75068E 02 0.13496E 04 0.25512E 03 0.83925E 03 0.52183E 03 0.24971E 03

0.75000     0.18424E 03 0.23288E 02-0.60208E 02 0.26747E 03 0.74982E 02 0.13496E 04 0.25509E 03 0.83928E 03 0.52184E 03 0.24971E 03
0.87500     0.14499E 03 0.25638E 02-0.72338E 02 0.35553E 03 0.21704E 03 0.11042E 04-0.66317E 01 0.41186E 03 0.61688E 03 0.25182E 03
1.00000     0.98181E 02 0.28661E 02-0.63848E 02 0.37360E 03 0.91265E 03 0.29906E 03-0.18580E 03-0.27714E 03 0.62480E 03 0.24100E 03


MAIN SHELL PART NO  6
1.57080     0.98160E 02 0.28661E 02-0.63848E 02 0.37361E 03 0.91256E 03 0.29908E 03-0.18583E 03-0.27712E 03 0.60931E 03 0.25638E 03
1.60352     0.93023E 02 0.25951E 02-0.62738E 02 0.36523E 03 0.10502E 04 0.15237E 03-0.19092E 03-0.38226E 03 0.60135E 03 0.25605E 03
1.63625     0.87925E 02 0.23396E 02-0.61658E 02 0.35408E 03 0.11673E 04 0.26180E 02-0.20159E 03-0.48003E 03 0.59128E 03 0.25644E 03

1.63625     0.87925E 02 0.23396E 02-0.61658E 02 0.35408E 03 0.11673E 04 0.26178E 02-0.20159E 03-0.48003E 03 0.59128E 03 0.25644E 03
1.66897     0.82922E 02 0.21016E 02-0.60606E 02 0.34051E 03 0.12653E 04-0.80959E 02-0.21693E 03-0.57015E 03 0.57928E 03 0.25737E 03
1.70170     0.78045E 02 0.18807E 02-0.59578E 02 0.32487E 03 0.13457E 04-0.17037E 03-0.23613E 03-0.65248E 03 0.56552E 03 0.25867E 03

1.70170     0.78045E 02 0.18807E 02-0.59578E 02 0.32487E 03 0.13457E 04-0.17037E 03-0.23613E 03-0.65248E 03 0.56552E 03 0.25867E 03
1.73442     0.73323E 02 0.16766E 02-0.58570E 02 0.30745E 03 0.14097E 04-0.24326E 03-0.25845E 03-0.72689E 03 0.55014E 03 0.26020E 03
1.76715     0.69781E 02 0.14837E 02-0.57579E 02 0.28852E 03 0.14586E 04-0.30076E 03-0.28320E 03-0.79330E 03 0.53327E 03 0.26183E 03

1.76715     0.68781E 02 0.14887E 02-0.57579E 02 0.28852E 03 0.14586E 04-0.30076E 03-0.28320E 03-0.79330E 03 0.53327E 03 0.26183E 03
1.79987     0.64444E 02 0.13166E 02-0.56603E 02 0.26831E 03 0.14936E 04-0.34391E 03-0.30977E 03-0.85162E 03 0.51501E 03 0.26345E 03
1.83260     0.60335E 02 0.11596E 02-0.55638E 02 0.24705E 03 0.15156E 04-0.37365E 03-0.33757E 03-0.90182E 03 0.49549E 03 0.26494E 03

1.83260     0.60335E 02 0.11596E 02-0.55638E 02 0.24705E 03 0.15156E 04-0.37365E 03-0.33757E 03-0.90182E 03 0.49549E 03 0.26494E 03
1.86532     0.56476E 02 0.10170E 02-0.54683E 02 0.22494E 03 0.15257E 04-0.39091E 03-0.36605E 03-0.94386E 03 0.47478E 03 0.26623E 03
1.89805     0.52890E 02 0.88815E 01-0.53735E 02 0.20217E 03 0.15249E 04-0.39654E 03-0.39467E 03-0.97772E 03 0.45297E 03 0.26722E 03

1.89805     0.52890E 02 0.88815E 01-0.53735E 02 0.20217E 03 0.15249E 04-0.39654E 03-0.39467E 03-0.97772E 03 0.45297E 03 0.26722E 03
1.93077     0.49598E 02 0.77215E 01-0.52791E 02 0.17892E 03 0.15141E 04-0.39135E 03-0.42295E 03-0.10034E 04 0.43015E 03 0.26783E 03
1.96350     0.46623E 02 0.66814E 01-0.51650E 02 0.15537E 03 0.14942E 04-0.37613E 03-0.45039E 03-0.10209E 04 0.40640E 03 0.26799E 03

1.96350     0.46623E 02 0.66814E 01-0.51850E 02 0.15537E 03 0.14942E 04-0.37613E 03-0.45039E 03-0.10209E 04 0.40640E 03 0.26799E 03
1.99622     0.43969E 02 0.57518E 01-0.50910E 02 0.13169E 03 0.14660E 04-0.35166E 03-0.47651E 03-0.10302E 04 0.38179E 03 0.26764E 03
2.02895     0.41717E 02 0.49225E 01-0.49969E 02 0.10804E 03 0.14306E 04-0.31869E 03-0.50087E 03-0.10314E 04 0.35640E 03 0.26669E 03

2.02895     0.41717E 02 0.49225E 01-0.49969E 02 0.10804E 03 0.14306E 04-0.31869E 03-0.50087E 03-0.10314E 04 0.35640E 03 0.26669E 03
2.06167     0.39833E 02 0.41828E 01-0.49026E 02 0.84587E 02 0.13987E 04-0.27799E 03-0.52300E 03-0.10246E 04 0.33030E 03 0.26511E 03
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2.09440    0.38361E 02 0.35214E 01-0.48079E 02 0.61505E 02 0.13413E 04-0.23031E 03-0.54247E 03-0.10097E 04 0.30359E 03 0.26283E 03

MAIN SHELL PART NO  7
2.09440    0.38361E 02 0.35214E 01-0.48079E 02 0.61505E 02 0.13413E 04-0.23031E 03-0.54247E 03-0.10097E 04 0.30359E 03 0.26283E 03
2.15725    0.36776E 02 0.24255E 01-0.46248E 02 0.18624E 02 0.12381E 04-0.12208E 03-0.57080E 03-0.95929E 03 0.25086E 03 0.25633E 03
2.22010    0.36973E 02 0.14613E 01-0.44399E 02-0.20665E 02 0.11247E 04 0.32250E 01-0.58484E 03-0.88103E 03 0.19689E 03 0.24687E 03

2.22010    0.36972E 02 0.14813E 01-0.44398E 02-0.20665E 02 0.11247E 04 0.32247E 01-0.58484E 03-0.88103E 03 0.19689E 03 0.24687E 03
2.28295    0.39114E 02 0.58924E 00-0.42536E 02-0.55737E 02 0.10084E 04 0.13956E 03-0.58210E 03-0.77675E 03 0.14252E 03 0.23442E 03
2.34580    0.43336E 02-0.35551E-00-0.40672E 02-0.85200E 02 0.89660E 03 0.28063E 03-0.56079E 03-0.64912E 03 0.88860E 02 0.21923E 03

2.34580    0.43336E 02-0.35552E-00-0.40672E 02-0.85200E 02 0.89660E 03 0.28063E 03-0.56079E 03-0.64912E 03 0.88860E 02 0.21923E 03
2.40865    0.49723E 02-0.14592E 01-0.38831E 02-0.10798E 03 0.79686E 03 0.42026E 03-0.52030E 03-0.50172E 03 0.37281E 02 0.20205E 03
2.47150    0.58274E 02-0.28227E 01-0.37048E 02-0.12325E 03 0.71593E 03 0.55305E 03-0.46184E 03-0.33920E 03-0.10450E 02 0.18432E 03

2.47150    0.58274E 02-0.28227E 01-0.37048E 02-0.12325E 03 0.71593E 03 0.55305E 03-0.46184E 03-0.33920E 03-0.10450E 02 0.18432E 03
2.53435    0.68860E 02-0.45315E 01-0.35373E 02-0.13052E 03 0.65867E 03 0.67553E 03-0.38940E 03-0.16702E 03-0.52057E 02 0.16852E 03
2.59720    0.81182E 02-0.66404E 01-0.33862E 02-0.12970E 03 0.62639E 03 0.78786E 03-0.31084E 03 0.88571E 01-0.84508E 02 0.15854E 03

2.59720    0.91183E 02-0.66405E 01-0.33862E 02-0.12970E 03 0.62638E 03 0.78786E 03-0.31083E 03 0.88682E 01-0.84507E 02 0.15854E 03
2.66005    0.94692E 02-0.91530E 01-0.32563E 02-0.12075E 03 0.61479E 03 0.69563E 03-0.23920E 03 0.18244E 03-0.10348E 03 0.15995E 03
2.72290    0.10644E 03-0.11990E 02-0.31483E 02-0.10285E 03 0.61268E 03 0.10110E 04-0.19415E 03 0.34749E 03-0.10223F 03 0.18002E 03

2.72290    0.10644E 03-0.11990E 02-0.31483E 02-0.10285E 03 0.61269E 03 0.10110E 04-0.19415E 03 0.34749E 03-0.10223E 03 0.18002E 03
2.78575    0.12067E 03-0.14941E 02-0.30421E 02-0.72609E 02 0.60327E 03 0.11509E 04-0.20320E 03 0.49495E 03-0.69157E 02 0.22663E 03
2.84860    0.12603E 03-0.17570E 02-0.29357E 02-0.23634E 02 0.57361E 03 0.13269E 04-0.30119E 03 0.60481E 03 0.16059E 02 0.30437E 03

2.84860    0.12603E 03-0.17570E 02-0.29357E 02-0.23636E 02 0.57364E 03 0.13269E 04-0.30119E 03 0.60480E 03 0.16050E 02 0.30437E 03
2.91145    0.12439E 03-0.19051E 02-0.27322E 02 0.53060E 02 0.54209E 03 0.15196E 04-0.52099E 03 0.63004E 03 0.18632E 03 0.40452E 03
2.97430    0.10134E 03-0.17949F 02-0.23300E 02 0.15212E 03 0.61614E 03 0.16366E 04-0.85623E 03 0.48033F 03 0.48041E 03 0.48429E 03

2.97430    0.10135E 03-0.17950E 02-0.23298E 02 0.15209E 03 0.61798E 03 0.16366E 04-0.85613E 03 0.48064E 03 0.48043E 03 0.48444E 03
3.03715    0.55733E 02-0.12104E 02-0.15411E 02 0.21684E 03 0.11167E 04 0.15010E 04-0.11500E 04 0.56117E 02 0.88298E 03 0.44629E 03
3.10000    0.15244E 02 0.61152E 00 0.33914E-01 0.14416E 01 0.37498E 04-0.49846E 03-0.40473E 03 0.14107E 04 0.79054E 03 0.19832E 04
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STRESS ANALYSIS PROGRAM OF SHELLS OF REVOLUTION UNDER STATIC LOADS, BY A. KALNINS, LEHIGH UNIV, BETHLEHEM, PA
        WRIGHT PATTERSON AIR FORCE BASE FLIGHT DYNAMICS LABORATORY VERSION, 22 JULY 1968

         STATIC ANALYSIS      PARTS=  1  BRANCHES=  0  NUMBER OF SUBCASES=  3

ANGLES OF ROTATION OF BOUNDARY CONDITIONS ARE ALFL= 0.30000E 02  ALFRS=-0.

        PART NO  1

SI= 0.          SX= 0.20000E 01   IPAP= 10   IMO=  3    SHELL TYPE 6   NTP= 0   LAYERS MLY= 1

CONICAL SHELL NO  6    H=  0.010000  PHI=   120.000 DEGREES    A=-0.30000E 01

LAYER NO  1  FROM Z=-0.50000E-02  TO Z= 0.50000E-02
    CONSISTS OF ISOTROPIC MATERIAL, YOUNGS MODULUS E= 0.10300E 08  POISSONS RATIO NU= 0.30000E-00
    COEFFICIENTS OF THERMAL EXPANSION  API= 0.12440E-04  ATHETA= 0.12440E-04  MASS DENSITY RHO= 0.25264E-03
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3

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            STRESS ANALYSIS PROGRAM OF SHELLS OF REVOLUTION UNDER STATIC LOADS, BY A. KALNINS, LEHIGH UNIV, BETHLEHEM, PA
                    WRIGHT PATTERSON AIR FORCE BASE FLIGHT DYNAMICS LABORATORY VERSION, 22 JULY 1968

                         SUBCASE NO  1   FOR FOURIER HARMONIC COS 0 THETA

BOUNDARY CONDITIONS AT STARTING EDGE      2-0.          3-0.          6-0.          7-0.

BOUNDARY CONDITIONS AT FINAL    EDGE      2-0.          4-0.          6-0.          8-0.

                    LOADS FOR PART NO  1  SUBCASE NO  1

RING LOADS AT END OF THIS PART ARE   Q=-0.        NPHI=-0.       MPHI=-0.       N=-0.

SURFACE AND TEMP LOADS ARE P1= 0.97500E-03   P2=-0.        PTHETA=-0.       TL=-0.       TU=-0.
```

STRESS ANALYSIS PROGRAM OF SHELLS OF REVOLUTION UNDER STATIC LOADS, BY A. KALNINS, LEHIGH UNIV, BETHLEHEM, PA
WRIGHT PATTERSON AIR FORCE BASE FLIGHT DYNAMICS LABORATORY VERSION, 22 JULY 1968

STATIC SOLUTION AT POINTS ALONG MERIDIAN FOR WAVE NUMBER NX= 0

MAIN SHELL PART NO 1

| S | W | Q | UPHI | NPHI | BPHI | MPHI | UTHETA | N | NTHETA | MTHETA |
|---|---|---|---|---|---|---|---|---|---|---|
| 0. | 0.26945E-06 | -0.65000E-03 | -0.15557E-06 | -0.11258E-02 | 0.36066E-05 | 0. | 0. | 0. | 0.21027E-01 | -0.10319E-05 |
| 0.06667 | 0.52830E-07 | -0.64562E-04 | -0.15921E-06 | -0.14220E-02 | 0.26449E-05 | -0.21402E-04 | 0. | 0. | 0.83767E-02 | -0.71947E-05 |
| 0.13333 | -0.73698E-07 | 0.13161E-03 | -0.16095E-06 | -0.14933E-02 | 0.11949E-05 | -0.17849E-04 | 0. | 0. | 0.74863E-03 | -0.57126E-05 |
| 0.20000 | -0.11863E-06 | 0.12790E-03 | -0.16172E-06 | -0.14490E-02 | 0.25975E-06 | -0.87830E-05 | 0. | 0. | -0.20441E-02 | -0.27145E-05 |
| 0.20000 | -0.11670E-06 | 0.12778E-03 | -0.16172E-06 | -0.14489E-02 | 0.26033E-06 | -0.87808E-05 | 0. | 0. | -0.20483E-02 | -0.27141E-05 |
| 0.26667 | -0.12126E-06 | 0.68300E-04 | -0.16220E-06 | -0.13721E-02 | -0.10719E-06 | -0.22791E-05 | 0. | 0. | -0.22141E-02 | -0.65007E-06 |
| 0.33333 | -0.11154E-06 | 0.20248E-04 | -0.16269E-06 | -0.13017E-02 | -0.15172E-06 | 0.52235E-06 | 0. | 0. | -0.15688E-02 | 0.20554E-06 |
| 0.40000 | -0.10328E-06 | -0.23329E-05 | -0.16327E-06 | -0.12457E-02 | -0.89746E-07 | 0.10087E-05 | 0. | 0. | -0.99271E-03 | 0.33223E-06 |
| 0.40000 | -0.10335E-06 | -0.22848E-05 | -0.16327E-06 | -0.12458E-02 | -0.89145E-07 | 0.10141E-05 | 0. | 0. | -0.99701E-03 | 0.33367E-06 |
| 0.46667 | -0.99555E-07 | -0.72384E-05 | -0.16390E-06 | -0.11997E-02 | -0.29001E-07 | 0.64582E-06 | 0. | 0. | -0.70693E-03 | 0.20357E-06 |
| 0.53333 | -0.98782E-07 | -0.50411E-05 | -0.16454E-06 | -0.11574E-02 | 0.89671E-09 | 0.22696E-06 | 0. | 0. | -0.62120E-03 | 0.67776E-07 |
| 0.60000 | -0.99160E-07 | -0.18902E-05 | -0.16515E-06 | -0.11154E-02 | 0.78079E-08 | 0.19128E-09 | 0. | 0. | -0.61784E-03 | -0.27350E-08 |
| 0.60000 | -0.99239E-07 | -0.19735E-05 | -0.16515E-06 | -0.11153E-02 | 0.91416E-08 | 0.13329E-07 | 0. | 0. | -0.62372E-03 | 0.72937E-09 |
| 0.66667 | -0.99807E-07 | -0.28445E-06 | -0.16574E-06 | -0.10721E-02 | 0.71117E-08 | -0.54316E-07 | 0. | 0. | -0.63659E-03 | -0.18911E-07 |
| 0.73333 | -0.10015E-06 | 0.23569E-06 | -0.16629E-06 | -0.10277E-02 | 0.31760E-08 | -0.52621E-07 | 0. | 0. | -0.63423E-03 | -0.16990E-07 |
| 0.80000 | -0.10025E-06 | 0.40629E-06 | -0.16682E-06 | -0.98278E-03 | 0.10523E-09 | -0.32552E-07 | 0. | 0. | -0.61412E-03 | -0.98067E-08 |
| 0.80000 | -0.10036E-06 | 0.25117E-06 | -0.16682E-06 | -0.98269E-03 | 0.19627E-08 | -0.17402E-07 | 0. | 0. | -0.62307E-03 | -0.59864E-08 |
| 0.86667 | -0.10046E-06 | 0.47424E-07 | -0.16732E-06 | -0.93705E-03 | 0.11670E-08 | -0.77688E-08 | 0. | 0. | -0.60381E-03 | -0.28001E-08 |
| 0.93333 | -0.10052E-06 | -0.60639E-07 | -0.16780E-06 | -0.89095E-03 | 0.61551E-09 | -0.93129E-08 | 0. | 0. | -0.58183E-03 | -0.30495E-08 |
| 1.00000 | -0.10054E-06 | 0.96214E-07 | -0.16825E-06 | -0.84443E-03 | -0.14314E-09 | -0.10491E-07 | 0. | 0. | -0.55658E-03 | -0.30860E-08 |
| 1.00000 | -0.10062E-06 | -0.66661E-07 | -0.16825E-06 | -0.84434E-03 | 0.11639E-08 | -0.42210E-10 | 0. | 0. | -0.56396E-03 | -0.51218E-09 |
| 1.06667 | -0.10070E-06 | -0.11868E-06 | -0.16867E-06 | -0.79707E-03 | 0.96910E-09 | -0.63971E-08 | 0. | 0. | -0.54472E-03 | -0.23494E-08 |
| 1.13333 | -0.10074E-06 | -0.10207E-06 | -0.16907E-06 | -0.74916E-03 | 0.22228E-09 | -0.14825E-07 | 0. | 0. | -0.52349E-03 | -0.45498E-08 |
| 1.20000 | -0.10071E-06 | 0.24210E-06 | -0.16944E-06 | -0.70066E-03 | -0.92354E-09 | -0.13651E-07 | 0. | 0. | -0.49648E-03 | -0.36549E-08 |
| 1.20000 | -0.10083E-06 | -0.83705E-07 | -0.16944E-06 | -0.70047E-03 | 0.84681E-09 | -0.24708E-08 | 0. | 0. | -0.50793E-03 | -0.11451E-08 |
| 1.26667 | -0.10088E-06 | -0.17631E-06 | -0.16978E-06 | -0.65086E-03 | 0.40571E-09 | -0.11321E-07 | 0. | 0. | -0.48893E-03 | -0.35971E-08 |
| 1.33333 | -0.10087E-06 | -0.13125E-06 | -0.17009E-06 | -0.60037E-03 | -0.85334E-09 | -0.23923E-07 | 0. | 0. | -0.46519E-03 | -0.67375E-08 |
| 1.40000 | -0.10075E-06 | 0.64839E-06 | -0.17038E-06 | -0.54922E-03 | -0.25300E-08 | -0.14161E-07 | 0. | 0. | -0.43039E-03 | -0.28910E-08 |
| 1.40000 | -0.10094E-06 | -0.11931E-06 | -0.17037E-06 | -0.54877E-03 | 0.32292E-09 | -0.21934E-08 | 0. | 0. | -0.45217E-03 | -0.83125E-09 |
| 1.46667 | -0.10095E-06 | -0.25635E-06 | -0.17063E-06 | -0.49587E-03 | -0.22991E-09 | -0.15145E-07 | 0. | 0. | -0.43282E-03 | -0.44149E-08 |
| 1.53333 | -0.10089E-06 | -0.13260E-06 | -0.17085E-06 | -0.44173E-03 | -0.19641E-08 | -0.32447E-07 | 0. | 0. | -0.40589E-03 | -0.85848E-08 |
| 1.60000 | -0.10068E-06 | 0.13536E-05 | -0.17104E-06 | -0.38678E-03 | -0.39265E-08 | -0.50712E-08 | 0. | 0. | -0.36183E-03 | 0.88596E-09 |
| 1.60000 | -0.10095E-06 | -0.14318E-06 | -0.17104E-06 | -0.38592E-03 | -0.18238E-09 | -0.36287E-08 | 0. | 0. | -0.39592E-03 | -0.97681E-09 |
| 1.66667 | -0.10092E-06 | -0.31494E-06 | -0.17119E-06 | -0.32819E-03 | -0.93162E-09 | -0.19457E-07 | 0. | 0. | -0.37642E-03 | -0.52373E-08 |
| 1.73333 | -0.10079E-06 | -0.23936E-07 | -0.17132E-06 | -0.26861E-03 | -0.30892E-08 | -0.37996E-07 | 0. | 0. | -0.34543E-03 | -0.93054E-08 |
| 1.80000 | -0.10051E-06 | 0.25666E-05 | -0.17141E-06 | -0.20788E-03 | -0.46364E-08 | 0.25677E-07 | 0. | 0. | -0.29204E-03 | 0.11020E-07 |
| 1.80000 | -0.10083E-06 | -0.21941E-06 | -0.17140E-06 | -0.20628E-03 | -0.77229E-09 | -0.57775E-08 | 0. | 0. | -0.34048E-03 | -0.11809E-08 |
| 1.86667 | -0.10075E-06 | -0.50944E-06 | -0.17145E-06 | -0.14099E-03 | -0.19793E-08 | -0.31364E-07 | 0. | 0. | -0.32007E-03 | -0.79100E-08 |
| 1.93333 | -0.10052E-06 | 0.33238E-06 | -0.17146E-06 | -0.72902E-04 | -0.53593E-08 | -0.54095E-07 | 0. | 0. | -0.27678E-03 | -0.11916E-07 |
| 2.00000 | -0.10009E-06 | 0.63416E-05 | -0.17144E-06 | -0.36612E-05 | -0.53211E-08 | 0.12627E-06 | 0. | 0. | -0.19968E-03 | 0.42447E-07 |

STRESS ANALYSIS PROGRAM OF SHELLS OF REVOLUTION UNDER STATIC LOADS, BY A. KALNINS, LEHIGH UNIV, BETHLEHEM, PA
WRIGHT PATTERSON AIR FORCE BASE FLIGHT DYNAMICS LABORATORY VERSION, 22 JULY 1968

STATIC SOLUTION AT POINTS ALONG MERIDIAN FOR WAVE NUMBER NX= 0

| S | W | UPHI | UTHETA | BPHI | SPHI IN | SPHI OUT | STHETA IN | STHETA OUT | SFITH IN | SFITH OUT |
|---|---|---|---|---|---|---|---|---|---|---|
| MAIN SHELL PART NO 1 | | | | | | | | | | |
| 0. | 0.26945E-06 | -0.15557E-06 | 0. | 0.36066E-05 | -0.11258E-00 | -0.11258E-00 | 0.21646E 01 | 0.20408E 01 | 0. | -0. |
| 0.06667 | 0.52830E-07 | -0.15921E-06 | 0. | 0.26449E-05 | 0.11419E 01 | -0.14263E 01 | 0.12694E 01 | 0.40599E-00 | 0. | -0. |
| 0.13333 | -0.73698E-07 | -0.16095E-06 | 0. | 0.11949E-05 | 0.92163E 00 | -0.12203E 01 | 0.41762E-00 | -0.26789E-00 | 0. | -0. |
| 0.20000 | -0.11863E-06 | -0.16172E-06 | 0. | 0.25975E-06 | 0.38208E-00 | -0.67188E 00 | -0.41537E-01 | -0.36728E-00 | 0. | -0. |
| 0.20000 | -0.11870E-06 | -0.16172E-06 | 0. | 0.26033E-06 | 0.38196E-00 | -0.67174E 00 | -0.41984E-01 | -0.36767E-00 | 0. | -0. |
| 0.26667 | -0.12126E-06 | -0.16220E-06 | 0. | -0.10719E-06 | -0.46536E-03 | -0.27396E-00 | -0.18241E-00 | -0.26042E-00 | 0. | -0. |
| 0.33333 | -0.11154E-06 | -0.16269E-06 | 0. | -0.15172E-06 | -0.16151E-00 | -0.98830E-01 | -0.16921E-00 | -0.14454E-00 | 0. | -0. |
| 0.40000 | -0.10328E-06 | -0.16327E-06 | 0. | -0.89746E-07 | -0.18509E-00 | -0.64052E-01 | -0.11920E-00 | -0.79337E-01 | 0. | -0. |
| 0.40000 | -0.10335E-06 | -0.16327E-06 | 0. | -0.89145E-07 | -0.18542E-00 | -0.63727E-01 | -0.11972E-00 | -0.79681E-01 | 0. | -0. |
| 0.46667 | -0.99555E-07 | -0.16390E-06 | 0. | -0.29001E-07 | -0.15872E-00 | -0.81218E-01 | -0.82907E-01 | -0.58478E-01 | 0. | -0. |
| 0.53333 | -0.98782E-07 | -0.16454E-06 | 0. | 0.89871E-09 | -0.12736E-00 | -0.10212E-00 | -0.66186E-01 | -0.58053E-01 | 0. | -0. |
| 0.60000 | -0.99160E-07 | -0.16515E-06 | 0. | 0.78079E-08 | -0.11155E-00 | -0.11152E-00 | -0.61620E-01 | -0.61948E-01 | 0. | -0. |
| 0.60000 | -0.99239E-07 | -0.16515E-06 | 0. | 0.91416E-08 | -0.11233E-00 | -0.11073E-00 | -0.62416E-01 | -0.62329E-01 | 0. | -0. |
| 0.66667 | -0.99807E-07 | -0.16574E-06 | 0. | 0.71117E-08 | -0.10395E-00 | -0.11046E-00 | -0.62524E-01 | -0.64793E-01 | 0. | -0. |
| 0.73333 | -0.10015E-06 | -0.16629E-06 | 0. | 0.31780E-08 | -0.99616E-01 | -0.10593E-00 | -0.62403E-01 | -0.64442E-01 | 0. | -0. |
| 0.80000 | -0.10025E-06 | -0.16682E-06 | 0. | 0.10523E-09 | -0.96325E-01 | -0.10023E-00 | -0.60824E-01 | -0.62001E-01 | 0. | -0. |
| 0.80000 | -0.10036E-06 | -0.16682E-06 | 0. | 0.19627E-08 | -0.97225E-01 | -0.99313E-01 | -0.61948E-01 | -0.62666E-01 | 0. | -0. |
| 0.86667 | -0.10046E-06 | -0.16732E-06 | 0. | 0.11670E-08 | -0.93239E-01 | -0.94171E-01 | -0.60213E-01 | -0.60549E-01 | 0. | -0. |
| 0.93333 | -0.10052E-06 | -0.16780E-06 | 0. | 0.61551E-09 | -0.88536E-01 | -0.89653E-01 | -0.58000E-01 | -0.58366E-01 | 0. | -0. |
| 1.00000 | -0.10054E-06 | -0.16825E-06 | 0. | -0.14314E-09 | -0.83814E-01 | -0.85072E-01 | -0.55472E-01 | -0.55843E-01 | 0. | -0. |
| 1.00000 | -0.10062E-06 | -0.16825E-06 | 0. | 0.11639E-08 | -0.84431E-01 | -0.84436E-01 | -0.56366E-01 | -0.56427E-01 | 0. | -0. |
| 1.06667 | -0.10070E-06 | -0.16867E-06 | 0. | 0.96910E-09 | -0.79323E-01 | -0.80091E-01 | -0.54331E-01 | -0.54613E-01 | 0. | -0. |
| 1.13333 | -0.10074E-06 | -0.16907E-06 | 0. | 0.22228E-09 | -0.74026E-01 | -0.75805E-01 | -0.52077E-01 | -0.52622E-01 | 0. | -0. |
| 1.20000 | -0.10071E-06 | -0.16944E-06 | 0. | -0.92354E-09 | -0.69247E-01 | -0.70885E-01 | -0.49429E-01 | -0.49867E-01 | 0. | -0. |
| 1.20000 | -0.10083E-06 | -0.16944E-06 | 0. | 0.84681E-09 | -0.69899E-01 | -0.70195E-01 | -0.50725E-01 | -0.50862E-01 | 0. | -0. |
| 1.26667 | -0.10088E-06 | -0.16978E-06 | 0. | 0.40571E-09 | -0.64407E-01 | -0.65765E-01 | -0.48677E-01 | -0.49108E-01 | 0. | -0. |
| 1.33333 | -0.10087E-06 | -0.17009E-06 | 0. | -0.85334E-09 | -0.58601E-01 | -0.61472E-01 | -0.46114E-01 | -0.46923E-01 | 0. | -0. |
| 1.40000 | -0.10075E-06 | -0.17038E-06 | 0. | -0.25300E-08 | -0.54072E-01 | -0.55771E-01 | -0.42866E-01 | -0.43213E-01 | 0. | -0. |
| 1.40000 | -0.10094E-06 | -0.17037E-06 | 0. | 0.32292E-09 | -0.54746E-01 | -0.55009E-01 | -0.45167E-01 | -0.45267E-01 | 0. | -0. |
| 1.46667 | -0.10095E-06 | -0.17063E-06 | 0. | -0.22991E-09 | -0.48678E-01 | -0.50496E-01 | -0.43017E-01 | -0.43547E-01 | 0. | -0. |
| 1.53333 | -0.10089E-06 | -0.17085E-06 | 0. | -0.19641E-08 | -0.42226E-01 | -0.46120E-01 | -0.40073E-01 | -0.41104E-01 | 0. | -0. |
| 1.60000 | -0.10068E-06 | -0.17104E-06 | 0. | -0.39265E-08 | -0.38374E-01 | -0.38982E-01 | -0.36236E-01 | -0.36130E-01 | 0. | -0. |
| 1.60000 | -0.10095E-06 | -0.17104E-06 | 0. | -0.18238E-09 | -0.38574E-01 | -0.38809E-01 | -0.39533E-01 | -0.39651E-01 | 0. | -0. |
| 1.66667 | -0.10092E-06 | -0.17119E-06 | 0. | -0.93162E-09 | -0.31651E-01 | -0.33986E-01 | -0.37328E-01 | -0.37956E-01 | 0. | -0. |
| 1.73333 | -0.10079E-06 | -0.17132E-06 | 0. | -0.30692E-08 | -0.24581E-01 | -0.29140E-01 | -0.33984E-01 | -0.35101E-01 | 0. | -0. |
| 1.80000 | -0.10051E-06 | -0.17141E-06 | 0. | -0.46364E-08 | -0.22329E-01 | -0.19248E-01 | -0.29865E-01 | -0.28543E-01 | 0. | -0. |
| 1.80000 | -0.10083E-06 | -0.17140E-06 | 0. | -0.77229E-09 | -0.20281E-01 | -0.20974E-01 | -0.33978E-01 | -0.34119E-01 | 0. | -0. |
| 1.86667 | -0.10075E-06 | -0.17145E-06 | 0. | -0.19793E-08 | -0.12217E-01 | -0.15980E-01 | -0.31532E-01 | -0.32482E-01 | 0. | -0. |
| 1.93333 | -0.10052E-06 | -0.17146E-06 | 0. | -0.53593E-08 | -0.40445E-02 | -0.10536E-01 | -0.26963E-01 | -0.28393E-01 | 0. | -0. |
| 2.00000 | -0.10009E-06 | -0.17144E-06 | 0. | -0.53211E-08 | -0.79421E-02 | 0.72098E-02 | -0.22515E-01 | -0.17421E-01 | 0. | -0. |

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STRESS ANALYSIS PROGRAM OF SHELLS OF REVOLUTION UNDER STATIC LOADS, BY A. KALNINS, LEHIGH UNIV, BETHLEHEM, PA
WRIGHT PATTERSON AIR FORCE BASE FLIGHT DYNAMICS LABORATORY VERSION, 22 JULY 1968

SUBCASE NO  2   FOR FOURIER HARMONIC COS 0 THETA

BOUNDARY CONDITIONS AT STARTING EDGE     2-0.10000E 03   3-0.        6-0.        7-0.

BOUNDARY CONDITIONS AT FINAL    EDGE     2-0.            4-0.        6-0.        8-0.

LOADS FOR PART NO  1  SUBCASE NO  2

RING LOADS AT END OF THIS PART ARE   U=-0.       HPHI=-0.       MPHI=-0.       N=-0.

SURFACE AND TEMP LOADS ARE  P=-0.       PFI=-0.       PTHETA=-0.       TL=-0.       TU=-0.
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STRESS ANALYSIS PROGRAM OF SHELLS OF REVOLUTION UNDER STATIC LOADS, BY A. KALNINS, LEHIGH UNIV, BETHLEHEM, PA
WRIGHT PATTERSON AIR FORCE BASE FLIGHT DYNAMICS LABORATORY VERSION, 22 JULY 1968

STATIC SOLUTION AT POINTS ALONG MERIDIAN FOR WAVE NUMBER NX= 0

MAIN SHELL PART NO 1

| S | W | U | UPHI | NPHI | BPHI | MPHI | UTHETA | N | NTHETA | MTHETA |
|---|---|---|---|---|---|---|---|---|---|---|
| 0. | 0.36564E-01 | -0.86603E 02 | -0.21110E-01 | 0.50000E 02 | 0.48012E-00 | 0. | 0. | 0. | 0.29141E 04 | -0.13737E-00 |
| 0.06667 | 0.77285E-02 | -0.86007E 01 | -0.21489E-01 | 0.49657E 01 | 0.35199E-00 | -0.28515E 01 | 0. | 0. | 0.12261E 04 | -0.95844E 00 |
| 0.13333 | -0.91026E-02 | 0.17535E 02 | -0.21618E-01 | -0.10124E 02 | 0.15880E-00 | -0.23780E 01 | 0. | 0. | 0.20723E 03 | -0.76096E 00 |
| 0.20000 | -0.15062E-01 | 0.17042E 02 | -0.21621E-01 | -0.98391E 01 | 0.34235E-01 | -0.11702E 01 | 0. | 0. | -0.16728E 03 | -0.36155E-00 |
| 0.20000 | -0.15072E-01 | 0.17026E 02 | -0.21621E-01 | -0.98297E 01 | 0.34304E-01 | -0.11697E 01 | 0. | 0. | -0.16792E 03 | -0.36141E-00 |
| 0.26667 | -0.15389E-01 | 0.91013E 01 | -0.21588E-01 | -0.52546E 01 | -0.14641E-01 | -0.30353E-00 | 0. | 0. | -0.19250E 03 | -0.86462E-01 |
| 0.33333 | -0.14070E-01 | 0.26997E 01 | -0.21560E-01 | -0.15586E 01 | -0.20563E-01 | 0.69760E-01 | 0. | 0. | -0.10903E 03 | 0.27547E-01 |
| 0.40000 | -0.12930E-01 | -0.30024E-00 | -0.21547E-01 | 0.17343E-00 | -0.12312E-01 | 0.13442E-00 | 0. | 0. | -0.34949E 02 | 0.44390E-01 |
| 0.40000 | -0.12956E-01 | -0.30409E-00 | -0.21547E-01 | 0.17564E-00 | -0.12209E-01 | 0.13541E-00 | 0. | 0. | -0.35325E 02 | 0.44655E-01 |
| 0.46667 | -0.12429E-01 | -0.96345E 00 | -0.21544E-01 | 0.55633E 00 | -0.41759E-02 | 0.66385E-01 | 0. | 0. | 0.82597E 00 | 0.27330E-01 |
| 0.53333 | -0.12306E-01 | -0.67039E 00 | -0.21545E-01 | 0.38713E-00 | -0.16789E-03 | 0.30601E-01 | 0. | 0. | 0.97270E 01 | 0.92567E-02 |
| 0.60000 | -0.12338E-01 | -0.25375E-00 | -0.21546E-01 | 0.14658E-00 | 0.77883E-03 | 0.59234E-03 | 0. | 0. | 0.76042E 01 | -0.10084E-03 |
| 0.60000 | -0.12347E-01 | -0.26224E-00 | -0.21546E-01 | 0.15148E-00 | 0.93889E-03 | 0.20574E-02 | 0. | 0. | 0.69127E 01 | 0.28145E-03 |
| 0.66667 | -0.12405E-01 | -0.36525E-01 | -0.21547E-01 | 0.21164E-01 | 0.68846E-03 | -0.68790E-02 | 0. | 0. | 0.26977E 01 | -0.23170E-02 |
| 0.73333 | -0.12434E-01 | 0.32862E-01 | -0.21548E-01 | -0.18966E-01 | 0.19138E-03 | -0.65437E-02 | 0. | 0. | 0.48089E-00 | -0.20356E-02 |
| 0.80000 | -0.12434E-01 | 0.52370E-01 | -0.21548E-01 | -0.30155E-01 | -0.18316E-03 | -0.38454E-02 | 0. | 0. | 0.55815E 00 | -0.10822E-02 |
| 0.80000 | -0.12447E-01 | 0.34143E-01 | -0.21548E-01 | -0.19643E-01 | 0.36253E-04 | -0.20318E-02 | 0. | 0. | -0.50329E 00 | -0.62369E-03 |
| 0.86667 | -0.12446E-01 | 0.76434E-02 | -0.21547E-01 | -0.43416E-02 | -0.49228E-04 | -0.67759E-03 | 0. | 0. | -0.43846E-00 | -0.18347E-03 |
| 0.93333 | -0.12441E-01 | -0.67936E-02 | -0.21547E-01 | 0.39959E-02 | -0.95644E-04 | -0.78076E-03 | 0. | 0. | -0.40253E-01 | -0.19450E-03 |
| 1.00000 | -0.12432E-01 | 0.10484E-01 | -0.21547E-01 | -0.59771E-02 | -0.16364E-03 | -0.92770E-03 | 0. | 0. | 0.72274E 00 | -0.20799E-03 |
| 1.00000 | -0.12442E-01 | -0.79965E-02 | -0.21547E-01 | 0.46824E-02 | -0.13734E-04 | 0.29682E-03 | 0. | 0. | -0.12390E-00 | 0.94940E-04 |
| 1.06667 | -0.12441E-01 | -0.13911E-01 | -0.21547E-01 | 0.80784E-02 | -0.16861E-04 | -0.45111E-03 | 0. | 0. | -0.61505E-01 | -0.12785E-03 |
| 1.13333 | -0.12438E-01 | -0.11779E-01 | -0.21547E-01 | 0.68697E-02 | -0.84120E-04 | -0.14247E-02 | 0. | 0. | 0.22010E-00 | -0.38873E-03 |
| 1.20000 | -0.12429E-01 | 0.27596E-01 | -0.21548E-01 | -0.15861E-01 | -0.19643E-03 | -0.12833E-02 | 0. | 0. | 0.11572E 01 | -0.29133E-03 |
| 1.20000 | -0.12442E-01 | -0.86644E-02 | -0.21547E-01 | 0.50592E-02 | 0.31941E-05 | 0.59574E-05 | 0. | 0. | -0.12765E-00 | 0.26412E-06 |
| 1.26667 | -0.12441E-01 | -0.17491E-01 | -0.21547E-01 | 0.10157E-01 | -0.24103E-04 | -0.88099E-03 | 0. | 0. | -0.97160E-01 | -0.25236E-03 |
| 1.33333 | -0.12437E-01 | -0.12247E-01 | -0.21547E-01 | 0.71325E-02 | -0.13178E-03 | -0.21013E-02 | 0. | 0. | 0.40116E-00 | -0.56251E-03 |
| 1.40000 | -0.12423E-01 | 0.65514E-01 | -0.21548E-01 | -0.37760E-01 | -0.27540E-03 | -0.10413E-02 | 0. | 0. | 0.19711E 01 | -0.16464E-03 |
| 1.40000 | -0.12442E-01 | -0.11200E-01 | -0.21547E-01 | 0.65122E-02 | 0.12841E-05 | 0.20544E-04 | 0. | 0. | -0.15763E-00 | 0.54744E-05 |
| 1.46667 | -0.12441E-01 | -0.22777E-01 | -0.21547E-01 | 0.13198E-01 | -0.33637E-04 | -0.11436E-02 | 0. | 0. | -0.93022E-01 | -0.32424E-03 |
| 1.53333 | -0.12435E-01 | -0.91473E-02 | -0.21547E-01 | 0.53312E-02 | -0.17138E-03 | -0.25930E-02 | 0. | 0. | 0.66002E 00 | -0.67760E-03 |
| 1.60000 | -0.12418E-01 | 0.12616E-00 | -0.21548E-01 | -0.72784E-01 | -0.31337E-03 | 0.16482E-03 | 0. | 0. | 0.28772E 01 | 0.24158E-03 |
| 1.60000 | -0.12442E-01 | -0.13823E-01 | -0.21547E-01 | 0.80144E-02 | 0.12757E-04 | -0.19289E-03 | 0. | 0. | -0.17462E-00 | -0.65690E-04 |
| 1.66667 | -0.12441E-01 | -0.30552E-01 | -0.21547E-01 | 0.17675E-01 | -0.47633E-04 | -0.17156E-02 | 0. | 0. | -0.10919E-00 | -0.48401E-03 |
| 1.73333 | -0.12432E-01 | 0.24124E-03 | -0.21547E-01 | -0.10207E-03 | -0.24273E-03 | -0.34522E-02 | 0. | 0. | 0.11422E 01 | -0.87116E-03 |
| 1.80000 | -0.12410E-01 | 0.26076E-00 | -0.21548E-01 | -0.15051E-00 | -0.35891E-03 | 0.32115E-02 | 0. | 0. | 0.45538E 01 | 0.12202E-02 |
| 1.80000 | -0.12442E-01 | -0.22339E-01 | -0.21547E-01 | 0.12918E-01 | 0.18451E-04 | -0.33728E-03 | 0. | 0. | -0.27695E-00 | -0.11438E-03 |
| 1.86667 | -0.12441E-01 | -0.52246E-01 | -0.21547E-01 | 0.30136E-01 | -0.84485E-04 | -0.29368E-02 | 0. | 0. | -0.11071E-00 | -0.81706E-03 |
| 1.93333 | -0.12426E-01 | 0.36107E-01 | -0.21547E-01 | -0.20323E-01 | -0.40731E-03 | -0.52047E-02 | 0. | 0. | 0.24483E 01 | -0.12336E-02 |
| 2.00000 | -0.12393E-01 | 0.66079E 00 | -0.21549E-01 | -0.38148E-00 | -0.36492E-03 | 0.13700E-01 | 0. | 0. | 0.84746E 01 | 0.44231E-02 |

STRESS ANALYSIS PROGRAM OF SHELLS OF REVOLUTION UNDER STATIC LOADS, BY A. KALNINS, LEHIGH UNIV, BETHLEHEM, PA
WRIGHT PATTERSON AIR FORCE BASE FLIGHT DYNAMICS LABORATORY VERSION, 22 JULY 1968

STATIC SOLUTION AT POINTS ALONG MERIDIAN FOR WAVE NUMBER NX= 0

| S | W | UPHI | UTHETA | BPHI | SPHI IN | SPHI OUT | STHETA IN | STHETA OUT | SFITH IN | SFITH OUT |
|---|---|---|---|---|---|---|---|---|---|---|
| MAIN SHELL PART NO 1 | | | | | | | | | | |
| 0. | 0.36564E-01 | -0.21110E-01 | 0. | 0.48012E-00 | 0.50000E 04 | 0.50000E 04 | 0.29965E 06 | 0.28317E 06 | 0. | -0. |
| 0.06667 | 0.77285E-02 | -0.21469E-01 | 0. | 0.35199E-00 | 0.17159E 06 | -0.17059E 06 | 0.18011E 06 | 0.65100E 05 | 0. | -0. |
| 0.13333 | -0.91026E-02 | -0.21618E-01 | 0. | 0.15080E-00 | 0.14167E 06 | -0.14369E 06 | 0.66381E 05 | -0.24934E 05 | 0. | -0. |
| 0.20000 | -0.15062E-01 | -0.21621E-01 | 0. | 0.34235E-01 | 0.69228E 05 | -0.71196E 05 | 0.49655E 04 | -0.38421E 05 | 0. | -0. |
| | | | | | | | | | | |
| 0.20000 | -0.15072E-01 | -0.21621E-01 | 0. | 0.34304E-01 | 0.69197E 05 | -0.71162E 05 | 0.48929E 04 | -0.38477E 05 | 0. | -0. |
| 0.26667 | -0.15389E-01 | -0.21568E-01 | 0. | -0.14641E-01 | 0.17686E 05 | -0.18737E 05 | -0.14062E 05 | -0.24438E 05 | 0. | -0. |
| 0.33333 | -0.14070E-01 | -0.21560E-01 | 0. | -0.20563E-01 | -0.43415E 04 | 0.40298E 04 | -0.12556E 05 | -0.92505E 04 | 0. | -0. |
| 0.40000 | -0.12950E-01 | -0.21547E-01 | 0. | -0.12312E-01 | -0.80478E 04 | 0.80825E 04 | -0.61583E 04 | -0.83145E 03 | 0. | -0. |
| | | | | | | | | | | |
| 0.40000 | -0.12956E-01 | -0.21547E-01 | 0. | -0.12209E-01 | -0.81073E 04 | 0.81424E 04 | -0.62118E 04 | -0.85321E 03 | 0. | -0. |
| 0.46667 | -0.12429E-01 | -0.21544E-01 | 0. | -0.41759E-02 | -0.51275E 04 | 0.52387E 04 | -0.15572E 04 | 0.17224E 04 | 0. | -0. |
| 0.53333 | -0.12306E-01 | -0.21545E-01 | 0. | -0.16789E-03 | -0.18010E 04 | 0.18784E 04 | 0.41729E 03 | 0.15281E 04 | 0. | -0. |
| 0.60000 | -0.12338E-01 | -0.21546E-01 | 0. | 0.77683E-03 | -0.20882E 02 | 0.50199E 02 | 0.76647E 03 | 0.75437E 03 | 0. | -0. |
| | | | | | | | | | | |
| 0.60000 | -0.12347E-01 | -0.21546E-01 | 0. | 0.93689E-03 | -0.10830E 03 | 0.13859E 03 | 0.67438E 03 | 0.70815E 03 | 0. | -0. |
| 0.66667 | -0.12405E-01 | -0.21547E-01 | 0. | 0.68846E-03 | 0.41466E 03 | -0.41063E 03 | 0.40879E 03 | 0.13075E 03 | 0. | -0. |
| 0.73333 | -0.12434E-01 | -0.21548E-01 | 0. | 0.19138E-03 | 0.39073E 03 | -0.39451E 03 | 0.17022E 03 | -0.74046E 02 | 0. | -0. |
| 0.80000 | -0.12434E-01 | -0.21548E-01 | 0. | -0.18316E-03 | 0.22771E 03 | -0.23374E 03 | 0.12074E 03 | -0.91142E 01 | 0. | -0. |
| | | | | | | | | | | |
| 0.80000 | -0.12447E-01 | -0.21548E-01 | 0. | 0.36253E-04 | 0.11995E 03 | -0.12387E 03 | -0.12908E 02 | -0.87751E 02 | 0. | -0. |
| 0.86667 | -0.12446E-01 | -0.21547E-01 | 0. | -0.49228E-04 | 0.40221E 02 | -0.41089E 02 | -0.32838E 02 | -0.54854E 02 | 0. | -0. |
| 0.93333 | -0.12441E-01 | -0.21547E-01 | 0. | -0.95644E-04 | 0.47245E 02 | -0.46446E 02 | 0.76449E 01 | -0.15696E 02 | 0. | -0. |
| 1.00000 | -0.12432E-01 | -0.21547E-01 | 0. | -0.16384E-03 | 0.55064E 02 | -0.56260E 02 | 0.84753E 02 | 0.59794E 02 | 0. | -0. |
| | | | | | | | | | | |
| 1.00000 | -0.12442E-01 | -0.21547E-01 | 0. | -0.13734E-04 | -0.17341E 02 | 0.18277E 02 | -0.18086E 02 | -0.66935E 01 | 0. | -0. |
| 1.06667 | -0.12441E-01 | -0.21547E-01 | 0. | -0.16661E-04 | 0.27676E 02 | -0.26257E 02 | 0.15203E 01 | -0.13821E 02 | 0. | -0. |
| 1.13333 | -0.12438E-01 | -0.21547E-01 | 0. | -0.84120E-04 | 0.86169E 02 | -0.84795E 02 | 0.45334E 02 | -0.13137E 01 | 0. | -0. |
| 1.20000 | -0.12429E-01 | -0.21548E-01 | 0. | -0.19643E-03 | 0.75414E 02 | -0.78586E 02 | 0.13320E 03 | 0.98242E 02 | 0. | -0. |
| | | | | | | | | | | |
| 1.20000 | -0.12442E-01 | -0.21547E-01 | 0. | 0.31941E-05 | 0.14848E-00 | 0.86337E 00 | -0.12781E 02 | -0.[illegible]749E 02 | 0. | -0. |
| 1.26667 | -0.12441E-01 | -0.21547E-01 | 0. | -0.24103E-04 | 0.53875E 02 | -0.51843E 02 | 0.54256E 01 | -0.24858E 02 | 0. | -0. |
| 1.33333 | -0.12437E-01 | -0.21547E-01 | 0. | -0.13178E-03 | 0.12679E 03 | -0.12536E 03 | 0.73866E 02 | 0.63650E 01 | 0. | -0. |
| 1.40000 | -0.12423E-01 | -0.21548E-01 | 0. | -0.27540E-03 | 0.58699E 02 | -0.66251E 02 | 0.20698E 03 | 0.18723E 03 | 0. | -0. |
| | | | | | | | | | | |
| 1.40000 | -0.12442E-01 | -0.21547E-01 | 0. | 0.12841E-05 | -0.58143E 00 | 0.18839E 01 | -0.16092E 02 | -0.15435E 02 | 0. | -0. |
| 1.46667 | -0.12441E-01 | -0.21547E-01 | 0. | -0.33637E-04 | 0.69935E 02 | -0.67295E 02 | 0.10152E 02 | -0.28757E 02 | 0. | -0. |
| 1.53333 | -0.12435E-01 | -0.21547E-01 | 0. | -0.17138E-03 | 0.15611E 03 | -0.15505E 03 | 0.10666E 03 | 0.25347E 02 | 0. | -0. |
| 1.60000 | -0.12418E-01 | -0.21548E-01 | 0. | -0.31339E-03 | -0.17168E 02 | 0.26110E 01 | 0.27322E 03 | 0.30221E 03 | 0. | -0. |
| | | | | | | | | | | |
| 1.60000 | -0.12442E-01 | -0.21547E-01 | 0. | 0.12757E-04 | 0.12375E 02 | -0.10772E 02 | -0.13520E 02 | -0.21403E 02 | 0. | -0. |
| 1.66667 | -0.12441E-01 | -0.21547E-01 | 0. | -0.47633E-04 | 0.10470E 03 | -0.10117E 03 | 0.18122E 02 | -0.39959E 02 | 0. | -0. |
| 1.73333 | -0.12432E-01 | -0.21547E-01 | 0. | -0.24273E-03 | 0.20712E 03 | -0.20714E 03 | 0.16649E 03 | 0.61951E 02 | 0. | -0. |
| 1.80000 | -0.12410E-01 | -0.21548E-01 | 0. | -0.35691E-03 | -0.20774E 03 | 0.17764E 03 | 0.38217E 03 | 0.52859E 03 | 0. | -0. |
| | | | | | | | | | | |
| 1.80000 | -0.12442E-01 | -0.21547E-01 | 0. | 0.18451E-04 | 0.21529E 02 | -0.18945E 02 | -0.20832E 02 | -0.34558E 02 | 0. | -0. |
| 1.86667 | -0.12441E-01 | -0.21547E-01 | 0. | -0.84485E-04 | 0.17923E 03 | -0.17319E 03 | 0.37953E 02 | -0.60095E 02 | 0. | -0. |
| 1.93333 | -0.12426E-01 | -0.21547E-01 | 0. | -0.40731E-03 | 0.31020E 03 | -0.31436E 03 | 0.31885E 03 | 0.17081E 03 | 0. | -0. |
| 2.00000 | -0.12393E-01 | -0.21549E-01 | 0. | -0.36492E-03 | -0.86013E 03 | 0.78384E 03 | 0.58208E 03 | 0.11129E 04 | 0. | -0. |

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        STRESS ANALYSIS PROGRAM OF SHELLS OF REVOLUTION UNDER STATIC LOADS, BY A. KALNINS, LEHIGH UNIV, BETHLEHEM, PA
                 WRIGHT PATTERSON AIR FORCE BASE FLIGHT DYNAMICS LABORATORY VERSION, 22 JULY 1968

                        SUBCASE NO  3   FOR FOURIER HARMONIC COS 0 THETA

BOUNDARY CONDITIONS AT STARTING EDGE       2-0.            3-0.            6-0.            7-0.

BOUNDARY CONDITIONS AT FINAL    EDGE       2-0.            4-0.            6-0.            8-0.

                  LOADS FOR PART NO  1  SUBCASE NO  3

RING LOADS AT END OF THIS PART ARE   Q=-0.           NPHI=-0.          MPHI=-0.          N=-0.

SURFACE AND TEMP LOADS ARE P1= 0.           P2=-0.          PTHETA=-0.          TL=-0.          TU=-0.

SHELL IS SPINNING ABOUT AXIS OF SYMMETRY WITH 0.10000E 03  RPM
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STRESS ANALYSIS PROGRAM OF SHELLS OF REVOLUTION UNDER STATIC LOADS, BY A. KALNINS, LEHIGH UNIV, BETHLEHEM, PA
WRIGHT PATTERSON AIR FORCE BASE FLIGHT DYNAMICS LABORATORY VERSION, 22 JULY 1968

STATIC SOLUTION AT POINTS ALONG MERIDIAN FOR WAVE NUMBER NX= 0

MAIN SHELL PART NO 1

| S | W | Q | UPHI | NPHI | BPHI | MPHI | UTHETA | N | NTHETA | MTHETA |
|---|---|---|---|---|---|---|---|---|---|---|
| 0. | 0.78274E-08 | 0.90950E-11 | -0.45192E-08 | 0.15753E-10 | 0.10635E-07 | 0. | 0. | 0. | 0.62063E-03 | -0.30428E-08 |
| 0.06667 | 0.71170E-08 | -0.52663E-07 | -0.46373E-08 | 0.30427E-07 | 0.10651E-07 | -0.21466E-08 | 0. | 0. | 0.59569E-03 | -0.37607E-08 |
| 0.13333 | 0.64122E-08 | -0.44290E-07 | -0.47504E-08 | 0.25593E-07 | 0.10451E-07 | -0.54867E-09 | 0. | 0. | 0.56974E-03 | -0.47753E-08 |
| 0.20000 | 0.57280E-08 | -0.18846E-07 | -0.48585E-08 | 0.10903E-07 | 0.10051E-07 | -0.75970E-08 | 0. | 0. | 0.54368E-03 | -0.53601E-08 |
| 0.20000 | 0.57254E-08 | -0.20513E-07 | -0.48585E-08 | 0.11862E-07 | 0.10095E-07 | -0.71898E-08 | 0. | 0. | 0.54352E-03 | -0.52514E-08 |
| 0.26667 | 0.50678E-08 | -0.48310E-08 | -0.49615E-08 | 0.26053E-08 | 0.96240E-08 | -0.79724E-08 | 0. | 0. | 0.51773E-03 | -0.54139E-08 |
| 0.33333 | 0.44428E-08 | 0.26966E-08 | -0.50596E-08 | -0.15372E-08 | 0.91257E-08 | -0.80086E-08 | 0. | 0. | 0.49265E-03 | -0.53399E-08 |
| 0.40000 | 0.38508E-08 | 0.78473E-08 | -0.51529E-08 | -0.45105E-08 | 0.66358E-08 | -0.76656E-08 | 0. | 0. | 0.46836E-03 | -0.51506E-08 |
| 0.40000 | 0.38490E-08 | 0.63621E-08 | -0.51529E-08 | -0.36545E-08 | 0.86674E-08 | -0.73816E-08 | 0. | 0. | 0.46823E-03 | -0.50758E-08 |
| 0.46667 | 0.32660E-08 | 0.69794E-08 | -0.52415E-08 | -0.40104E-08 | 0.82274E-08 | -0.69227E-08 | 0. | 0. | 0.44451E-03 | -0.48644E-08 |
| 0.53333 | 0.27513E-08 | 0.61484E-08 | -0.53255E-08 | -0.35301E-08 | 0.78189E-08 | -0.64544E-08 | 0. | 0. | 0.42136E-03 | -0.46571E-08 |
| 0.60000 | 0.22429E-08 | -0.65960E-09 | -0.54051E-08 | 0.40106E-09 | 0.74358E-08 | -0.61967E-08 | 0. | 0. | 0.39869E-03 | -0.45183E-08 |
| 0.60000 | 0.22466E-08 | 0.40691E-08 | -0.54052E-08 | -0.23321E-08 | 0.73789E-08 | -0.66557E-08 | 0. | 0. | 0.39897E-03 | -0.46357E-08 |
| 0.66667 | 0.17681E-08 | 0.50366E-08 | -0.54605E-08 | -0.28902E-08 | 0.69798E-08 | -0.63396E-08 | 0. | 0. | 0.37711E-03 | -0.44694E-08 |
| 0.73333 | 0.13155E-08 | 0.46595E-08 | -0.55517E-08 | -0.26720E-08 | 0.66033E-08 | -0.59846E-08 | 0. | 0. | 0.35581E-03 | -0.42959E-08 |
| 0.80000 | 0.88721E-09 | -0.18189E-08 | -0.56187E-08 | 0.10688E-08 | 0.62447E-08 | -0.58184E-08 | 0. | 0. | 0.33500E-03 | -0.41819E-08 |
| 0.80000 | 0.89024E-09 | 0.38792E-08 | -0.56187E-08 | -0.22238E-08 | 0.62016E-08 | -0.61123E-08 | 0. | 0. | 0.33525E-03 | -0.42533E-08 |
| 0.86667 | 0.48911E-09 | 0.48323E-08 | -0.56819E-08 | -0.27736E-08 | 0.58354E-08 | -0.58079E-08 | 0. | 0. | 0.31523E-03 | -0.40902E-08 |
| 0.93333 | 0.11170E-09 | 0.41391E-08 | -0.57412E-08 | -0.23729E-08 | 0.54907E-08 | -0.54755E-08 | 0. | 0. | 0.29578E-03 | -0.39230E-08 |
| 1.00000 | -0.24336E-09 | -0.31262E-08 | -0.57968E-08 | 0.18223E-08 | 0.51610E-08 | -0.53680E-08 | 0. | 0. | 0.27683E-03 | -0.38254E-08 |
| 1.00000 | -0.24069E-09 | 0.39138E-08 | -0.57968E-08 | -0.22438E-08 | 0.51236E-08 | -0.55269E-08 | 0. | 0. | 0.27706E-03 | -0.38570E-08 |
| 1.06667 | -0.57116E-09 | 0.48975E-08 | -0.58488E-08 | -0.28111E-08 | 0.47937E-08 | -0.52194E-08 | 0. | 0. | 0.25890E-03 | -0.36940E-08 |
| 1.13333 | -0.88035E-09 | 0.37227E-08 | -0.58974E-08 | -0.21322E-08 | 0.44855E-08 | -0.48897E-08 | 0. | 0. | 0.24127E-03 | -0.35294E-08 |
| 1.20000 | -0.11696E-08 | -0.66662E-08 | -0.59426E-08 | 0.38665E-08 | 0.41394E-08 | -0.48917E-08 | 0. | 0. | 0.22413E-03 | -0.34652E-08 |
| 1.20000 | -0.11664E-08 | 0.46205E-08 | -0.59426E-08 | -0.26560E-08 | 0.41498E-08 | -0.49717E-08 | 0. | 0. | 0.22445E-03 | -0.34703E-08 |
| 1.26667 | -0.14331E-08 | 0.70779E-08 | -0.59846E-08 | -0.40744E-08 | 0.38569E-08 | -0.45655E-08 | 0. | 0. | 0.20812E-03 | -0.32795E-08 |
| 1.33333 | -0.16814E-08 | 0.52349E-08 | -0.60234E-08 | -0.30099E-08 | 0.35956E-08 | -0.40789E-08 | 0. | 0. | 0.19228E-03 | -0.30754E-08 |
| 1.40000 | -0.19130E-08 | -0.16823E-07 | -0.60593E-08 | 0.97259E-08 | 0.33502E-08 | -0.42605E-08 | 0. | 0. | 0.17677E-03 | -0.30754E-08 |
| 1.40000 | -0.19079E-08 | 0.38243E-08 | -0.60593E-08 | -0.21994E-08 | 0.32807E-08 | -0.44520E-08 | 0. | 0. | 0.17734E-03 | -0.30956E-08 |
| 1.46667 | -0.21177E-08 | 0.54756E-08 | -0.60923E-08 | -0.31524E-08 | 0.30169E-08 | -0.41255E-08 | 0. | 0. | 0.16285E-03 | -0.29264E-08 |
| 1.53333 | -0.23107E-08 | 0.36784E-08 | -0.61226E-08 | -0.21145E-08 | 0.27772E-08 | -0.37615E-08 | 0. | 0. | 0.14891E-03 | -0.27537E-08 |
| 1.60000 | -0.24683E-08 | -0.13295E-07 | -0.61502E-08 | 0.76856E-08 | 0.25469E-08 | -0.39312E-08 | 0. | 0. | 0.13540E-03 | -0.27409E-08 |
| 1.60000 | -0.24853E-08 | 0.38754E-08 | -0.61502E-08 | -0.22314E-08 | 0.25133E-08 | -0.39082E-08 | 0. | 0. | 0.13578E-03 | -0.27133E-08 |
| 1.66667 | -0.26451E-08 | 0.51152E-08 | -0.61754E-08 | -0.29469E-08 | 0.22827E-08 | -0.35866E-08 | 0. | 0. | 0.12313E-03 | -0.25455E-08 |
| 1.73333 | -0.27902E-08 | 0.14492E-08 | -0.61981E-08 | -0.82999E-09 | 0.20741E-08 | -0.32975E-08 | 0. | 0. | 0.11102E-03 | -0.23947E-08 |
| 1.80000 | -0.29217E-08 | -0.19681E-07 | -0.62185E-08 | 0.11370E-07 | 0.18626E-08 | -0.37563E-08 | 0. | 0. | 0.99400E-04 | -0.24591E-08 |
| 1.80000 | -0.29192E-08 | 0.10890E-08 | -0.62186E-08 | -0.62219E-09 | 0.18431E-08 | -0.35255E-08 | 0. | 0. | 0.99767E-04 | -0.23760E-08 |
| 1.86667 | -0.30349E-08 | 0.77751E-08 | -0.62369E-08 | -0.44820E-08 | 0.16293E-08 | -0.32866E-08 | 0. | 0. | 0.89091E-04 | -0.22200E-08 |
| 1.93333 | -0.31373E-08 | 0.32165E-07 | -0.62532E-08 | -0.18563E-07 | 0.14584E-08 | -0.20534E-08 | 0. | 0. | 0.79095E-04 | -0.17896E-08 |
| 2.00000 | -0.32328E-08 | 0.44640E-07 | -0.62676E-08 | -0.25765E-07 | 0.14413E-08 | 0.10686E-08 | 0. | 0. | 0.68823E-04 | -0.91656E-09 |

STRESS ANALYSIS PROGRAM OF SHELLS OF REVOLUTION UNDER STATIC LOADS, BY A. KALNINS, LEHIGH UNIV, BETHLEHEM, PA
WRIGHT PATTERSON AIR FORCE BASE FLIGHT DYNAMICS LABORATORY VERSION, 22 JULY 1968

STATIC SOLUTION AT POINTS ALONG MERIDIAN FOR WAVE NUMBER NX= 0

MAIN SHELL PART NO 1

| S | W | UPHI | UTHETA | BPHI | SPHI IN | SPHI OUT | STHETA IN | STHETA OUT | SFITH IN | SFITH OUT |
|---|---|---|---|---|---|---|---|---|---|---|
| 0. | 0.78274E-08 | -0.45192E-08 | 0. | 0.10635E-07 | 0.18626E-08 | 0.18626E-08 | 0.62246E-01 | 0.61881E-01 | 0. | -0. |
| 0.06667 | 0.71170E-08 | -0.46373E-08 | 0. | 0.10651E-07 | 0.13184E-03 | -0.12575E-03 | 0.59794E-01 | 0.59343E-01 | 0. | -0. |
| 0.13333 | 0.64122E-08 | -0.47504E-08 | 0. | 0.10451E-07 | 0.33176E-03 | -0.32664E-03 | 0.57261E-01 | 0.56688E-01 | 0. | -0. |
| 0.20000 | 0.57280E-08 | -0.48565E-08 | 0. | 0.10051E-07 | 0.45691E-03 | -0.45473E-03 | 0.54690E-01 | 0.54047E-01 | 0. | -0. |
| 0.20000 | 0.57254E-08 | -0.48565E-08 | 0. | 0.10095E-07 | 0.43257E-03 | -0.43020E-03 | 0.54667E-01 | 0.54037E-01 | 0. | -0. |
| 0.26667 | 0.50678E-08 | -0.49615E-08 | 0. | 0.96240E-08 | 0.47862E-03 | -0.47806E-03 | 0.52098E-01 | 0.51448E-01 | 0. | -0. |
| 0.33333 | 0.44428E-08 | -0.50596E-08 | 0. | 0.91257E-08 | 0.48036E-03 | -0.48067E-03 | 0.49585E-01 | 0.48944E-01 | 0. | -0. |
| 0.40000 | 0.38508E-08 | -0.51529E-08 | 0. | 0.86358E-08 | 0.45949E-03 | -0.46039E-03 | 0.47145E-01 | 0.46527E-01 | 0. | -0. |
| 0.40000 | 0.38490E-08 | -0.51529E-08 | 0. | 0.86674E-08 | 0.44253E-03 | -0.44326E-03 | 0.47128E-01 | 0.46519E-01 | 0. | -0. |
| 0.46667 | 0.32860E-08 | -0.52415E-08 | 0. | 0.82274E-08 | 0.41496E-03 | -0.41576E-03 | 0.44743E-01 | 0.44159E-01 | 0. | -0. |
| 0.53333 | 0.27513E-08 | -0.53255E-08 | 0. | 0.78189E-08 | 0.38691E-03 | -0.38762E-03 | 0.42416E-01 | 0.41857E-01 | 0. | -0. |
| 0.60000 | 0.22429E-08 | -0.54051E-08 | 0. | 0.74358E-08 | 0.37184E-03 | -0.37176E-03 | 0.40140E-01 | 0.39598E-01 | 0. | -0. |
| 0.60000 | 0.22466E-08 | -0.54052E-08 | 0. | 0.73789E-08 | 0.39911E-03 | -0.39957E-03 | 0.40175E-01 | 0.39619E-01 | 0. | -0. |
| 0.66667 | 0.17661E-08 | -0.54805E-08 | 0. | 0.69798E-08 | 0.38009E-03 | -0.38067E-03 | 0.37979E-01 | 0.37443E-01 | 0. | -0. |
| 0.73333 | 0.13155E-08 | -0.55517E-08 | 0. | 0.66033E-08 | 0.35881E-03 | -0.35935E-03 | 0.35838E-01 | 0.35323E-01 | 0. | -0. |
| 0.80000 | 0.86721E-09 | -0.56187E-08 | 0. | 0.62447E-08 | 0.34921E-03 | -0.34899E-03 | 0.33751E-01 | 0.33249E-01 | 0. | -0. |
| 0.80000 | 0.89024E-09 | -0.56187E-08 | 0. | 0.62016E-08 | 0.36651E-03 | -0.36696E-03 | 0.33780E-01 | 0.33270E-01 | 0. | -0. |
| 0.86667 | 0.48911E-09 | -0.56813E-08 | 0. | 0.58354E-08 | 0.34820E-03 | -0.34875E-03 | 0.31768E-01 | 0.31278E-01 | 0. | -0. |
| 0.93333 | 0.11170E-09 | -0.57412E-08 | 0. | 0.54907E-08 | 0.32829E-03 | -0.32877E-03 | 0.29813E-01 | 0.29342E-01 | 0. | -0. |
| 1.00000 | -0.24336E-09 | -0.57968E-08 | 0. | 0.51610E-08 | 0.32226E-03 | -0.32190E-03 | 0.27912E-01 | 0.27453E-01 | 0. | -0. |
| 1.00000 | -0.24069E-09 | -0.57968E-08 | 0. | 0.51236E-08 | 0.33139E-03 | -0.33184E-03 | 0.27938E-01 | 0.27475E-01 | 0. | -0. |
| 1.06667 | -0.57116E-09 | -0.58488E-08 | 0. | 0.47937E-08 | 0.31288E-03 | -0.31344E-03 | 0.26111E-01 | 0.25668E-01 | 0. | -0. |
| 1.13333 | -0.88035E-09 | -0.58974E-08 | 0. | 0.44855E-08 | 0.29317E-03 | -0.29359E-03 | 0.24339E-01 | 0.23915E-01 | 0. | -0. |
| 1.20000 | -0.11696E-08 | -0.59426E-08 | 0. | 0.41894E-08 | 0.29389E-03 | -0.29311E-03 | 0.22621E-01 | 0.22205E-01 | 0. | -0. |
| 1.20000 | -0.11664E-08 | -0.59426E-08 | 0. | 0.41498E-08 | 0.29804E-03 | -0.29857E-03 | 0.22653E-01 | 0.22236E-01 | 0. | -0. |
| 1.26667 | -0.14331E-08 | -0.59846E-08 | 0. | 0.38569E-08 | 0.27352E-03 | -0.27434E-03 | 0.21009E-01 | 0.20615E-01 | 0. | -0. |
| 1.33333 | -0.16814E-08 | -0.60234E-08 | 0. | 0.35956E-08 | 0.24443E-03 | -0.24503E-03 | 0.19412E-01 | 0.19043E-01 | 0. | -0. |
| 1.40000 | -0.19130E-08 | -0.60593E-08 | 0. | 0.33502E-08 | 0.25660E-03 | -0.25466E-03 | 0.17861E-01 | 0.17492E-01 | 0. | -0. |
| 1.40000 | -0.19074E-08 | -0.60593E-08 | 0. | 0.32807E-08 | 0.26690E-03 | -0.26734E-03 | 0.17919E-01 | 0.17548E-01 | 0. | -0. |
| 1.46667 | -0.21177E-08 | -0.60923E-08 | 0. | 0.30169E-08 | 0.24721E-03 | -0.24784E-03 | 0.16461E-01 | 0.16110E-01 | 0. | -0. |
| 1.53333 | -0.23107E-08 | -0.61226E-08 | 0. | 0.27772E-08 | 0.22548E-03 | -0.22590E-03 | 0.15056E-01 | 0.14725E-01 | 0. | -0. |
| 1.60000 | -0.24863E-08 | -0.61502E-08 | 0. | 0.25469E-08 | 0.23664E-03 | -0.23510E-03 | 0.13704E-01 | 0.13375E-01 | 0. | -0. |
| 1.60000 | -0.24853E-08 | -0.61502E-08 | 0. | 0.25133E-08 | 0.23427E-03 | -0.23471E-03 | 0.13740E-01 | 0.13415E-01 | 0. | -0. |
| 1.66667 | -0.26451E-08 | -0.61754E-08 | 0. | 0.22827E-08 | 0.21490E-03 | -0.21549E-03 | 0.12466E-01 | 0.12160E-01 | 0. | -0. |
| 1.73333 | -0.27902E-08 | -0.61981E-08 | 0. | 0.20741E-08 | 0.19776E-03 | -0.19793E-03 | 0.11246E-01 | 0.10958E-01 | 0. | -0. |
| 1.80000 | -0.29217E-08 | -0.62185E-08 | 0. | 0.18726E-08 | 0.22651E-03 | -0.22424E-03 | 0.10088E-01 | 0.97924E-02 | 0. | -0. |
| 1.80000 | -0.29132E-08 | -0.62186E-08 | 0. | 0.18431E-08 | 0.21147E-03 | -0.21159E-03 | 0.10119E-01 | 0.98342E-02 | 0. | -0. |
| 1.86667 | -0.30349E-08 | -0.62369E-08 | 0. | 0.16293E-08 | 0.19675E-03 | -0.19765E-03 | 0.90423E-02 | 0.87759E-02 | 0. | -0. |
| 1.93333 | -0.31373E-08 | -0.62532E-08 | 0. | 0.14584E-08 | 0.12135E-03 | -0.12506E-03 | 0.80169E-02 | 0.78021E-02 | 0. | -0. |
| 2.00000 | -0.32326E-08 | -0.62676E-08 | 0. | 0.14413E-08 | -0.66690E-04 | 0.61537E-04 | 0.69373E-02 | 0.68273E-02 | 0. | -0. |

## C. Cylindrical Shell Under Thermal Loads

This case is intended to illustrate the use of thermal loads. Two subcases for a cylindrical shell are considered. One has a temperature gradient through the thickness and the other has a gradient along the generator of the shell. Both of these cases have been solved analytically and the solutions are given in Timoshenko and Woinowsky-Krieger, "Theory of Plates and Shells", on pp. 497-501, of the second edition.

According formula (e) on p. 498, the maximum hoop stress for the subcase No. 1 is given by $\sigma_\theta$=22,740 psi. The example on p. 501 shows that for the subcase No. 2 the axial stress at the end of Part No. 1 should be $\sigma_x$ = 6516 psi. The corresponding values given by the program are 23,189 and 6249 psi.

Data sheets and the appropriate output for Case C follow.

CYLINDRICAL SHELL - THERMAL LOAD

SUBCASE (1) THERMAL GRADIENT THRU THICKNESS
SUBCASE (2) THERMAL GRADIENT ALONG MERIDIAN

```
    2    2    0    2
 0.0
 0.0        4.25          4     3     2     0     1
 1.375      9.6875       90.0
14000000.      0.3                               0.0000101 0.0000101
   -0.6875     0.6875
    4.25       10.0          6     3     2     0     1
 1.375      9.6875       90.0
14000000.      0.3                               0.0000101 0.0000101
   -0.6875     0.6875
    0    0
    2               3               6               7
    2               4               6               8

 0.0                               200.0       20.0

 0.0                               200.0       20.0
    0    0
    2               3               6               7
    2               4               6               8
  0.0
 0.0        0.0         0.0        20.0        20.0        2
    4    5                    1
T-IN     2
 0.0        20.0        4.25        200.0
T-OUT    2
 C.0        20.0        4.25        200.0
 0.0
 0.0                               200.0       200.0
    0
```

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STRESS ANALYSIS PROGRAM OF SHELLS OF REVOLUTION UNDER STATIC LOADS, BY A. KALNINS, LEHIGH UNIV, BETHLEHEM, PA
          WRIGHT PATTERSON AIR FORCE BASE FLIGHT DYNAMICS LABORATORY VERSION, 22 JULY 1968

          STATIC ANALYSIS     PARTS=  2  BRANCHES=  0  NUMBER OF SUBCASES=  2

ANGLES OF ROTATION OF BOUNDARY CONDITIONS ARE ALFL= 0.           ALFRS=-0.

     PART NO  1

SI= 0.          SX= 0.42500E 01   IPAR=  4   ING=  3    SHELL TYPE 2    NTP= 0   LAYERS MLY= 1

CYLINDRICAL SHELL NO  2     H= 0.13750E 01     R= 0.96875E 01  PHI=     90.000  DEGREES

LAYER NO  1  FROM Z=-0.68750E 00  TO Z= 0.68700E 00
     CONSISTS OF ISOTROPIC MATERIAL, YOUNGS MODULUS E= 0.14000E 08  POISSONS RATIO NU= 0.30000E-00
     COEFFICIENTS OF THERMAL EXPANSION.  AFI= 0.10100E-04  ATHETA= 0.10100E-04  MASS DENSITY RHO=-0.

     PART NO  2

SI= 0.42500E 01   SX= 0.10000E 02   IPAR=  6   ING=  3    SHELL TYPE 2    NTP= 0   LAYERS MLY= 1

CYLINDRICAL SHELL NO  2     H= 0.13750E 01     R= 0.96875E 01  PHI=     90.000  DEGREES

LAYER NO  1  FROM Z=-0.68750E 00  TO Z= 0.68700E 00
     CONSISTS OF ISOTROPIC MATERIAL, YOUNGS MODULUS E= 0.14000E 08  POISSONS RATIO NU= 0.30000E-00
     COEFFICIENTS OF THERMAL EXPANSION  AFI= 0.10100E-04  ATHETA= 0.10100E-04  MASS DENSITY RHO=-0.
```

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        STRESS ANALYSIS PROGRAM OF SHELLS OF REVOLUTION UNDER STATIC LOADS, BY A. KALNINS, LEHIGH UNIV, BETHLEHEM, PA
               WRIGHT PATTERSON AIR FORCE BASE FLIGHT DYNAMICS LABORATORY VERSION, 22 JULY 1968

                          SUBCASE NO  1   FOR FOURIER HARMONIC COS 0 THETA

BOUNDARY CONDITIONS AT STARTING EDGE      2-0.          3-0.          6-0.          7-0.

BOUNDARY CONDITIONS AT FINAL    EDGE      2-0.          4-0.          6-0.          8-0.

                 LOADS FOR PART NO  1  SUBCASE NO  1

RING LOADS AT END OF THIS PART ARE   Q=-0.         NPHI=-0.         MPHI=-0.         N=-0.

SURFACE AND TEMP LOADS ARE  P= 0.          PFI=-0.          PTHETA=-0.          TL= 0.20000E 03  TU= 0.20000E 02

                 LOADS FOR PART NO  2  SUBCASE NO  1

RING LOADS AT END OF THIS PART ARE   Q=-0.         NPHI=-0.         MPHI=-0.         N=-0.

SURFACE AND TEMP LOADS ARE  P= 0.          PFI=-0.          PTHETA=-0.          TL= 0.20000E 03  TU= 0.20000E 02
```

STRESS ANALYSIS PROGRAM OF SHELLS OF REVOLUTION UNDER STATIC LOADS, BY A. KALNINS, LEHIGH UNIV, BETHLEHEM, PA
WRIGHT PATTERSON AIR FORCE BASE FLIGHT DYNAMICS LABORATORY VERSION, 22 JULY 1968

STATIC SOLUTION AT POINTS ALONG MERIDIAN FOR WAVE NUMBER NX= 0

| S | W | Q | UPHI | NPHI | BPHI | MPHI | UTHETA | N | NTHETA | MTHETA |
|---|---|---|---|---|---|---|---|---|---|---|
| MAIN SHELL PART NO 1 | | | | | | | | | | |
| 0. | 0.18003E-01 | 0. | 0. | -0.98877E-02 | 0.52734E-02 | 0. | 0. | 0. | 0.14381E 05 | 0.40035E 04 |
| 0.35417 | 0.16243E-01 | 0.46055E 03 | 0.32377E-03 | -0.98877E-02 | 0.46675E-02 | 0.85328E 02 | 0. | 0. | 0.10885E 05 | 0.40300E 04 |
| 0.70833 | 0.14694E-01 | 0.80101E 03 | 0.66568E-03 | -0.98877E-02 | 0.40786E-02 | 0.31205E 03 | 0. | 0. | 0.78097E 04 | 0.40988E 04 |
| 1.06250 | 0.13350E-01 | 0.10365E 04 | 0.10235E-02 | -0.98877E-02 | 0.35196E-02 | 0.64032E 03 | 0. | 0. | 0.51390E 04 | 0.41979E 04 |
| 1.06250 | 0.13350E-01 | 0.10365E 04 | 0.10235E-02 | -0.88196E-02 | 0.35196E-02 | 0.64033E 03 | 0. | 0. | 0.51390E 04 | 0.41979E 04 |
| 1.41667 | 0.12197E-01 | 0.11814E 04 | 0.13949E-02 | -0.88196E-02 | 0.29993E-02 | 0.10356E 04 | 0. | 0. | 0.28485E 04 | 0.43171E 04 |
| 1.77083 | 0.11220E-01 | 0.12491E 04 | 0.17781E-02 | -0.88196E-02 | 0.25233E-02 | 0.14680E 04 | 0. | 0. | 0.90859E 03 | 0.44473E 04 |
| 2.12500 | 0.10404E-01 | 0.12517E 04 | 0.21710E-02 | -0.88196E-02 | 0.20941E-02 | 0.19126E 04 | 0. | 0. | -0.71281E 03 | 0.45811E 04 |
| 2.12500 | 0.10404E-01 | 0.12517E 04 | 0.21710E-02 | -0.75836E-02 | 0.20941E-02 | 0.19126E 04 | 0. | 0. | -0.71283E 03 | 0.45811E 04 |
| 2.47917 | 0.97313E-02 | 0.12004E 04 | 0.25722E-02 | -0.75836E-02 | 0.17119E-02 | 0.23463E 04 | 0. | 0. | -0.20489E 04 | 0.47121E 04 |
| 2.83333 | 0.91860E-02 | 0.11050E 04 | 0.29800E-02 | -0.75836E-02 | 0.13748E-02 | 0.27577E 04 | 0. | 0. | -0.31321E 04 | 0.48352E 04 |
| 3.18750 | 0.87526E-02 | 0.97411E 03 | 0.33931E-02 | -0.75836E-02 | 0.10793E-02 | 0.31268E 04 | 0. | 0. | -0.39931E 04 | 0.49461E 04 |
| 3.18750 | 0.87526E-02 | 0.97411E 03 | 0.33931E-02 | -0.64850E-02 | 0.10793E-02 | 0.31269E 04 | 0. | 0. | -0.39931E 04 | 0.49461E 04 |
| 3.54167 | 0.84172E-02 | 0.81540E 03 | 0.38105E-02 | -0.64850E-02 | 0.82033E-03 | 0.34445E 04 | 0. | 0. | -0.46593E 04 | 0.50416E 04 |
| 3.89583 | 0.81679E-02 | 0.63552E 03 | 0.42311E-02 | -0.64850E-02 | 0.59205E-03 | 0.37019E 04 | 0. | 0. | -0.51545E 04 | 0.51190E 04 |
| 4.25000 | 0.79950E-02 | 0.44036E 03 | 0.46540E-02 | -0.64850E-02 | 0.38766E-03 | 0.38928E 04 | 0. | 0. | -0.54979E 04 | 0.51763E 04 |
| MAIN SHELL PART NO 2 | | | | | | | | | | |
| 4.25000 | 0.79950E-02 | 0.44036E 03 | 0.46540E-02 | -0.56534E-02 | 0.38765E-03 | 0.38928E 04 | 0. | 0. | -0.54979E 04 | 0.51763E 04 |
| 4.56944 | 0.78986E-02 | 0.25561E 03 | 0.50368E-02 | -0.56534E-02 | 0.21772E-03 | 0.40042E 04 | 0. | 0. | -0.56894E 04 | 0.52098E 04 |
| 4.88889 | 0.78551E-02 | 0.66293E 02 | 0.54202E-02 | -0.56534E-02 | 0.55611E-04 | 0.40556E 04 | 0. | 0. | -0.57759E 04 | 0.52252E 04 |
| 5.20833 | 0.78628E-02 | -0.12419E 03 | 0.58039E-02 | -0.56534E-02 | -0.10447E-03 | 0.40464E 04 | 0. | 0. | -0.57604E 04 | 0.52224E 04 |
| 5.20833 | 0.78628E-02 | -0.12419E 03 | 0.58039E-02 | -0.45013E-02 | -0.10447E-03 | 0.40464E 04 | 0. | 0. | -0.57604E 04 | 0.52224E 04 |
| 5.52778 | 0.79222E-02 | -0.31248E 03 | 0.61872E-02 | -0.45013E-02 | -0.26835E-03 | 0.39765E 04 | 0. | 0. | -0.56425E 04 | 0.52015E 04 |
| 5.84722 | 0.80353E-02 | -0.49514E 03 | 0.65696E-02 | -0.45013E-02 | -0.44180E-03 | 0.38473E 04 | 0. | 0. | -0.54178E 04 | 0.51627E 04 |
| 6.16667 | 0.82061E-02 | -0.66853E 03 | 0.69507E-02 | -0.45013E-02 | -0.63040E-03 | 0.36612E 04 | 0. | 0. | -0.50786E 04 | 0.51067E 04 |
| 6.16667 | 0.82061E-02 | -0.66853E 03 | 0.69507E-02 | -0.32196E-02 | -0.63040E-03 | 0.36612E 04 | 0. | 0. | -0.50786E 04 | 0.51067E 04 |
| 6.48611 | 0.84403E-02 | -0.82869E 03 | 0.73297E-02 | -0.32196E-02 | -0.83946E-03 | 0.34216E 04 | 0. | 0. | -0.46135E 04 | 0.50347E 04 |
| 6.80556 | 0.87451E-02 | -0.97125E 03 | 0.77061E-02 | -0.32196E-02 | -0.10739E-02 | 0.31336E 04 | 0. | 0. | -0.40079E 04 | 0.49482E 04 |
| 7.12500 | 0.91295E-02 | -0.10913E 04 | 0.80790E-02 | -0.32196E-02 | -0.13380E-02 | 0.28035E 04 | 0. | 0. | -0.32444E 04 | 0.48490E 04 |
| 7.12500 | 0.91295E-02 | -0.10913E 04 | 0.80790E-02 | -0.22125E-02 | -0.13380E-02 | 0.28035E 04 | 0. | 0. | -0.32444E 04 | 0.48490E 04 |
| 7.44444 | 0.96035E-02 | -0.11833E 04 | 0.84478E-02 | -0.22125E-02 | -0.16355E-02 | 0.24394E 04 | 0. | 0. | -0.23028E 04 | 0.47395E 04 |
| 7.76389 | 0.10178E-01 | -0.12410E 04 | 0.88113E-02 | -0.22125E-02 | -0.19692E-02 | 0.20512E 04 | 0. | 0. | -0.11611E 04 | 0.46227E 04 |
| 8.08333 | 0.10866E-01 | -0.12574E 04 | 0.91686E-02 | -0.22125E-02 | -0.23408E-02 | 0.16509E 04 | 0. | 0. | 0.20427E 03 | 0.45023E 04 |
| 8.08333 | 0.10866E-01 | -0.12574E 04 | 0.91686E-02 | -0.16632E-02 | -0.23408E-02 | 0.16509E 04 | 0. | 0. | 0.20428E 03 | 0.45023E 04 |
| 8.40278 | 0.11678E-01 | -0.12248E 04 | 0.95185E-02 | -0.16632E-02 | -0.27508E-02 | 0.12531E 04 | 0. | 0. | 0.18177E 04 | 0.43826E 04 |
| 8.72222 | 0.12627E-01 | -0.11345E 04 | 0.98596E-02 | -0.16632E-02 | -0.31983E-02 | 0.87459E 03 | 0. | 0. | 0.37032E 04 | 0.42685E 04 |
| 9.04167 | 0.13725E-01 | -0.97728E 03 | 0.10191E-01 | -0.16632E-02 | -0.36803E-02 | 0.53537E 03 | 0. | 0. | 0.58838E 04 | 0.41662E 04 |
| 9.04167 | 0.13725E-01 | -0.97728E 03 | 0.10191E-01 | -0.97656E-03 | -0.36803E-02 | 0.53537E 03 | 0. | 0. | 0.58839E 04 | 0.41662E 04 |
| 9.36111 | 0.14982E-01 | -0.74300E 03 | 0.10510E-01 | -0.97656E-03 | -0.41921E-02 | 0.25842E 03 | 0. | 0. | 0.83801E 04 | 0.40825E 04 |
| 9.68056 | 0.16406E-01 | -0.42096E 03 | 0.10816E-01 | -0.97656E-03 | -0.47264E-02 | 0.70023E 02 | 0. | 0. | 0.11209E 05 | 0.40253E 04 |
| 10.00000 | 0.18003E-01 | -0.29221E-02 | 0.11107E-01 | -0.97656E-03 | -0.52734E-02 | 0.10386E-02 | 0. | 0. | 0.14381E 05 | 0.40035E 04 |

STRESS ANALYSIS PROGRAM OF SHELLS OF REVOLUTION UNDER STATIC LOADS, BY A. KALNINS, LEHIGH UNIV. BETHLEHEM, PA
WRIGHT PATTERSON AIR FORCE BASE FLIGHT DYNAMICS LABORATORY VERSION, 22 JULY 1968

STATIC SOLUTION AT POINTS ALONG MERIDIAN FOR WAVE NUMBER NX= 0

| S | W | UPHI | UTHETA | BPHI | SPHI IN | SPHI OUT | STHETA IN | STHETA OUT | SFITH IN | SFITH OUT |
|---|---|---|---|---|---|---|---|---|---|---|
| MAIN SHELL PART NO 1 | | | | | | | | | | |
| 0. | 0.18003E-01 | 0. | 0. | 0.52734E-02 | -0.78125E-02 | -0.71716E-02 | -0.22633E 04 | 0.23189E 05 | -0. | 0. |
| 0.35417 | 0.16243E-01 | 0.32377E-03 | 0. | 0.46675E-02 | -0.27100E 03 | 0.27098E 03 | -0.48882E 04 | 0.20726E 05 | -0. | 0. |
| 0.70833 | 0.14694E-01 | 0.66566E-03 | 0. | 0.40786E-02 | -0.99103E 03 | 0.99101E 03 | -0.73415E 04 | 0.18705E 05 | -0. | 0. |
| 1.06250 | 0.13350E-01 | 0.10235E-02 | 0. | 0.35196E-02 | -0.20336E 04 | 0.20336E 04 | -0.95972E 04 | 0.17075E 05 | -0. | 0. |
| | | | | | | | | | | |
| 1.06250 | 0.13350E-01 | 0.10235E-02 | 0. | 0.35196E-02 | -0.20336E 04 | 0.20336E 04 | -0.95972E 04 | 0.17075E 05 | -0. | 0. |
| 1.41667 | 0.12197E-01 | 0.13949E-02 | 0. | 0.29993E-02 | -0.32888E 04 | 0.32888E 04 | -0.11640E 05 | 0.15785E 05 | -0. | 0. |
| 1.77083 | 0.11220E-01 | 0.17761E-02 | 0. | 0.25233E-02 | -0.46623E 04 | 0.46623E 04 | -0.13464E 05 | 0.14786E 05 | -0. | 0. |
| 2.12500 | 0.10404E-01 | 0.21710E-02 | 0. | 0.20941E-02 | -0.60743E 04 | 0.60743E 04 | -0.15067E 05 | 0.14030E 05 | -0. | 0. |
| | | | | | | | | | | |
| 2.12500 | 0.10404E-01 | 0.21710E-02 | 0. | 0.20941E-02 | -0.60743E 04 | 0.60743E 04 | -0.15067E 05 | 0.14030E 05 | -0. | 0. |
| 2.47917 | 0.97313E-02 | 0.25722E-02 | 0. | 0.17419E-02 | -0.74580E 04 | 0.74580E 04 | -0.16454E 05 | 0.13473E 05 | -0. | 0. |
| 2.83333 | 0.91660E-02 | 0.29800E-02 | 0. | 0.13748E-02 | -0.87582E 04 | 0.87582E 04 | -0.17632E 05 | 0.13075E 05 | -0. | 0. |
| 3.18750 | 0.87526E-02 | 0.33931E-02 | 0. | 0.10793E-02 | -0.99305E 04 | 0.99304E 04 | -0.18610E 05 | 0.12800E 05 | -0. | 0. |
| | | | | | | | | | | |
| 3.18750 | 0.87526E-02 | 0.33931E-02 | 0. | 0.10793E-02 | -0.99305E 04 | 0.99305E 04 | -0.18610E 05 | 0.12800E 05 | -0. | 0. |
| 3.54167 | 0.84172E-02 | 0.38105E-02 | 0. | 0.82033E-03 | -0.10939E 05 | 0.10939E 05 | -0.19398E 05 | 0.12618E 05 | -0. | 0. |
| 3.89583 | 0.81679E-02 | 0.42311E-02 | 0. | 0.59205E-03 | -0.11757E 05 | 0.11757E 05 | -0.20003E 05 | 0.12503E 05 | -0. | 0. |
| 4.25000 | 0.79950E-02 | 0.46540E-02 | 0. | 0.38766E-03 | -0.12363E 05 | 0.12363E 05 | -0.20435E 05 | 0.12435E 05 | -0. | 0. |
| | | | | | | | | | | |
| MAIN SHELL PART NO 2 | | | | | | | | | | |
| 4.25000 | 0.79950E-02 | 0.46540E-02 | 0. | 0.38765E-03 | -0.12363E 05 | 0.12363E 05 | -0.20435E 05 | 0.12435E 05 | -0. | 0. |
| 4.56944 | 0.78966E-02 | 0.50368E-02 | 0. | 0.21772E-03 | -0.12717E 05 | 0.12717E 05 | -0.20680E 05 | 0.12402E 05 | -0. | 0. |
| 4.88889 | 0.78551E-02 | 0.54242E-02 | 0. | 0.55611E-04 | -0.12680E 05 | 0.12680E 05 | -0.20792E 05 | 0.12388E 05 | -0. | 0. |
| 5.20833 | 0.78628E-02 | 0.58039E-02 | 0. | -0.10447E-03 | -0.12851E 05 | 0.12851E 05 | -0.20772E 05 | 0.12390E 05 | -0. | 0. |
| | | | | | | | | | | |
| 5.20833 | 0.78628E-02 | 0.58039E-02 | 0. | -0.10447E-03 | -0.12851E 05 | 0.12851E 05 | -0.20772E 05 | 0.12390E 05 | -0. | 0. |
| 5.52778 | 0.79222E-02 | 0.61672E-02 | 0. | -0.26835E-03 | -0.12629E 05 | 0.12629E 05 | -0.20620E 05 | 0.12410E 05 | -0. | 0. |
| 5.84722 | 0.80353E-02 | 0.65696E-02 | 0. | -0.44180E-03 | -0.12219E 05 | 0.12219E 05 | -0.20333E 05 | 0.12450E 05 | -0. | 0. |
| 6.16667 | 0.82061E-02 | 0.69507E-02 | 0. | -0.63040E-03 | -0.11627E 05 | 0.11627E 05 | -0.19909E 05 | 0.12519E 05 | -0. | 0. |
| | | | | | | | | | | |
| 6.16667 | 0.82061E-02 | 0.69507E-02 | 0. | -0.63040E-03 | -0.11627E 05 | 0.11627E 05 | -0.19909E 05 | 0.12519E 05 | -0. | 0. |
| 6.48611 | 0.84403E-02 | 0.73297E-02 | 0. | -0.83946E-03 | -0.10867E 05 | 0.10867E 05 | -0.19342E 05 | 0.12630E 05 | -0. | 0. |
| 6.80556 | 0.87451E-02 | 0.77061E-02 | 0. | -0.10739E-02 | -0.99519E 04 | 0.99519E 04 | -0.18627E 05 | 0.12796E 05 | -0. | 0. |
| 7.12500 | 0.91295E-02 | 0.80790E-02 | 0. | -0.13380E-02 | -0.89036E 04 | 0.89035E 04 | -0.17757E 05 | 0.13037E 05 | -0. | 0. |
| | | | | | | | | | | |
| 7.12500 | 0.91295E-02 | 0.80790E-02 | 0. | -0.13380E-02 | -0.89035E 04 | 0.89035E 04 | -0.17757E 05 | 0.13037E 05 | -0. | 0. |
| 7.44444 | 0.96035E-02 | 0.84478E-02 | 0. | -0.16355E-02 | -0.77471E 04 | 0.77471E 04 | -0.16726E 05 | 0.13375E 05 | -0. | 0. |
| 7.76389 | 0.10178E-01 | 0.88113E-02 | 0. | -0.19692E-02 | -0.65143E 04 | 0.65143E 04 | -0.15525E 05 | 0.13836E 05 | -0. | 0. |
| 8.08333 | 0.10866E-01 | 0.91686E-02 | 0. | -0.23408E-02 | -0.52432E 04 | 0.52432E 04 | -0.14150E 05 | 0.14448E 05 | -0. | 0. |
| | | | | | | | | | | |
| 8.08333 | 0.10866E-01 | 0.91686E-02 | 0. | -0.23408E-02 | -0.52432E 04 | 0.52432E 04 | -0.14150E 05 | 0.14448E 05 | -0. | 0. |
| 8.40278 | 0.11678E-01 | 0.95185E-02 | 0. | -0.27508E-02 | -0.39796E 04 | 0.39796E 04 | -0.12597E 05 | 0.15242E 05 | -0. | 0. |
| 8.72222 | 0.12627E-01 | 0.98596E-02 | 0. | -0.31993E-02 | -0.27776E 04 | 0.27776E 04 | -0.10865E 05 | 0.16253E 05 | -0. | 0. |
| 9.04167 | 0.13725E-01 | 0.10191E-01 | 0. | -0.36803E-02 | -0.17003E 04 | 0.17003E 04 | -0.89554E 04 | 0.17517E 05 | -0. | 0. |
| | | | | | | | | | | |
| 9.04167 | 0.13725E-01 | 0.10191E-01 | 0. | -0.36803E-02 | -0.17003E 04 | 0.17003E 04 | -0.89554E 04 | 0.17517E 05 | -0. | 0. |
| 9.36111 | 0.14982E-01 | 0.10510E-01 | 0. | -0.41921E-02 | -0.82069E 03 | 0.82069E 03 | -0.68754E 04 | 0.19069E 05 | -0. | 0. |
| 9.68056 | 0.16406E-01 | 0.10816E-01 | 0. | -0.47264E-02 | -0.22236E 03 | 0.22236E 03 | -0.46380E 04 | 0.20947E 05 | -0. | 0. |
| 10.00000 | 0.18003E-01 | 0.11107E-01 | 0. | -0.52744E-02 | -0.48828E-02 | 0.25940E-02 | -0.22633E 04 | 0.23189E 05 | -0. | 0. |

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        STRESS ANALYSIS PROGRAM OF SHELLS OF REVOLUTION UNDER STATIC LOADS, BY A. KALNINS, LEHIGH UNIV, BETHLEHEM, PA
                WRIGHT PATTERSON AIR FORCE BASE FLIGHT DYNAMICS LABORATORY VERSION, 22 JULY 1968

                    SUBCASE NO  2   FOR FOURIER HARMONIC COS 0 THETA

BOUNDARY CONDITIONS AT STARTING EDGE       2-0.          3-0.          6-0.          7-0.

BOUNDARY CONDITIONS AT FINAL    EDGE       2-0.          4-0.          6-0.          8-0.


               LOADS FOR PART NO  1  SUBCASE NO  2

RING LOADS AT END OF THIS PART ARE   Q= 0.          NPHI=-0.        MPHI=-0.        N=-0.

SURFACE AND TEMP LOADS ARE  P= 0.         PFI= 0.         PTHETA= 0.         TL= 0.20000E 02  TU= 0.20000E 02

VARIAPLE LOAD PARAMETERS ARE    4    5   -0   -0   -0

        T-IN   LINEAR FUNCTION GENERATOR NO.   1 FROM   2 POINTS

Y COORDINATES     20.00000 200.00000

X COORDINATES      0.       4.25000

        T-OUT  LINEAR FUNCTION GENERATOR NO.   2 FROM   2 POINTS

Y COORDINATES     20.00000 200.00000

X COORDINATES      0.       4.25000


               LOADS FOR PART NO  2  SUBCASE NO  2

RING LOADS AT END OF THIS PART ARE   Q= 0.          NPHI=-0.        MPHI=-0.        N=-0.

SURFACE AND TEMP LOADS ARE  P= 0.         PFI=-0.         PTHETA=-0.         TL= 0.20000E 03  TU= 0.20000E 03
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STRESS ANALYSIS PROGRAM OF SHELLS OF REVOLUTION UNDER STATIC LOADS. BY A. KALNINS, LEHIGH UNIV, BETHLEHEM, PA
WRIGHT PATTERSON AIR FORCE BASE FLIGHT DYNAMICS LABORATORY VERSION, 22 JULY 1968

STATIC SOLUTION AT POINTS ALONG MERIDIAN FOR WAVE NUMBER NX= 0

| S | W | Q | UPHI | WPHI | BPHI | MPHI | UTHETA | N | NTHETA | MTHETA |
|---|---|---|---|---|---|---|---|---|---|---|
| MAIN SHELL PART NO 1 | | | | | | | | | | |
| 0. | 0.45692E-02 | -0. | -0. | -0.93384E-02 | -0.31283E-02 | -0. | 0. | 0. | 0.51890E 04 | -0.12973E 01 |
| 0.35417 | 0.56770E-02 | 0.17664E 03 | 0.71692E-04 | -0.93384E-02 | -0.31271E-02 | 0.32052E 02 | 0. | 0. | 0.44743E 04 | 0.84969E 01 |
| 0.70833 | 0.67835E-02 | 0.32712E 03 | 0.20099E-03 | -0.93384E-02 | -0.31194E-02 | 0.12203E 03 | 0. | 0. | 0.37568E 04 | 0.35671E 02 |
| 1.06250 | 0.78851E-02 | 0.45122E 03 | 0.38794E-03 | -0.93384E-02 | -0.30994E-02 | 0.26065E 03 | 0. | 0. | 0.30298E 04 | 0.77437E 02 |
| 1.06250 | 0.78851E-02 | 0.45122E 03 | 0.38794E-03 | -0.90637E-02 | -0.30994E-02 | 0.26065E 03 | 0. | 0. | 0.30298E 04 | 0.77438E 02 |
| 1.41667 | 0.89769E-02 | 0.54642E 03 | 0.63261E-03 | -0.90637E-02 | -0.30626E-02 | 0.43847E 03 | 0. | 0. | 0.22831E 04 | 0.13097E 03 |
| 1.77083 | 0.10052E-01 | 0.61776E 03 | 0.93514E-03 | -0.90637E-02 | -0.30051E-02 | 0.64583E 03 | 0. | 0. | 0.15034E 04 | 0.19337E 03 |
| 2.12500 | 0.11103E-01 | 0.65776E 03 | 0.12958E-02 | -0.90637E-02 | -0.29245E-02 | 0.87259E 03 | 0. | 0. | 0.67528E 03 | 0.26161E 03 |
| 2.12500 | 0.11103E-01 | 0.65776E 03 | 0.12958E-02 | -0.84229E-02 | -0.29245E-02 | 0.87259E 03 | 0. | 0. | 0.67528E 03 | 0.26161E 03 |
| 2.47917 | 0.12121E-01 | 0.66634E 03 | 0.17148E-02 | -0.84229E-02 | -0.28191E-02 | 0.11080E 04 | 0. | 0. | -0.21823E 03 | 0.33246E 03 |
| 2.83333 | 0.13097E-01 | 0.64079E 03 | 0.21927E-02 | -0.84229E-02 | -0.26886E-02 | 0.13406E 04 | 0. | 0. | -0.11946E 04 | 0.40247E 03 |
| 3.18750 | 0.14022E-01 | 0.57776E 03 | 0.27298E-02 | -0.84229E-02 | -0.25345E-02 | 0.15575E 04 | 0. | 0. | -0.22713E 04 | 0.44782E 03 |
| 3.18750 | 0.14022E-01 | 0.57776E 03 | 0.27298E-02 | -0.74539E-02 | -0.25345E-02 | 0.15575E 04 | 0. | 0. | -0.22713E 04 | 0.44782E 03 |
| 3.54167 | 0.14889E-01 | 0.47329E 03 | 0.33269E-02 | -0.74539E-02 | -0.23585E-02 | 0.17449E 04 | 0. | 0. | -0.34643E 04 | 0.52434E 03 |
| 3.89583 | 0.15691E-01 | 0.32267E 03 | 0.39646E-02 | -0.74539E-02 | -0.21648E-02 | 0.18873E 04 | 0. | 0. | -0.47877E 04 | 0.56740E 03 |
| 4.25000 | 0.16421E-01 | 0.12151E 03 | 0.47037E-02 | -0.74539E-02 | -0.19591E-02 | 0.19676E 04 | 0. | 0. | -0.62519E 04 | 0.59184E 03 |
| MAIN SHELL PART NO 2 | | | | | | | | | | |
| 4.25000 | 0.16421E-01 | 0.12151E 03 | 0.47037E-02 | -0.67673E-02 | -0.19591E-02 | 0.19676E 04 | 0. | 0. | -0.62519E 04 | 0.59184E 03 |
| 4.56944 | 0.17017E-01 | -0.64813E 02 | 0.53772E-02 | -0.67673E-02 | -0.17695E-02 | 0.19756E 04 | 0. | 0. | -0.50689E 04 | 0.59395E 03 |
| 4.86889 | 0.17552E-01 | -0.21411E 03 | 0.60450E-02 | -0.67673E-02 | -0.15817E-02 | 0.19301E 04 | 0. | 0. | -0.40059E 04 | 0.58004E 03 |
| 5.20833 | 0.18028E-01 | -0.33029E 03 | 0.67079E-02 | -0.67673E-02 | -0.14004E-02 | 0.18424E 04 | 0. | 0. | -0.30602E 04 | 0.55347E 03 |
| 5.20833 | 0.18026E-01 | -0.33029E 03 | 0.67079E-02 | -0.55084E-02 | -0.14004E-02 | 0.18424E 04 | 0. | 0. | -0.30602E 04 | 0.55347E 03 |
| 5.52778 | 0.18448E-01 | -0.41716E 03 | 0.73663E-02 | -0.55084E-02 | -0.12292E-02 | 0.17222E 04 | 0. | 0. | -0.22265E 04 | 0.51723E 03 |
| 5.84722 | 0.18815E-01 | -0.47628E 03 | 0.80209E-02 | -0.55084E-02 | -0.10767E-02 | 0.15786E 04 | 0. | 0. | -0.14976E 04 | 0.47395E 03 |
| 6.16667 | 0.19133E-01 | -0.51698E 03 | 0.86720E-02 | -0.55084E-02 | -0.92675E-03 | 0.14191E 04 | 0. | 0. | -0.86468E 03 | 0.42593E 03 |
| 6.16667 | 0.19133E-01 | -0.51698E 03 | 0.86720E-02 | -0.43030E-02 | -0.92675E-03 | 0.14191E 04 | 0. | 0. | -0.86467E 03 | 0.42593E 03 |
| 6.48611 | 0.19409E-01 | -0.53626E 03 | 0.93203E-02 | -0.43030E-02 | -0.79663E-03 | 0.12503E 04 | 0. | 0. | -0.31812E 03 | 0.37518E 03 |
| 6.80556 | 0.19645E-01 | -0.53680E 03 | 0.99660E-02 | -0.43030E-02 | -0.68691E-03 | 0.10782E 04 | 0. | 0. | 0.15232E 03 | 0.32343E 03 |
| 7.12500 | 0.19849E-01 | -0.52693E 03 | 0.10609E-01 | -0.43030E-02 | -0.59167E-03 | 0.90765E 03 | 0. | 0. | 0.55710E 03 | 0.27215E 03 |
| 7.12500 | 0.19849E-01 | -0.52693E 03 | 0.10609E-01 | -0.30670E-02 | -0.59166E-03 | 0.90765E 03 | 0. | 0. | 0.55711E 03 | 0.27215E 03 |
| 7.44444 | 0.20025E-01 | -0.50266E 03 | 0.11251E-01 | -0.30670E-02 | -0.51254E-03 | 0.74289E 03 | 0. | 0. | 0.90660E 03 | 0.22264E 03 |
| 7.76389 | 0.20178E-01 | -0.46764E 03 | 0.11891E-01 | -0.30670E-02 | -0.44680E-03 | 0.58764E 03 | 0. | 0. | 0.12108E 04 | 0.17599E 03 |
| 8.08333 | 0.20313E-01 | -0.42320E 03 | 0.12530E-01 | -0.30670E-02 | -0.39934E-03 | 0.44512E 03 | 0. | 0. | 0.14792E 04 | 0.13317E 03 |
| 8.08333 | 0.20313E-01 | -0.42320E 03 | 0.12530E-01 | -0.20943E-02 | -0.39936E-03 | 0.44512E 03 | 0. | 0. | 0.14792E 04 | 0.13317E 03 |
| 8.40278 | 0.20435E-01 | -0.37039E 03 | 0.13167E-01 | -0.20943E-02 | -0.36293E-03 | 0.31815E 03 | 0. | 0. | 0.17204E 04 | 0.95015E 02 |
| 8.72222 | 0.20546E-01 | -0.30496E 03 | 0.13803E-01 | -0.20943E-02 | -0.33773E-03 | 0.20929E 03 | 0. | 0. | 0.19421E 04 | 0.62302E 02 |
| 9.04167 | 0.20652E-01 | -0.24245E 03 | 0.14438E-01 | -0.20943E-02 | -0.32206E-03 | 0.12088E 03 | 0. | 0. | 0.21510E 04 | 0.35725E 02 |
| 9.04167 | 0.20652E-01 | -0.24245E 03 | 0.14438E-01 | -0.13261E-02 | -0.32206E-03 | 0.12087E 03 | 0. | 0. | 0.21510E 04 | 0.35725E 02 |
| 9.36111 | 0.20753E-01 | -0.16818E 03 | 0.15072E-01 | -0.13261E-02 | -0.31382E-03 | 0.55111E 02 | 0. | 0. | 0.23524E 04 | 0.15945E 02 |
| 9.68056 | 0.20853E-01 | -0.87345E 02 | 0.15705E-01 | -0.13261E-02 | -0.31059E-03 | 0.14124E 02 | 0. | 0. | 0.25503E 04 | 0.35997E 01 |
| 10.00000 | 0.20952E-01 | -0.39148E-03 | 0.16337E-01 | -0.13261E-02 | -0.31024E-03 | 0.31081E-03 | 0. | 0. | 0.27472E 04 | -0.68672E 00 |

```
STRESS ANALYSIS PROGRAM OF SHELLS OF REVOLUTION UNDER STATIC LOADS, BY A. KALNINS, LEHIGH UNIV. BETHLEHEM, PA
WRIGHT PATTERSON AIR FORCE BASE FLIGHT DYNAMICS LABORATORY VERSION, 22 JULY 1968

STATIC SOLUTION AT POINTS ALONG MERIDIAN FOR WAVE NUMBER NN=  0

   S          W          UPHI       UTHETA     BPHI        SPHI IN     SPHI OUT    STHETA IN   STHETA OUT  SFITH IN    SFITH OUT

MAIN SHELL PART NO  1
0.          0.45692E-02-0.          0.          -0.31293E-02-0.68054E-02-0.68054E-02 0.37752E 04 0.37752E 04 0.          -0.
0.35417     0.56770E-02 0.71692E-04 0.          -0.31271E-02-0.10150E 03 0.10178E 03 0.32247E 04 0.32857E 04-0.           0.
0.70833     0.67835E-02 0.20099E-03 0.          -0.31194E-02-0.38757E 03 0.38756E 03 0.26169E 04 0.28495E 04-0.           0.
1.06250     0.78851E-02 0.36794E-03 0.          -0.30994E-02-0.82779E 03 0.82778E 03 0.19560E 04 0.24526E 04-0.           0.

1.06250     0.78851E-02 0.38794E-03 0.          -0.30994E-02-0.82780E 03 0.82778E 03 0.19560E 04 0.24526E 04-0.           0.
1.41667     0.89769E-02 0.63261E-03 0.          -0.30626E-02-0.13925E 04 0.13925E 04 0.12433E 04 0.20788E 04-0.           0.
1.77083     0.10032E-01 0.93514E-03 0.          -0.30051E-02-0.20511E 04 0.20510E 04 0.47849E 03 0.17091E 04-0.           0.
2.12500     0.11103E-01 0.12958E-02 0.          -0.29245E-02-0.27712E 04 0.27712E 04-0.34008E 03 0.13227E 04-0.           0.

2.12500     0.11103E-01 0.12958E-02 0.          -0.29245E-02-0.27712E 04 0.27712E 04-0.34008E 03 0.13227E 04-0.           0.
2.47917     0.12121E-01 0.17148E-02 0.          -0.28191E-02-0.35190E 04 0.35190E 04-0.12145E 04 0.89692E 03-0.           0.
2.83333     0.13097E-01 0.21927E-02 0.          -0.26888E-02-0.42574E 04 0.42574E 04-0.21464E 04 0.40808E 03-0.           0.
3.18750     0.14022E-01 0.27298E-02 0.          -0.25345E-02-0.49464E 04 0.49464E 04-0.31364E 04-0.16853E 03-0.           0.

3.18750     0.14022E-01 0.27298E-02 0.          -0.25345E-02-0.49464E 04 0.49466E 04-0.31364E 04-0.16853E 03-0.           0.
3.54167     0.14889E-01 0.33269E-02 0.          -0.23585E-02-0.55416E 04 0.55416E 04-0.41829E 04-0.85795E 03-0.           0.
3.89583     0.15631E-01 0.39846E-02 0.          -0.21648E-02-0.59939E 04 0.59939E 04-0.52814E 04-0.16850E 04-0.           0.
4.25000     0.16421E-01 0.47037E-02 0.          -0.19591E-02-0.62488E 04 0.62488E 04-0.64231E 04-0.26738E 04-0.           0.

MAIN SHELL PART NO  2
4.25000     0.16421E-01 0.47037E-02 0.          -0.19591E-02-0.62488E 04 0.62488E 04-0.64231E 04-0.26738E 04-0.           0.
4.56944     0.17017E-01 0.53772E-02 0.          -0.17695E-02-0.62743E 04 0.62743E 04-0.55701E 04-0.18055E 04-0.           0.
4.88889     0.17552E-01 0.60450E-02 0.          -0.15817E-02-0.61299E 04 0.61298E 04-0.47534E 04-0.10755E 04-0.           0.
5.20833     0.18028E-01 0.67079E-02 0.          -0.14004E-02-0.58511E 04 0.58511E 04-0.39817E 04-0.47110E 03-0.           0.

5.20833     0.18028E-01 0.67079E-02 0.          -0.14004E-02-0.58511E 04 0.58511E 04-0.39817E 04-0.47109E 03-0.           0.
5.52778     0.18448E-01 0.73663E-02 0.          -0.12292E-02-0.54696E 04 0.54696E 04-0.32608E 04 0.20998E 02-0.           0.
5.84722     0.18815E-01 0.80209E-02 0.          -0.10707E-02-0.50133E 04 0.50133E 04-0.25936E 04 0.41445E 03-0.           0.
6.16667     0.19133E-01 0.86720E-02 0.          -0.92675E-03-0.45067E 04 0.45067E 04-0.19811E 04 0.72293E 03-0.           0.

6.16667     0.19133E-01 0.86720E-02 0.          -0.92675E-03-0.45067E 04 0.45067E 04-0.19811E 04 0.72293E 03-0.           0.
6.48611     0.19409E-01 0.93203E-02 0.          -0.79863E-03-0.39709E 04 0.39709E 04-0.14227E 04 0.95983E 03-0.           0.
6.80556     0.19645E-01 0.99660E-02 0.          -0.68691E-03-0.34243E 04 0.34243E 04-0.91647E 03 0.11381E 04-0.           0.
7.12500     0.19849E-01 0.10609E-01 0.          -0.59167E-03-0.28826E 04 0.28826E 04-0.45946E 03 0.12701E 04-0.           0.

7.12500     0.19849E-01 0.10609E-01 0.          -0.59166E-03-0.28826E 04 0.28826E 04-0.45945E 03 0.12701E 04-0.           0.
7.44444     0.20025E-01 0.11251E-01 0.          -0.51254E-03-0.23593E 04 0.23593E 04-0.48209E 02 0.13674E 04-0.           0.
7.76389     0.20178E-01 0.11891E-01 0.          -0.44880E-03-0.18663E 04 0.18663E 04 0.32103E 03 0.14408E 04-0.           0.
8.08333     0.20313E-01 0.12530E-01 0.          -0.39936E-03-0.14136E 04 0.14136E 04 0.65207E 03 0.15003E 04-0.           0.

8.08333     0.20313E-01 0.12530E-01 0.          -0.39936E-03-0.14136E 04 0.14136E 04 0.65207E 03 0.15003E 04-0.           0.
8.40278     0.20435E-01 0.13167E-01 0.          -0.36238E-03-0.10104E 04 0.10104E 04 0.94852E 03 0.15548E 04-0.           0.
8.72222     0.20546E-01 0.13803E-01 0.          -0.33773E-03-0.66468E 03 0.66468E 03 0.12135E 04 0.16124E 04-0.           0.
9.04167     0.20652E-01 0.14438E-01 0.          -0.32206E-03-0.38309E 03 0.38308E 03 0.14498E 04 0.16801E 04-0.           0.

9.04167     0.20652E-01 0.14438E-01 0.          -0.32206E-03-0.38308E 03 0.38308E 03 0.14498E 04 0.16801E 04-0.           0.
9.36111     0.20753E-01 0.15072E-01 0.          -0.31381E-03-0.17503E 03 0.17502E 03 0.16589E 04 0.17640E 04-0.           0.
9.68056     0.20853E-01 0.15705E-01 0.          -0.31069E-03-0.44858E 02 0.44855E 02 0.18420E 04 0.18689E 04-0.           0.
10.00000    0.20952E-01 0.16337E-01 0.          -0.31024E-03-0.24414E-02 0.                0.19987E 04 0.19987E 04-0.           0.

IBRM=0 WHICH INDICATES END OF JOB. EXIT IS CALLED
```

## 2. Axisymmetric Eigenvalue Program

Three cases are considered. Two are for stability analysis and one for free vibration. Of the stability cases, one does and the other does not agree with experimental results. The free vibration case has been verified experimentally.

## D. Stability of Ellipsoidal Shell

This case finds the buckling load for an ellipsoidal shell of revolution subjected to external pressure. The prestress state is variable along the meridian. It was calculated separately by the Static Program for an ellipsoidal shell with an external pressure $p/E=1$.

Experimental collapse pressures have been reported for the same case by Hyman and Healey*, and their value is 137 psi for a three-lobed buckling mode. Our stability program gives an eigenvalue of XMR=0.0004284. The pressure of the prestressed state must be multiplied by this eigenvalue in order to obtain the buckling load. For E=325,000 psi, this gives a buckling pressure of p=139.23 psi as predicted by the program.

## E. Stability of Axially Compressed Cylinder

This is the classical case of a cylindrical shell under

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*B. I. Hyman and J. J. Healey, "Buckling of Prolate Spheroidal Shells under Hydrostatic Pressure," AIAA Journal, v. 5, 1967, p. 1471.

axial load. The obtained eigenvalue agrees with the one obtained from Figure 10 on page 427, as given in Flugge, "Stress in Shells", Springer Verlag, 1966 Edition. The actual collapse load of such a cylindrical shell is about one-third of the theoretical value.

## F. Free Vibration of a Cylindrical Shell

This case predicts a two-lobe mode of free vibration at $\omega$=179 cps for a cylindrical shell, fixed at one end and free at the other. In vibration experiments with mylar cylindrical shells, conducted at the Shell Vibration Laboratory of Lehigh University, the experimentally found value was $\omega$=180 cps.

Data sheets and the appropriate output for these three test cases of the Axisymmetric Eigenvalue Program follow.

TEST CASES FOR AXISYMMETRIC EIGENVALUE PROGRAM

CYLINDRICAL SHELL - FREE VIBRATION

```
  1    2    0    1    0    0

 0.0     12.0          17    3    2    0    1
 0.125     3.938       90.0
5000.0    0.3                                                      0.0001131
-0.0625   0.0625

 175.0      2.0         183.0     1    2
  1              3              5              7
  2              4              6              8
```

ELLIPSOIDAL SHELL - STABILITY UNDER EXTERNAL PRESSURE

```
  2    1    0    1    1    0

.034091     1.26159    10    2    5    0    1
 .063       1.0        3.0       1.0
1.          .4
-.0315      .0315
 1   26     1
          0.03409100          -0.17330917          -0.06932367          0.
          0.08319136          -0.12340079          -0.11218234          0.
          0.13229171          -0.11780769          -0.12356622          0.
          0.18139207          -0.11841565          -0.13461012          0.
          0.23049243          -0.12148704          -0.14675558          0.
          0.27959279          -0.12598061          -0.15989803          0.
          0.32869314          -0.13145042          -0.17381085          0.
          0.37779351          -0.13765848          -0.18831594          0.
          0.42689386          -0.14446341          -0.20330207          0.
          0.47599422          -0.15177891          -0.21872125          0.
          0.52509458          -0.15955574          -0.23458390          0.
          0.57419493          -0.16777322          -0.25095647          0.
          0.62329528          -0.17643541          -0.26796139          0.
          0.67239564          -0.18556996          -0.28577882          0.
          0.72149599          -0.19522839          -0.30464904          0.
          0.77059635          -0.20548749          -0.32487352          0.
          0.81969670          -0.21645092          -0.34681326          0.
          0.86879706          -0.22825070          -0.37088130          0.
          0.91789741          -0.24104732          -0.39752666          0.
          0.96699777          -0.25502740          -0.42720736          0.
          1.01609811          -0.27039685          -0.46035320          0.
          1.06519847          -0.28736734          -0.49732129          0.
          1.11429882          -0.30613334          -0.53835284          0.
          1.16339917          -0.32683595          -0.58353489          0.
          1.21249953          -0.34950922          -0.63274185          0.
          1.26159999          -0.37398630          -0.68549491          0.
1.26159     1.5708     10    2    5    0    1
 .061       1.0        3.0       1.0
1.          .4
```

```
 -.0315      .0315
  2   26      1
           1.26159999            -0.37398630            -0.68549441           0.
           1.27396798            -0.38039021            -0.69922471           0.
           1.28633597            -0.38686968            -0.71310013           0.
           1.29870397            -0.39341102            -0.72707722           0.
           1.31107196            -0.39999837            -0.74112779           0.
           1.32343996            -0.40661343            -0.75521371           0.
           1.33580795            -0.41323544            -0.76929166           0.
           1.34817594            -0.41984094            -0.78331296           0.
           1.36054394            -0.42640371            -0.79722317           0.
           1.37291193            -0.43289479            -0.81096204           0.
           1.38527992            -0.43928246            -0.82446393           0.
           1.39764792            -0.44553235            -0.83765754           0.
           1.41001591            -0.45160764            -0.85046694           0.
           1.42238390            -0.45746931            -0.86281182           0.
           1.43475190            -0.46307663            -0.87460845           0.
           1.44711989            -0.46838756            -0.88577066           0.
           1.45948789            -0.47335947            -0.89621090           0.
           1.47185588            -0.47794979            -0.90584194           0.
           1.48422387            -0.48211694            -0.91457823           0.
           1.49659167            -0.48582111            -0.92233810           0.
           1.50895986            -0.48902526            -0.92904619           0.
           1.52132785            -0.49169610            -0.93463475           0.
           1.53369585            -0.49380495            -0.93904556           0.
           1.54606384            -0.49532847            -0.94223177           0.
           1.55843183            -0.49624953            -0.94415797           0.
           1.57080               -0.49656               -0.94460              0.
 0.00039    0.00002   0.00049          1     3
     1               3                 5               7
     2               3                 5               6
```

3. CYLINDRICAL SHELL - STABILITY UNDER AXIAL COMPRESSION

```
     1     1     0     3     1     0
    0.0
    0.0       2.0         10     2     2     0     1
   0.01095      1.0       90.0
    1.0       0.167
  -0.005475 0.005475
     1     2    -1
    0.0                             -1.0
    2.0                             -1.0
 0.00005    0.00001    0.00009         1     5
     1               4                 6               7
     1               4                 6               7
 0.00005    0.00001    0.00009         1     4
     1               4                 6               7
     1               4                 6               7
 0.00005    0.00001    0.00009         1     6
     1               4                 6               7
     1               4                 6               7
     0
 $EOF
```

```
        STABILITY AND FREE VIBRATION OF SHELLS WITH AXISYMMETRIC PRESTRESS, BY A. KALNINS, LEHIGH UNIV, BETHLEHEM, PA
                 WRIGHT PATTERSON AIR FORCE BASE FLIGHT DYNAMICS LABORATORY VERSION, 22 JULY 1968

                  FREE VIBRATION      PARTS=  1  BRANCHES=  0  NUMBER OF SUBCASES=  1

ANGLES OF ROTATION OF BOUNDARY CONDITIONS ARE ALFL=-0.           ALFRS=-0.

        PART NO  1

SI= 0.            SX= 0.12000E 02   IPAR= 17   ING=  3    SHELL TYPE 2   NTP= 0   LAYERS NLY= 1

CYLINDRICAL SHELL NO  2     H= 0.12500E-00    R= 0.39380E 01  PHI=    90.000  DEGREES

LAYER NO  1  FROM Z=-0.62500E-01  TO Z= 0.62500E-01
     CONSISTS OF ISOTROPIC MATERIAL, YOUNGS MODULUS E= 0.36500E 06  POISSONS RATIO NU= 0.30000E-00
     COEFFICIENTS OF THERMAL EXPANSION  AFI=-0.          ATHETA=-0.           MASS DENSITY RHO= 0.11310E-03

LCST=  -0  SO THAT PRESTRESS IS ZERO OVER THIS PART
```

STABILITY AND FREE VIBRATION OF SHELLS WITH AXISYMMETRIC PRESTRESS, BY A. KALNINS, LEHIGH UNIV, BETHLEHEM, PA
WRIGHT PATTERSON AIR FORCE BASE FLIGHT DYNAMICS LABORATORY VERSION, 22 JULY 1968

SUBCASE NO 1 WITH WAVE NUMBER 2

STARTING OMEGA= 0.17500E 03 INCREMENT= 0.20000E 01 FINAL OMEGA= 0.18300E 03 1 EIGENVALUES

| | | | | |
|---|---|---|---|---|
| BOUNDARY CONDITIONS AT STARTING EDGE | 1-0. | 3-0. | 5-0. | 7-0. |
| BOUNDARY CONDITIONS AT FINAL EDGE | 2-0. | 4-0. | 6-0. | 8-0. |

```
                 STABILITY AND FREE VIBRATION OF SHELLS WITH AXISYMMETRIC PRESTRESS, BY A. KALNINS, LEHIGH UNIV, BETHLEHEM, PA
                          WRIGHT PATTERSON AIR FORCE BASE FLIGHT DYNAMICS LABORATORY VERSION, 22 JULY 1968

 SOLUTION AT POINTS ALONG MERIDIAN AT EIGENVALUE PARAMETER OMEGA= 0.1750000E 03  FOR WAVE NUMBER NX=  2

      S          W          Q          UPHI       NPHI       BPHI       MPHI       UTHETA      N          NTHETA     MTHETA

   MAIN SHELL PART NO  1
 OMEGA= 0.17500E 03  OMSQ= 0.12090E 07  XMR= 0.10000E 01  LC= -0  PRESTRESS=   0.              0.

   0.      -0.          0.32390E 12-0.          -0.64364E 13-0.          -0.11110E 12-0.          -0.87317E 12-0.19309E 13-0.33331E 11
   0.23529 0.37390E 08 0.20757E 12-0.29926E 08-0.63180E 13-0.28096E 09-0.49537E 11-0.15074E 08-0.11036E 13-0.18115E 13-0.14404E 11
   0.47059 0.11881E 09 0.11233E 12-0.59701E 08-0.61738E 13-0.39020E 09-0.14277E 11-0.36585E 08-0.12926E 13-0.13234E 13-0.27429E 10
   0.70588 0.21411E 09 0.48575E 11-0.89492E 08-0.60107E 13-0.41046E 09 0.19536E 10-0.63777E 08-0.14187E 13-0.80033E 12 0.33783E 10

   0.70588 0.21412E 09 0.48576E 11-0.89492E 08-0.60107E 13-0.41050E 09 0.19478E 10-0.63778E 08-0.14187E 13-0.80024E 12 0.33767E 10
   0.94118 0.30937E 09 0.13058E 11-0.11914E 09-0.58358E 13-0.39656E 09 0.66639E 10-0.95816E 08-0.14877E 13-0.38665E 12 0.60056E 10
   1.17647 0.40048E 09-0.22916E 10-0.14840E 09-0.56553E 13-0.37870E 09 0.56872E 10-0.13201E 09-0.15157E 13-0.11552E 12 0.68314E 10
   1.41176 0.48832E 09-0.55839E 10-0.17705E 09-0.54729E 13-0.36998E 09 0.27591E 10-0.17185E 09-0.15187E 13 0.33757E 11 0.69938E 10

   1.41176 0.48833E 09-0.55861E 10-0.17705E 09-0.54729E 13-0.37000E 09 0.27555E 10-0.17185E 09-0.15187E 13 0.33800E 11 0.69928E 10
   1.64706 0.57556E 09-0.30110E 10-0.20491E 09-0.52909E 13-0.37337E 09-0.12482E 09-0.21501E 09-0.15092E 13 0.98813E 11 0.71346E 10
   1.88235 0.66483E 09 0.14960E 10-0.23190E 09-0.51103E 13-0.38689E 09-0.21449E 10-0.26129E 09-0.14949E 13 0.11502E 12 0.75420E 10
   2.11765 0.75815E 09 0.58602E 10-0.25797E 09-0.49313E 13-0.40710E 09-0.31727E 10-0.31054E 09-0.14800E 13 0.10863E 12 0.82862E 10

   2.11765 0.75815E 09 0.58489E 10-0.25797E 09-0.49313E 13-0.40712E 09-0.31797E 10-0.31054E 09-0.14800E 13 0.10867E 12 0.82841E 10
   2.35294 0.85669E 09 0.92537E 10-0.28310E 09-0.47539E 13-0.43081E 09-0.33950E 10-0.36266E 09-0.14662E 13 0.95896E 11 0.93302E 10
   2.58824 0.96096E 09 0.11586E 11-0.30730E 09-0.45780E 13-0.45555E 09-0.30560E 10-0.41755E 09-0.14535E 13 0.84895E 11 0.10609E 11
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       STABILITY AND FREE VIBRATION OF SHELLS WITH AXISYMMETRIC PRESTRESS, BY A. KALNINS, LEHIGH UNIV, BETHLEHEM, PA
                  WRIGHT PATTERSON AIR FORCE BASE FLIGHT DYNAMICS LABORATORY VERSION, 22 JULY 1968

SOLUTION AT POINTS ALONG MERIDIAN AT EIGENVALUE PARAMETER OMEGA= 0.1750000E 03  FOR WAVE NUMBER NX=  2

    S          W          UPHI       UTHETA      BPHI       SPHI IN    SPHI OUT   STHETA IN  STHETA OUT  SFITH IN   SFITH OUT

  MAIN SHELL PART NO  1
OMEGA= 0.17300E 03  OMSQ= 0.12090E 07  XMR= 0.10000E 01  LC= -0  PRESTRESS=   0.              0.

 0.      -0.        -0.        -0.        -0.        -0.88282E 13-0.94155E 14-0.26485E 13-0.28247E 14-0.60739E 13-0.70956E 13

 0.23529 0.37390E 08-0.29926E 08-0.15074E 08-0.28096E 09-0.31522E 14-0.69566E 14-0.89610E 13-0.20023E 14-0.11171E 14-0.65112E 13

 0.47059 0.11881E 09-0.59701E 08-0.36585E 08-0.39020E 09-0.43908E 14-0.54873E 14-0.95339E 13-0.11640E 14-0.13604E 14-0.71125E 13

 0.70588 0.21411E 09-0.89492E 08-0.63777E 08-0.41046E 09-0.48835E 14-0.47335E 14-0.77000E 13-0.51054E 13-0.14744E 14-0.79907E 13


 0.70588 0.21412E 09-0.89492E 08-0.63778E 08-0.41050E 09-0.48833E 14-0.47337E 14-0.76986E 13-0.51053E 13-0.14744E 14-0.79903E 13

 0.94118 0.30937E 09-0.11914E 09-0.95816E 08-0.39656E 09-0.49246E 14-0.44128E 14-0.53994E 13-0.78706E 12-0.15129E 14-0.87084E 13

 1.17647 0.40048E 09-0.14840E 09-0.13201E 09-0.37870E 09-0.47426E 14-0.43058E 14-0.35474E 13 0.16991E 13-0.15156E 14-0.91271E 13

 1.41176 0.48832E 09-0.17705E 09-0.171[illegible]5E 09-0.36998E 09-0.44843E 14-0.42724E 14-0.24156E 13 0.29557E 13-0.15069E 14-0.92612E 13


 1.41176 0.48833E 09-0.17705E 09-0.17185E 09-0.37000E 09-0.44841E 14-0.42725E 14-0.24148E 13 0.29556E 13-0.15069E 14-0.92610E 13

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 2.58824 0.96096E 09-0.30730E 09-0.41755E 09-0.45555E 09-0.35451E 14-0.37798E 14-0.33948E 13 0.47531E 13-0.15174E 14-0.81195E 13

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 3.29412 0.13077E 10-0.37446E 09-0.59785E 09-0.52490E 09-0.32181E 14-0.32751E 14-0.52103E 13 0.64903E 13-0.15439E 14-0.72910E 13

 3.52941 0.14336E 10-0.39506E 09-0.66281E 09-0.54523E 09-0.31149E 14-0.31054E 14-0.58193E 13 0.71805E 13-0.15496E 14-0.70295E 13


 3.52941 0.14336E 10-0.39508E 09-0.66281E 09-0.54530E 09-0.31141E 14-0.31062E 14-0.58161E 13 0.71790E 13-0.15497E 14-0.70294E 13

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|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 9.41176 | 0.54826E | 10-0.66315E | 09-0.27214E | 10-0.74220E | 09-0.12509E | 14 | 0.40666E | 13-0.24315E | 14 | 0.29165E | 14-0.11005E | 14 | 0.55786E | 12 |
| 9.64706 | 0.56568E | 10-0.66615E | 09-0.28092E | 10-0.73871E | 09-0.12304E | 14 | 0.51053E | 13-0.25154E | 14 | 0.30110E | 14-0.10577E | 14 | 0.92928E | 12 |
| 9.88235 | 0.58301E | 10-0.66876E | 09-0.28967E | 10-0.73357E | 09-0.12164E | 14 | 0.61127E | 13-0.26039E | 14 | 0.31009E | 14-0.10128E | 14 | 0.12939E | 13 |
| 9.88235 | 0.58302E | 10-0.66875E | 09-0.28968E | 10-0.73402E | 09-0.12115E | 14 | 0.60653E | 13-0.26021E | 14 | 0.30999E | 14-0.10135E | 14 | 0.12942E | 13 |
| 10.11765 | 0.60022E | 10-0.67101E | 09-0.29839E | 10-0.72778E | 09-0.11978E | 14 | 0.69788E | 13-0.26943E | 14 | 0.31821E | 14-0.96753E | 13 | 0.16509E | 13 |
| 10.35294 | 0.61726E | 10-0.67292E | 09-0.30706E | 10-0.72064E | 09-0.11782E | 14 | 0.77371E | 13-0.27907E | 14 | 0.32521E | 14-0.92149E | 13 | 0.19925E | 13 |
| 10.58823 | 0.63413E | 10-0.67451E | 09-0.31568E | 10-0.71419E | 09-0.11387E | 14 | 0.82025E | 13-0.28873E | 14 | 0.33056E | 14-0.87784E | 13 | 0.23226E | 13 |
| 10.58823 | 0.63414E | 10-0.67451E | 09-0.31568E | 10-0.71468E | 09-0.11333E | 14 | 0.81505E | 13-0.28853E | 14 | 0.33044E | 14-0.87867E | 13 | 0.23230E | 13 |
| 10.82353 | 0.65092E | 10-0.67579E | 09-0.32426E | 10-0.71269E | 09-0.10517E | 14 | 0.81070E | 13-0.29694E | 14 | 0.33415E | 14-0.84142E | 13 | 0.26684E | 13 |
| 11.05882 | 0.66775E | 10-0.67680E | 09-0.33280E | 10-0.71993E | 09-0.90838E | 13 | 0.73632E | 13-0.30226E | 14 | 0.33709E | 14-0.81447E | 13 | 0.30756E | 13 |
| 11.29412 | 0.68493E | 10-0.67760E | 09-0.34129E | 10-0.74345E | 09-0.68851E | 13 | 0.57730E | 13-0.30143E | 14 | 0.34177E | 14-0.80125E | 13 | 0.36360E | 13 |
| 11.29412 | 0.68493E | 10-0.67760E | 09-0.34129E | 10-0.74398E | 09-0.68257E | 13 | 0.57161E | 13-0.30121E | 14 | 0.34164E | 14-0.80215E | 13 | 0.36364E | 13 |
| 11.52941 | 0.70295E | 10-0.67835E | 09-0.34972E | 10-0.79188E | 09-0.39510E | 13 | 0.33607E | 13-0.29009E | 14 | 0.35331E | 14-0.80269E | 13 | 0.44967E | 13 |
| 11.76471 | 0.72242E | 10-0.67931E | 09-0.35806E | 10-0.86789E | 09-0.11256E | 13 | 0.92976E | 12-0.26506E | 14 | 0.38057E | 14-0.80670E | 13 | 0.58297E | 13 |
| 12.00000 | 0.74397E | 10-0.68087E | 09-0.36631E | 10-0.96554E | 09-0.64251E | 11 | 0.61603E | 11-0.22502E | 14 | 0.43534E | 14-0.78708E | 13 | 0.77973E | 13 |

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             STABILITY AND FREE VIBRATION OF SHELLS WITH AXISYMMETRIC PRESTRESS, BY A. KALNINS, LEHIGH UNIV, BETHLEHEM, PA
                       WRIGHT PATTERSON AIR FORCE BASE FLIGHT DYNAMICS LABORATORY VERSION, 22 JULY 1968

SOLUTION AT POINTS ALONG MERIDIAN AT EIGENVALUE PARAMETER OMEGA= 0.1770000E 03   FOR WAVE NUMBER NX=  2

    S          W           G           UPHI        NPHI         BPHI        MPHI        UTHETA        N          NTHETA       MTHETA

  MAIN SHELL PART NO  1
OMEGA= 0.17700E 03  CMSC= 0.12368E 07  XMR= 0.10000E 01  LC= -0  PRESTRESS=    0.                 0.

  0.      -0.          0.32523E 12-0.          -0.64530E 13-0.          -0.11165E 12-0.          -0.88545E 12-0.19359E 13-0.33495E 11
  0.23529 0.37581E 08 0.20854E 12-0.29998E 08-0.63330E 13-0.26242E 09-0.49816E 11-0.15248E 08-0.11166E 13-0.18178E 13-0.14486E 11
  0.47059 0.11943E 09 0.11293E 12-0.59837E 08-0.61872E 13-0.39233E 09-0.14382E 11-0.36951E 08-0.13063E 13-0.13286E 13-0.27676E 10
  0.70588 0.21527E 09 0.48898E 11-0.89686E 08-0.60224E 13-0.41279E 09 0.19440E 10-0.64350E 08-0.14329E 13-0.80372E 12 0.33808E 10

  0.70588 0.21528E 09 0.48899E 11-0.89686E 08-0.60224E 13-0.41283E 09 0.19387E 10-0.64351E 08-0.14329E 13-0.80364E 12 0.33873E 10
  0.94118 0.31107E 09 0.13204E 11-0.11939E 09-0.58458E 13-0.39885E 09 0.66966E 10-0.96609E 08-0.15022E 13-0.38828E 12 0.60355E 10
  1.17647 0.40271E 09-0.22391E 10-0.14870E 09-0.56635E 13-0.38087E 09 0.57308E 10-0.13303E 09-0.15303E 13-0.11580E 12 0.68708E 10
  1.41176 0.49105E 09-0.55681E 10-0.17738E 09-0.54794E 13-0.37204E 09 0.27996E 10-0.17310E 09-0.15333E 13 0.34371E 11 0.70381E 10

  1.41176 0.49106E 09-0.55719E 10-0.17738E 09-0.54794E 13-0.37206E 09 0.27960E 10-0.17310E 09-0.15333E 13 0.34415E 11 0.70371E 10
  1.64706 0.57876E 09-0.30013E 10-0.20527E 09-0.52956E 13-0.37534E 09-0.91588E 08-0.21650E 09-0.15236E 13 0.99957E 11 0.71822E 10
  1.88235 0.66849E 09 0.15154E 10-0.23229E 09-0.51133E 13-0.38879E 09-0.21177E 10-0.26302E 09-0.15091E 13 0.11646E 12 0.75929E 10
  2.11765 0.76224E 09 0.58927E 10-0.25837E 09-0.49326E 13-0.40896E 09-0.31484E 10-0.31250E 09-0.14940E 13 0.11024E 12 0.83412E 10

  2.11765 0.76225E 09 0.58810E 10-0.25837E 09-0.49326E 13-0.40899E 09-0.31555E 10-0.31250E 09-0.14940E 13 0.11028E 12 0.83392E 10
  2.35294 0.86122E 09 0.92970E 10-0.28350E 09-0.47535E 13-0.43265E 09-0.33706E 10-0.36486E 09-0.14799E 13 0.97632E 11 0.93902E 10
  2.58824 0.96593E 09 0.11637E 11-0.30770E 09-0.45760E 13-0.45736E 09-0.30291E 10-0.41998E 09-0.14670E 13 0.86753E 11 0.10675E 11
  2.82353 0.10764E 10 0.13104E 11-0.33097E 09-0.43998E 13-0.48165E 09-0.23658E 10-0.47777E 09-0.14549E 13 0.80611E 11 0.12124E 11

  2.82353 0.10764E 10 0.13081E 11-0.33097E 09-0.43998E 13-0.48170E 09-0.23793E 10-0.47777E 09-0.14549E 13 0.80692E 11 0.12120E 11
  3.05882 0.11925E 10 0.13964E 11-0.35333E 09-0.42249E 13-0.50486E 09-0.15671E 10-0.53813E 09-0.14429E 13 0.79582E 11 0.13680E 11
  3.29412 0.13139E 10 0.14511E 11-0.37480E 09-0.40514E 13-0.52658E 09-0.70096E 09-0.60094E 09-0.14305E 13 0.82346E 11 0.15319E 11
  3.52941 0.14402E 10 0.14896E 11-0.39538E 09-0.38793E 13-0.54684E 09 0.16987E 09-0.66610E 09-0.14173E 13 0.87615E 11 0.17016E 11

  3.52941 0.14402E 10 0.14861E 11-0.39538E 09-0.38793E 13-0.54692E 09 0.14864E 09-0.66610E 09-0.14173E 13 0.87729E 11 0.17010E 11
  3.76471 0.15712E 10 0.15195E 11-0.41509E 09-0.37088E 13-0.56588E 09 0.99834E 09-0.73348E 09-0.14030E 13 0.94620E 11 0.18755E 11
  4.00000 0.17064E 10 0.15526E 11-0.43393E 09-0.35399E 13-0.58361E 09 0.18364E 10-0.80297E 09-0.13875E 13 0.10223E 12 0.20547E 11
  4.23529 0.18457E 10 0.15863E 11-0.45192E 09-0.33729E 13-0.60019E 09 0.26699E 10-0.87444E 09-0.13707E 13 0.11003E 12 0.22384E 11

  4.23529 0.18457E 10 0.15813E 11-0.45192E 09-0.33729E 13-0.60029E 09 0.26399E 10-0.87444E 09-0.13708E 13 0.11018E 12 0.22375E 11
  4.47059 0.19888E 10 0.16165E 11-0.46906E 09-0.32079E 13-0.61591E 09 0.34638E 10-0.94778E 09-0.13527E 13 0.11825E 12 0.24253E 11
  4.70588 0.21355E 10 0.16518E 11-0.48537E 09-0.30450E 13-0.63050E 09 0.42975E 10-0.10229E 10-0.13333E 13 0.12633E 12 0.26175E 11
  4.94118 0.22855E 10 0.16860E 11-0.50086E 09-0.28845E 13-0.64407E 09 0.51450E 10-0.10996E 10-0.13125E 13 0.13428E 12 0.28139E 11

  4.94118 0.22855E 10 0.16792E 11-0.50086E 09-0.28845E 13-0.64421E 09 0.51052E 10-0.10996E 10-0.13127E 13 0.13446E 12 0.28128E 11
  5.17647 0.24386E 10 0.17123E 11-0.51554E 09-0.27264E 13-0.65692E 09 0.59509E 10-0.11778E 10-0.12906E 13 0.14272E 12 0.30128E 11
  5.41176 0.25945E 10 0.17435E 11-0.52943E 09-0.25710E 13-0.66866E 09 0.68130E 10-0.12575E 10-0.12670E 13 0.15102E 12 0.32166E 11
  5.64706 0.27532E 10 0.17720E 11-0.54253E 09-0.24185E 13-0.67939E 09 0.76910E 10-0.13384E 10-0.12422E 13 0.15924E 12 0.34240E 11

  5.64706 0.27532E 10 0.17633E 11-0.54253E 09-0.24184E 13-0.67957E 09 0.76407E 10-0.13384E 10-0.12423E 13 0.15946E 12 0.34226E 11
  5.88235 0.29143E 10 0.17901E 11-0.55487E 09-0.22688E 13-0.68950E 09 0.85117E 10-0.14205E 10-0.12160E 13 0.16809E 12 0.36326E 11
  6.11765 0.30776E 10 0.18148E 11-0.56646E 09-0.21224E 13-0.69846E 09 0.93972E 10-0.15037E 10-0.11883E 13 0.17678E 12 0.38457E 11
  6.35294 0.32429E 10 0.18366E 11-0.57731E 09-0.19794E 13-0.70644E 09 0.10296E 11-0.15878E 10-0.11591E 13 0.18534E 12 0.40615E 11

  6.35294 0.32429E 10 0.18258E 11-0.57731E 09-0.19793E 13-0.70666E 09 0.10235E 11-0.15878E 10-0.11593E 13 0.18560E 12 0.40597E 11
  6.58823 0.34101E 10 0.18460E 11-0.58745E 09-0.18398E 13-0.71390E 09 0.11119E 11-0.16728E 10-0.11286E 13 0.19459E 12 0.42772E 11
  6.82353 0.35788E 10 0.18642E 11-0.59689E 09-0.17039E 13-0.72022E 09 0.12016E 11-0.17586E 10-0.10964E 13 0.20360E 12 0.44970E 11
  7.05882 0.37489E 10 0.18795E 11-0.60566E 09-0.15720E 13-0.72558E 09 0.12924E 11-0.18449E 10-0.10627E 13 0.21238E 12 0.47187E 11

  7.05882 0.37489E 10 0.18665E 11-0.60566E 09-0.15719E 13-0.72584E 09 0.12850E 11-0.18449E 10-0.10630E 13 0.21260E 12 0.47166E 11
  7.29412 0.39203E 10 0.18802E 11-0.61377E 09-0.14440E 13-0.73056E 09 0.13735E 11-0.19319E 10-0.10278E 13 0.22193E 12 0.49391E 11
  7.52941 0.40927E 10 0.18922E 11-0.62124E 09-0.13204E 13-0.73441E 09 0.14629E 11-0.20193E 10-0.99106E 12 0.23119E 12 0.51631E 11
  7.76471 0.42658E 10 0.19013E 11-0.62811E 09-0.12012E 13-0.73737E 09 0.15531E 11-0.21070E 10-0.95280E 12 0.24018E 12 0.53803E 11
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 8.23529 0.46141E 10 0.19033E 11-0.64009E 09-0.97678E 12-0.74175E 09 0.17198E 11-0.22832E 10-0.87185E 12 0.25991E 12 0.58370E 11
 8.47059 0.47888E 10 0.19121E 11-0.64527E 09-0.87205E 12-0.74257E 09 0.18094E 11-0.23715E 10-0.82883E 12 0.26958E 12 0.60638E 11

 8.47059 0.47888E 10 0.18945E 11-0.64526E 09-0.87195E 12-0.74292E 09 0.17995E 11-0.23715E 10-0.82918E 12 0.26994E 12 0.60609E 11
 8.70588 0.49637E 10 0.19073E 11-0.64993E 09-0.77239E 12-0.74328E 09 0.18869E 11-0.24597E 10-0.78448E 12 0.28052E 12 0.62875E 11
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 9.17647 0.53132E 10 0.19497E 11-0.65783E 09-0.58976E 12-0.74136E 09 0.20744E 11-0.26359E 10-0.68997E 12 0.30152E 12 0.67442E 11

 9.17647 0.53132E 10 0.19296E 11-0.65783E 09-0.58965E 12-0.74176E 09 0.20631E 11-0.26359E 10-0.69037E 12 0.30194E 12 0.67409E 11
 9.41176 0.54875E 10 0.19586E 11-0.66113E 09-0.50694E 12-0.73958E 09 0.21618E 11-0.27237E 10-0.64057E 12 0.31197E 12 0.69703E 11
 9.64706 0.56611E 10 0.19838E 11-0.66403E 09-0.43027E 12-0.73606E 09 0.22684E 11-0.28112E 10-0.58926E 12 0.31963E 12 0.72014E 11
 9.88235 0.58338E 10 0.19843E 11-0.66656E 09-0.35978E 12-0.73095E 09 0.23608E 11-0.28983E 10-0.53687E 12 0.32233E 12 0.74329E 11

 9.88235 0.58338E 10 0.19618E 11-0.66656E 09-0.35966E 12-0.73139E 09 0.23682E 11-0.28983E 10-0.53732E 12 0.32278E 12 0.74292E 11
10.11765 0.60052E 10 0.19057E 11-0.66874E 09-0.29541E 12-0.72513E 09 0.24723E 11-0.29850E 10-0.48450E 12 0.31898E 12 0.76566E 11
10.35294 0.61750E 10 0.17418E 11-0.67060E 09-0.23745E 12-0.71775E 09 0.25551E 11-0.30712E 10-0.43227E 12 0.30530E 12 0.78755E 11
10.58823 0.63430E 10 0.14065E 11-0.67214E 09-0.18562E 12-0.71051E 09 0.25843E 11-0.31570E 10-0.38193E 12 0.28069E 12 0.80760E 11

10.58823 0.63430E 10 0.13818E 11-0.67213E 09-0.18548E 12-0.71100E 09 0.25705E 11-0.31570E 10-0.38242E 12 0.28119E 12 0.80720E 11
10.82353 0.65098E 10 0.82497E 10-0.67338E 09-0.13945E 12-0.70726E 09 0.24922E 11-0.32422E 10-0.33494E 12 0.25159E 12 0.82387E 11
11.05882 0.66765E 10 0.46828E 09-0.67436E 09-0.98903E 11-0.71127E 09 0.22559E 11-0.33270E 10-0.29030E 12 0.23061E 12 0.83583E 11
11.29412 0.68456E 10-0.82460E 10-0.67514E 09-0.63624E 11-0.72968E 09 0.18150E 11-0.34113E 10-0.24587E 12 0.24820E 12 0.84206E 11

11.29412 0.68457E 10-0.65180E 10-0.67513E 09-0.63471E 11-0.73021E 09 0.17999E 11-0.34113E 10-0.24642E 12 0.24872E 12 0.84162E 11
11.52941 0.70219E 10-0.14423E 11-0.67583E 09-0.33742E 11-0.77129E 09 0.11532E 11-0.34950E 10-0.19474E 12 0.35790E 12 0.84279E 11
11.76471 0.72109E 10-0.10062E 11-0.67669E 09-0.11300E 11-0.84013E 09 0.43862E 10-0.35781E 10-0.12050E 12 0.63095E 12 0.84396E 11
12.00000 0.74192E 10 0.16811E 11-0.67805E 09-0.16542E 09-0.93386E 09 0.16351E 09-0.36599E 10 0.58886E 09 0.11509E 13 0.85694E 11
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STABILITY AND FREE VIBRATION OF SHELLS WITH AXISYMMETRIC PRESTRESS, BY A. KALNINS, LEHIGH UNIV, BETHLEHEM, PA
WRIGHT PATTERSON AIR FORCE BASE FLIGHT DYNAMICS LABORATORY VERSION, 22 JULY 1968

SOLUTION AT POINTS ALONG MERIDIAN AT EIGENVALUE PARAMETER OMEGA= 0.1770000E 03 FOR WAVE NUMBER NX= 2

MAIN SHELL PART NO 1
OMEGA= 0.17700E 03 OMSQ= 0.12368E 07 XMK= 0.10000E 01 LC= -0 PRESTRESS= 0. 0.

| S | W | UPHI | UTHETA | BPHI | SPHI IN | SPHI OUT | STHETA IN | STHETA OUT | SFITH IN | SFITH OUT |
|---|---|---|---|---|---|---|---|---|---|---|
| 0. | -0. | -0. | -0. | -0. | -0.87502E 13 | -0.94497E 14 | -0.26251E 13 | -0.28349E 14 | -0.69706E 13 | -0.71954E 13 |
| 0.23529 | 0.37581E 08 | -0.29998E 08 | -0.15248E 08 | -0.28242E 09 | -0.31535E 14 | -0.69793E 14 | -0.89799E 13 | -0.20105E 14 | -0.11286E 14 | -0.66039E 13 |
| 0.47059 | 0.11943E 09 | -0.59837E 08 | -0.36951E 08 | -0.39233E 09 | -0.43775E 14 | -0.55021E 14 | -0.95664E 13 | -0.11692E 14 | -0.13731E 14 | -0.72051E 13 |
| 0.70588 | 0.21527E 09 | -0.89686E 08 | -0.64350E 08 | -0.41279E 09 | -0.48925E 14 | -0.47432E 14 | -0.77310E 13 | -0.51284E 13 | -0.14876E 14 | -0.80857E 13 |
| 0.70588 | 0.21528E 09 | -0.89686E 08 | -0.64351E 08 | -0.41283E 09 | -0.48923E 14 | -0.47434E 14 | -0.77299E 13 | -0.51284E 13 | -0.14876E 14 | -0.80855E 13 |
| 0.94118 | 0.31107E 09 | -0.11939E 09 | -0.96609E 08 | -0.39885E 09 | -0.49336E 14 | -0.44195E 14 | -0.54239E 13 | -0.78864E 12 | -0.15263E 14 | -0.88063E 13 |
| 1.17647 | 0.40271E 09 | -0.14870E 09 | -0.13303E 09 | -0.38087E 09 | -0.47509E 14 | -0.43107E 14 | -0.35648E 13 | 0.17120E 13 | -0.15290E 14 | -0.92267E 13 |
| 1.41176 | 0.49105E 09 | -0.17736E 09 | -0.17310E 09 | -0.37204E 09 | -0.44910E 14 | -0.42760E 14 | -0.24277E 13 | 0.29776E 13 | -0.15202E 14 | -0.93616E 13 |
| 1.41176 | 0.49106E 09 | -0.17738E 09 | -0.17310E 09 | -0.37206E 09 | -0.44909E 14 | -0.42761E 14 | -0.24269E 13 | 0.29776E 13 | -0.15202E 14 | -0.93615E 13 |
| 1.64706 | 0.57876E 09 | -0.20527E 09 | -0.21650E 09 | -0.37534E 09 | -0.42330E 14 | -0.42400E 14 | -0.19583E 13 | 0.35576E 13 | -0.15123E 14 | -0.92850E 13 |
| 1.88235 | 0.66849E 09 | -0.23229E 09 | -0.26302E 09 | -0.38879E 09 | -0.40093E 14 | -0.41719E 14 | -0.19840E 13 | 0.38474E 13 | -0.15099E 14 | -0.90785E 13 |
| 2.11765 | 0.76224E 09 | -0.25837E 09 | -0.31250E 09 | -0.40896E 09 | -0.38252E 14 | -0.40670E 14 | -0.23211E 13 | 0.40849E 13 | -0.15131E 14 | -0.88064E 13 |
| 2.11765 | 0.76225E 09 | -0.25837E 09 | -0.31250E 09 | -0.40899E 09 | -0.38249E 14 | -0.40672E 14 | -0.23200E 13 | 0.40845E 13 | -0.15132E 14 | -0.88063E 13 |
| 2.35294 | 0.86122E 09 | -0.28350E 09 | -0.36486E 09 | -0.43265E 09 | -0.36734E 14 | -0.39322E 14 | -0.28248E 13 | 0.43869E 13 | -0.15204E 14 | -0.85104E 13 |
| 2.58824 | 0.96593E 09 | -0.30770E 09 | -0.41998E 09 | -0.45736E 09 | -0.35444E 14 | -0.37771E 14 | -0.34051E 13 | 0.47931E 13 | -0.15296E 14 | -0.82137E 13 |
| 2.82353 | 0.10764E 10 | -0.33097E 09 | -0.47777E 09 | -0.48165E 09 | -0.34290E 14 | -0.36107E 14 | -0.40107E 13 | 0.53005E 13 | -0.15392E 14 | -0.79259E 13 |
| 2.82353 | 0.10764E 10 | -0.33097E 09 | -0.47777E 09 | -0.48170E 09 | -0.34284E 14 | -0.36112E 14 | -0.40086E 13 | 0.52996E 13 | -0.15393E 14 | -0.79259E 13 |
| 3.05882 | 0.11925E 10 | -0.35333E 09 | -0.53813E 09 | -0.50486E 09 | -0.33197E 14 | -0.34401E 14 | -0.46165E 13 | 0.58899E 13 | -0.15480E 14 | -0.76481E 13 |
| 3.29412 | 0.13139E 10 | -0.37480E 09 | -0.60094E 09 | -0.52658E 09 | -0.32142E 14 | -0.32680E 14 | -0.52236E 13 | 0.65411E 13 | -0.15553E 14 | -0.73788E 13 |
| 3.52941 | 0.14402E 10 | -0.39538E 09 | -0.66610E 09 | -0.54684E 09 | -0.31100E 14 | -0.30969E 14 | -0.58333E 13 | 0.72351E 13 | -0.15606E 14 | -0.71140E 13 |
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| 3.76471 | 0.15712E 10 | -0.41509E 09 | -0.73348E 09 | -0.56588E 09 | -0.30053E 14 | -0.29287E 14 | -0.64450E 13 | 0.79589E 13 | -0.15643E 14 | -0.68525E 13 |
| 4.00000 | 0.17064E 10 | -0.43393E 09 | -0.80297E 09 | -0.58361E 09 | -0.29024E 14 | -0.27614E 14 | -0.70721E 13 | 0.87078E 13 | -0.15658E 14 | -0.65905E 13 |
| 4.23529 | 0.18457E 10 | -0.45192E 09 | -0.87444E 09 | -0.60019E 09 | -0.28009E 14 | -0.25958E 14 | -0.77151E 13 | 0.94755E 13 | -0.15654E 14 | -0.63274E 13 |

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          STABILITY AND FREE VIBRATION OF SHELLS WITH AXISYMMETRIC PRESTRESS, BY A. KALNINS, LEHIGH UNIV, BETHLEHEM, PA
                    WRIGHT PATTERSON AIR FORCE BASE FLIGHT DYNAMICS LABORATORY VERSION, 22 JULY 1968

SOLUTION AT POINTS ALONG MERIDIAN AT EIGENVALUE PARAMETER OMEGA= 0.1790000E 03  FOR WAVE NUMBER NX=  2

     S          W          Q           UPHI        NPHI        BPHI        MPHI       UTHETA        N          NTHETA      MTHETA

  MAIN SHELL PART NO  1
OMEGA= 0.17900E 03  OMSQ= 0.12649E 07  XMR= 0.10000E 01  LC= -0  PRESTRESS=   0.               0.

  0.      -0.          0.32658E 12-0.          -0.64698E 13-0.          -0.11221E 12-0.          -0.89793E 12-0.19409E 13-0.33662E 11
  0.23529 0.37775E 08 0.20952E 12-0.30072E 08-0.63483E 13-0.28391E 09-0.50100E 11-0.15424E 08-0.11298E 13-0.18242E 13-0.14569E 11
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 9.41176 0.54925E 10 0.19444E 11-0.65907E 09-0.48591E 12-0.73691E 09 0.21653E 11-0.27261E 10-0.63189E 12 0.32092E 12 0.69772E 11
 9.64706 0.56655E 10 0.19724E 11-0.66188E 09-0.41031E 12-0.73337E 09 0.22701E 11-0.28132E 10-0.57918E 12 0.32968E 12 0.72070E 11
 9.88235 0.58375E 10 0.19827E 11-0.66433E 09-0.34107E 12-0.72826E 09 0.23821E 11-0.28999E 10-0.52522E 12 0.33413E 12 0.74377E 11

 9.88235 0.58375E 10 0.19600E 11-0.66432E 09-0.34094E 12-0.72872E 09 0.23693E 11-0.28999E 10-0.52567E 12 0.33461E 12 0.74340E 11
10.11765 0.60083E 10 0.19251E 11-0.66644E 09-0.27814E 12-0.72243E 09 0.24763E 11-0.29862E 10-0.47101E 12 0.33327E 12 0.76617E 11
10.35294 0.61774E 10 0.17974E 11-0.66623E 09-0.22186E 12-0.71479E 09 0.25690E 11-0.30719E 10-0.41662E 12 0.32245E 12 0.78829E 11
10.58823 0.63446E 10 0.15134E 11-0.66973E 09-0.17200E 12-0.70676E 09 0.26186E 11-0.31571E 10-0.36378E 12 0.30015E 12 0.80887E 11

10.58823 0.63447E 10 0.14884E 11-0.66972E 09-0.17186E 12-0.70727E 09 0.26045E 11-0.31572E 10-0.36427E 12 0.30069E 12 0.80846E 11
10.82353 0.65104E 10 0.98988E 10-0.67093E 09-0.12811E 12-0.70175E 09 0.25604E 11-0.32419E 10-0.31413E 12 0.27086E 12 0.82604E 11
11.05882 0.66754E 10 0.25258E 10-0.67189E 09-0.90146E 11-0.70248E 09 0.23720E 11-0.33260E 10-0.26716E 12 0.24368E 12 0.83922E 11
11.29412 0.68420E 10-0.64485E 10-0.67264E 09-0.57666E 11-0.71570E 09 0.19843E 11-0.34097E 10-0.22174E 12 0.24415E 12 0.84670E 11

11.29412 0.68420E 10-0.67226E 10-0.67263E 09-0.57511E 11-0.71625E 09 0.19689E 11-0.34097E 10-0.22229E 12 0.24471E 12 0.84625E 11
11.52941 0.70141E 10-0.14372E 11-0.67328E 09-0.30547E 11-0.75042E 09 0.13571E 11-0.34928E 10-0.17248E 12 0.32020E 12 0.84789E 11
11.76471 0.71973E 10-0.14378E 11-0.67402E 09-0.10351E 11-0.81199E 09 0.61201E 10-0.35753E 10-0.10555E 12 0.53874E 12 0.84730E 11
12.00000 0.73985E 10 0.42534E 10-0.67519E 09-0.16752E 09-0.90173E 09 0.16642E 09-0.36567E 10 0.59508E 09 0.98513E 12 0.85402E 11
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STABILITY AND FREE VIBRATION OF SHELLS WITH AXISYMMETRIC PRESTRESS, BY A. KALNINS, LEHIGH UNIV, BETHLEHEM, PA
       WRIGHT PATTERSON AIR FORCE BASE FLIGHT DYNAMICS LABORATORY VERSION, 22 JULY 1968

SOLUTION AT POINTS ALONG MERIDIAN AT EIGENVALUE PARAMETER OMEGA= 0.1790000E 03  FOR WAVE NUMBER NX=  2

   S          W            UPHI          UTHETA        BPHI         SPHI IN     SPHI OUT   STHETA IN    STHETA OUT    SFITH IN     SFITH OUT

  MAIN SHELL PART NO  1
OMEGA= 0.17900E 03  OMSQ= 0.12649E 07  XMR= 0.10000E 01  LC= -0  PRESTRESS=   0.                0.
 0.      -0.          -0.          -0.          -0.          -0.86710E 13-0.94845E 14-0.26013E 13-0.28454E 14-0.70688E 13-0.72968E 13
 0.23529 0.37775E 08-0.30072E 08-0.15424E 08-0.28391E 09-0.31548E 14-0.70025E 14-0.89993E 13-0.20189E 14-0.11403E 14-0.66981E 13
 0.47059 0.12007E 09-0.59976E 08-0.37322E 08-0.39450E 09-0.44044E 14-0.55171E 14-0.95994E 13-0.11744E 14-0.13859E 14-0.72992E 13
 0.70588 0.21645E 09-0.89884E 08-0.64932E 08-0.41516E 09-0.49018E 14-0.47532E 14-0.77628E 13-0.51519E 13-0.15011E 14-0.81823E 13

 0.70588 0.21646E 09-0.89884E 08-0.64934E 08-0.41520E 09-0.49016E 14-0.47534E 14-0.77617E 13-0.51519E 13-0.15011E 14-0.81822E 13
 0.94118 0.31281E 09-0.11964E 09-0.97415E 08-0.40118E 09-0.49432E 14-0.44264E 14-0.54489E 13-0.79029E 12-0.15400E 14-0.89057E 13
 1.17647 0.40497E 09-0.14899E 09-0.13406E 09-0.38308E 09-0.47593E 14-0.43158E 14-0.35826E 13 0.17250E 13-0.15426E 14-0.93281E 13
 1.41176 0.49382E 09-0.17772E 09-0.17438E 09-0.37413E 09-0.44979E 14-0.42797E 14-0.24401E 13 0.29998E 13-0.15337E 14-0.94637E 13

 1.41176 0.49383E 09-0.17771E 09-0.17438E 09-0.37415E 09-0.44977E 14-0.42796E 14-0.24393E 13 0.29998E 13-0.15337E 14-0.94636E 13
 1.64706 0.58201E 09-0.20564E 09-0.21802E 09-0.37733E 09-0.42381E 14-0.42426E 14-0.19676E 13 0.35854E 13-0.15256E 14-0.93869E 13
 1.88235 0.67220E 09-0.23267E 09-0.26477E 09-0.39073E 09-0.40128E 14-0.41733E 14-0.19922E 13 0.38790E 13-0.15230E 14-0.91794E 13
 2.11765 0.76641E 09-0.25877E 09-0.31450E 09-0.41086E 09-0.38272E 14-0.40670E 14-0.23295E 13 0.41196E 13-0.15260E 14-0.89058E 13

 2.11765 0.76641E 09-0.25877E 09-0.31450E 09-0.41089E 09-0.38269E 14-0.40673E 14-0.23284E 13 0.41192E 13-0.15260E 14-0.89058E 13
 2.35294 0.86583E 09-0.28391E 09-0.36709E 09-0.43451E 09-0.36740E 14-0.39309E 14-0.28340E 13 0.44245E 13-0.15331E 14-0.86081E 13
 2.58824 0.97097E 09-0.30810E 09-0.42244E 09-0.45920E 09-0.35438E 14-0.37743E 14-0.34155E 13 0.48339E 13-0.15421E 14-0.83095E 13
 2.82353 0.10819E 10-0.33136E 09-0.48046E 09-0.48346E 09-0.34272E 14-0.36064E 14-0.40224E 13 0.53447E 13-0.15514E 14-0.80197E 13

 2.82353 0.10819E 10-0.33136E 09-0.48047E 09-0.48351E 09-0.34266E 14-0.36070E 14-0.40201E 13 0.53439E 13-0.15514E 14-0.80196E 13
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 3.29412 0.13202E 10-0.37514E 09-0.60408E 09-0.52829E 09-0.32102E 14-0.32609E 14-0.52371E 13 0.65929E 13-0.15668E 14-0.74680E 13
 3.52941 0.14469E 10-0.39569E 09-0.66945E 09-0.54848E 09-0.31050E 14-0.30883E 14-0.58475E 13 0.72908E 13-0.15719E 14-0.72016E 13

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 3.76471 0.15782E 10-0.41535E 09-0.73703E 09-0.56744E 09-0.29993E 14-0.29187E 14-0.64596E 13 0.80186E 13-0.15751E 14-0.69367E 13
 4.00000 0.17139E 10-0.43414E 09-0.80670E 09-0.58507E 09-0.28954E 14-0.27501E 14-0.70870E 13 0.87713E 13-0.15762E 14-0.66718E 13
 4.23529 0.18535E 10-0.45207E 09-0.67835E 09-0.60154F 09-0.27929E 14-0.25832E 14-0.77301E 13 0.95429E 13-0.15753E 14-0.64058E 13
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4.70588 0.21438E 10-0.48539E 09-0.10271E 10-0.63161E 09-0.25913E 14-0.22560E 14-0.90552E 13 0.11136E 14-0.15682E 14-0.58640E 13
4.94118 0.22941E 10-0.50080E 09-0.11039E 10-0.64504E 09-0.24946E 14-0.20939E 14-0.97455E 13 0.11958E 14-0.15616E 14-0.55888E 13

4.94118 0.22941E 10-0.50080E 09-0.11039E 10-0.64519E 09-0.24930E 14-0.20954E 14-0.97394E 13 0.11955E 14-0.15619E 14-0.55886E 13
5.17647 0.24474E 10-0.51540E 09-0.11823E 10-0.65776E 09-0.23983E 14-0.19354E 14-0.10441E 14 0.12792E 14-0.15536E 14-0.53087E 13
5.41176 0.26035E 10-0.52920E 09-0.12620E 10-0.66933E 09-0.23063E 14-0.17770E 14-0.11156E 14 0.13645E 14-0.15433E 14-0.50253E 13
5.64706 0.27623E 10-0.54221E 09-0.13430E 10-0.67990E 09-0.22173E 14-0.16204E 14-0.11886E 14 0.14510E 14-0.15312E 14-0.47384E 13

5.64706 0.27623E 10-0.54221E 09-0.13430E 10-0.68008E 09-0.22153E 14-0.16223E 14-0.11878E 14 0.14506E 14-0.15315E 14-0.47382E 13
5.88235 0.29235E 10-0.55445E 09-0.14252E 10-0.68984E 09-0.21285E 14-0.14683E 14-0.12615E 14 0.15384E 14-0.15175E 14-0.44456E 13
6.11765 0.30869E 10-0.56593E 09-0.15084E 10-0.69862E 09-0.20448E 14-0.13165E 14-0.13362E 14 0.16275E 14-0.15016E 14-0.41489E 13
6.35294 0.32522E 10-0.57669E 09-0.15925E 10-0.70641E 09-0.19645E 14-0.11668E 14-0.14121E 14 0.17174E 14-0.14838E 14-0.38481E 13

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7.76471 0.42742E 10-0.62681E 09-0.21112E 10-0.73606E 09-0.15416E 14-0.34129E 13-0.18762E 14 0.22717E 14-0.13378E 14-0.19193E 13

7.76471 0.42743E 10-0.62680E 09-0.21112E 10-0.73638E 09-0.15382E 14-0.34460E 13-0.18749E 14 0.22710E 14-0.13383E 14-0.19191E 13
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8.23529 0.46218E 10-0.63856E 09-0.22870E 10-0.73998E 09-0.14263E 14-0.98011E 12-0.20320E 14 0.24598E 14-0.12745E 14-0.12223E 13
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8.70588 0.49704E 10-0.64818E 09-0.24631E 10-0.74106E 09-0.13271E 14 0.12856E 13-0.21877E 14 0.26490E 14-0.12038E 14-0.49503E 12
8.94118 0.51447E 10-0.65225E 09-0.25510E 10-0.74040E 09-0.12868E 14 0.23790E 13-0.22660E 14 0.27448E 14-0.11653E 14-0.11870E 12
9.17647 0.53187E 10-0.65587E 09-0.26387E 10-0.73878E 09-0.12531E 14 0.34453E 13-0.23451E 14 0.28408E 14-0.11246E 14 0.26378E 12

9.17647 0.53188E 10-0.65587E 09-0.26387E 10-0.73919E 09-0.12486E 14 0.34021E 13-0.23435E 14 0.28398E 14-0.11253E 14 0.26414E 12
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9.41176 0.54925E 10-0.65907E 09-0.27261E 10-0.73691E 09-0.12202E 14 0.44275E 13-0.24225E 14 0.29360E 14-0.10827E 14 0.65554E 12
9.64706 0.56655E 10-0.66188E 09-0.28132E 10-0.73337E 09-0.12000E 14 0.54348E 13-0.25037E 14 0.30312E 14-0.10377E 14 0.10493E 13
9.88235 0.58375E 10-0.66433E 09-0.28999E 10-0.72826E 09-0.11876E 14 0.64187E 13-0.25888E 14 0.31234E 14-0.99033E 13 0.14398E 13

9.88235 0.58375E 10-0.66432E 09-0.28999E 10-0.72872E 09-0.11826E 14 0.63706E 13-0.25870E 14 0.31223E 14-0.99109E 13 0.14402E 13
10.11765 0.60083E 10-0.66644E 09-0.29862E 10-0.72243E 09-0.11734E 14 0.72841E 13-0.26755E 14 0.32087E 14-0.94219E 13 0.18262E 13
10.35294 0.61774E 10-0.66823E 09-0.30719E 10-0.71479E 09-0.11640E 14 0.80899E 13-0.27691E 14 0.32850E 14-0.89232E 13 0.21985E 13
10.58823 0.63446E 10-0.66973E 09-0.31571E 10-0.70676E 09-0.11431E 14 0.86794E 13-0.28660E 14 0.33462E 14-0.84336E 13 0.25550E 13

10.58823 0.63447E 10-0.66972E 09-0.31572E 10-0.70727E 09-0.11376E 14 0.86263E 13-0.28639E 14 0.33450E 14-0.84420E 13 0.25555E 13
10.82353 0.65104E 10-0.67093E 09-0.32419E 10-0.70175E 09-0.10857E 14 0.88072E 13-0.29553E 14 0.33887E 14-0.79965E 13 0.29126E 13
11.05882 0.66754E 10-0.67189E 09-0.33260E 10-0.70248E 09-0.98296E 13 0.83872E 13-0.30277E 14 0.34176E 14-0.76321E 13 0.32997E 13
11.29412 0.68420E 10-0.67264E 09-0.34097E 10-0.71570E 09-0.80809E 13 0.71583E 13-0.30560E 14 0.34466E 14-0.73922E 13 0.37852E 13

11.29412 0.68420E 10-0.67263E 09-0.34097E 10-0.71625E 09-0.80206E 13 0.71004E 13-0.30538E 14 0.34454E 14-0.74014E 13 0.37856E 13
11.52941 0.70141E 10-0.67328E 09-0.34928E 10-0.75042E 09-0.54557E 13 0.49670E 13-0.29997E 14 0.35121E 14-0.73147E 13 0.44925E 13
11.76471 0.71973E 10-0.67402E 09-0.35753E 10-0.81199E 09-0.24329E 13 0.22673E 13-0.28226E 14 0.36846E 14-0.73386E 13 0.55814E 13
12.00000 0.73985E 10-0.67519E 09-0.36567E 10-0.90173E 09-0.65247E 11 0.62567E 11-0.24913E 14 0.40675E 14-0.73056E 13 0.72381E 13
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STABILITY AND FREE VIBRATION OF SHELLS WITH AXISYMMETRIC PRESTRESS, BY A. KALNINS, LEHIGH UNIV, BETHLEHEM, PA
WRIGHT PATTERSON AIR FORCE BASE FLIGHT DYNAMICS LABORATORY VERSION, 22 JULY 1968

SOLUTION AT POINTS ALONG MERIDIAN AT EIGENVALUE PARAMETER OMEGA= 0.1810000E 03 FOR WAVE NUMBER NX= 2

MAIN SHELL PART NO 1
OMEGA= 0.18100E 03 OMSQ= 0.12934E 07 XMR= 0.10000E 01 LC= -0 PRESTRESS= 0. 0.

| S | W | Q | UPHI | NPHI | BPHI | MPHI | UTHETA | N | NTHETA | MTHETA |
|---|---|---|---|---|---|---|---|---|---|---|
| 0. | -0. | 0.32796E 12 | -0. | -0.64869E 13 | -0. | -0.11277E 12 | -0. | -0.91068E 12 | -0.19461E 13 | -0.33832E 11 |
| 0.23529 | 0.37973E 08 | 0.21052E 12 | -0.30146E 08 | -0.63638E 13 | -0.28542E 09 | -0.50388E 11 | -0.15604E 08 | -0.11433E 13 | -0.18308E 13 | -0.14654E 11 |
| 0.47059 | 0.12072E 09 | 0.11417E 12 | -0.60117E 08 | -0.62147E 13 | -0.39671E 09 | -0.14596E 11 | -0.37702E 08 | -0.13344E 13 | -0.13394E 13 | -0.28180E 10 |
| 0.70588 | 0.21764E 09 | 0.49562E 11 | -0.90085E 08 | -0.60464E 13 | -0.41756E 09 | 0.19261E 10 | -0.65526E 08 | -0.14620E 13 | -0.81070E 12 | 0.34108E 10 |
| 0.70586 | 0.21765E 09 | 0.49563E 11 | -0.90085E 08 | -0.60464E 13 | -0.41760E 09 | 0.19201E 10 | -0.65527E 08 | -0.14620E 13 | -0.81061E 12 | 0.34091E 10 |
| 0.94118 | 0.31457E 09 | 0.13504E 11 | -0.11989E 09 | -0.58663E 13 | -0.40355E 09 | 0.67639E 10 | -0.98234E 08 | -0.15320E 13 | -0.39163E 12 | 0.60967E 10 |
| 1.17647 | 0.40728E 09 | -0.21326E 10 | -0.14929E 09 | -0.56804E 13 | -0.38533E 09 | 0.58204E 10 | -0.13512E 09 | -0.15602E 13 | -0.11638E 12 | 0.69517E 10 |
| 1.41176 | 0.49664E 09 | -0.55381E 10 | -0.17806E 09 | -0.54927E 13 | -0.37626E 09 | 0.26824E 10 | -0.17567E 09 | -0.15631E 13 | 0.35622E 11 | 0.71290E 10 |
| 1.41176 | 0.49665E 09 | -0.55401E 10 | -0.17806E 09 | -0.54926E 13 | -0.37628E 09 | 0.28790E 10 | -0.17567E 09 | -0.15631E 13 | 0.35661E 11 | 0.71280E 10 |
| 1.64706 | 0.58532E 09 | -0.29803E 10 | -0.20601E 09 | -0.53053E 13 | -0.37937E 09 | -0.23161E 08 | -0.21956E 09 | -0.15532E 13 | 0.10230E 12 | 0.72798E 10 |
| 1.88235 | 0.67598E 09 | 0.15552E 10 | -0.23307E 09 | -0.51194E 13 | -0.39270E 09 | -0.20614E 10 | -0.26656E 09 | -0.15383E 13 | 0.11942E 12 | 0.76975E 10 |
| 2.11765 | 0.77065E 09 | 0.59590E 10 | -0.25918E 09 | -0.49352E 13 | -0.41279E 09 | -0.30981E 10 | -0.31653E 09 | -0.15228E 13 | 0.11356E 12 | 0.84542E 10 |
| 2.11765 | 0.77065E 09 | 0.59472E 10 | -0.25918E 09 | -0.49352E 13 | -0.41281E 09 | -0.31053E 10 | -0.31653E 09 | -0.15228E 13 | 0.11360E 12 | 0.84522E 10 |
| 2.35294 | 0.87052E 09 | 0.93861E 10 | -0.28432E 09 | -0.47527E 13 | -0.43641E 09 | -0.33201E 10 | -0.36936E 09 | -0.15082E 13 | 0.10121E 12 | 0.95132E 10 |
| 2.58824 | 0.97610E 09 | 0.11742E 11 | -0.30851E 09 | -0.45717E 13 | -0.46107E 09 | -0.29735E 10 | -0.42495E 09 | -0.14948E 13 | 0.90584E 11 | 0.10809E 11 |
| 2.82353 | 0.10875E 10 | 0.13216E 11 | -0.33176E 09 | -0.43922E 13 | -0.48529E 09 | -0.23019E 10 | -0.48321E 09 | -0.14820E 13 | 0.84726E 11 | 0.12271E 11 |
| 2.82353 | 0.10875E 10 | 0.13194E 11 | -0.33176E 09 | -0.43922E 13 | -0.48534E 09 | -0.23157E 10 | -0.48321E 09 | -0.14821E 13 | 0.84811E 11 | 0.12267E 11 |
| 3.05882 | 0.12044E 10 | 0.14079E 11 | -0.35408E 09 | -0.42140E 13 | -0.50842E 09 | -0.14939E 10 | -0.54402E 09 | -0.14694E 13 | 0.84034E 11 | 0.13839E 11 |
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| 3.52941 | 0.14537E 10 | 0.15006E 11 | -0.39600E 09 | -0.38621E 13 | -0.55014E 09 | 0.26347E 09 | -0.67285E 09 | -0.14423E 13 | 0.92857E 11 | 0.17200E 11 |
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| 4.23529 | 0.18614E 10 | 0.15956E 11 | -0.45223E 09 | -0.33469E 13 | -0.60292E 09 | 0.27912E 10 | -0.88232E 09 | -0.13931E 13 | 0.11664E 12 | 0.22600E 11 |
| 4.23529 | 0.18614E 10 | 0.15905E 11 | -0.45223E 09 | -0.33469E 13 | -0.60303E 09 | 0.27609E 10 | -0.88232E 09 | -0.13932E 13 | 0.11679E 12 | 0.22591E 11 |
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11.76471 0.71836E 10-0.18755E 11-0.67132E 09-0.93885E 10-0.78350E 09 0.78737E 10-0.35725E 10-0.90416E 11 0.44530E 12 0.85067E 11
12.00000 0.73774E 10-0.84692E 10-0.67228E 09-0.16759E 09-0.86921E 09 0.16570E 09-0.36534E 10 0.59227E 09 0.81720E 12 0.85104E 11

MAX OF 1ST PRESCRIBED VARIABLE AT FINAL EDGE IS 0.32796E 12 DET= 0.87665E 10   ACC= 0.10000E-01

THIS WAS ITERATION NO  1  ACCURACY NOT ATTAINED
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STABILITY AND FREE VIBRATION OF SHELLS WITH AXISYMMETRIC PRESTRESS, BY A. KALNINS, LEHIGH UNIV, BETHLEHEM, PA
WRIGHT PATTERSON AIR FORCE BASE FLIGHT DYNAMICS LABORATORY VERSION, 22 JULY 1968

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SOLUTION AT POINTS ALONG MERIDIAN AT EIGENVALUE PARAMETER OMEGA= 0.1810000E 03   FOR WAVE NUMBER NX=  2

   S        W          UPHI        UTHETA       BPHI        SPHI IN     SPHI OUT    STHETA IN   STHETA OUT   SFITH IN    SFITH OUT

 MAIN SHELL PART NO  1
OMEGA= 0.18100E 03  OMSQ= 0.12934E 07  XMR= 0.10000E 01  LC= -0  PRESTRESS=   0.            0.

 0.      -0.         -0.         -0.         -0.         -0.85901E 13-0.95200E 14-0.25770E 13-0.28560E 14-0.71692E 13-0.74004E 13
 0.23529 0.37973E 08-0.30146E 08-0.15604E 08-0.28542E 09-0.31561E 14-0.70260E 14-0.90190E 13-0.20273E 14-0.11523E 14-0.67943E 13
 0.47059 0.12072E 09-0.60117E 08-0.37702E 08-0.39671E 09-0.44113E 14-0.55323E 14-0.96332E 13-0.11797E 14-0.13991E 14-0.73953E 13
 0.70588 0.21764E 09-0.90085E 08-0.65526E 08-0.41756E 09-0.49111E 14-0.47632E 14-0.77954E 13-0.51759E 13-0.15148E 14-0.82810E 13

 0.70588 0.21765E 09-0.90085E 08-0.65527E 08-0.41760E 09-0.49109E 14-0.47634E 14-0.77940E 13-0.51757E 13-0.15148E 14-0.92806E 13
 0.94118 0.31457E 09-0.11989E 09-0.98234E 08-0.40355E 09-0.49528E 14-0.44333E 14-0.54742E 13-0.79189E 12-0.15539E 14-0.90069E 13
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 1.41176 0.49664E 09-0.17806E 09-0.17567E 09-0.37626E 09-0.45048E 14-0.42834E 14-0.24526E 13 0.30225E 13-0.15474E 14-0.95675E 13

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8.94118 0.51510E 10-0.65036E 09-0.25540E 10-0.73794E 09-0.12712E 14 0.25915E 13-0.22632E 14 0.27549E 14-0.11584E 14-0.87429E 11
9.17647 0.53244E 10-0.65388E 09-0.26414E 10-0.73617E 09-0.12374E 14 0.36450E 13-0.23416E 14 0.28507E 14-0.11166E 14 0.30390E 12

9.17647 0.53245E 10-0.65386E 09-0.26414E 10-0.73658E 09-0.12329E 14 0.36019E 13-0.23399E 14 0.28497E 14-0.11172E 14 0.30424E 12
```

| | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 9.41176 | 0.54975E | 10-0.65698E | 09-0.27285E | 10-0.73421E | 09-0.12045E | 14 | 0.46118E | 13-0.24180E | 14 | 0.29459E | 14-0.10735E | 14 | 0.70555E | 12 |
| 9.64706 | 0.56699E | 10-0.65970E | 09-0.28152E | 10-0.73064E | 09-0.11844E | 14 | 0.56029E | 13-0.24978E | 14 | 0.30416E | 14-0.10274E | 14 | 0.11107E | 13 |
| 9.88235 | 0.58412E | 10-0.66206E | 09-0.29015E | 10-0.72555E | 09-0.11728E | 14 | 0.65745E | 13-0.25811E | 14 | 0.31348E | 14-0.97884E | 13 | 0.15144E | 13 |
| | | | | | | | | | | | | | | |
| 9.88235 | 0.58413E | 10-0.66206E | 09-0.29015E | 10-0.72601E | 09-0.11678E | 14 | 0.65264E | 13-0.25793E | 14 | 0.31338E | 14-0.97961E | 13 | 0.15148E | 13 |
| 10.11765 | 0.60114E | 10-0.66409E | 09-0.29873E | 10-0.71969E | 09-0.11610E | 14 | 0.74397E | 13-0.26659E | 14 | 0.32222E | 14-0.92923E | 13 | 0.19157E | 13 |
| 10.35294 | 0.61799E | 10-0.66563E | 09-0.30726E | 10-0.71180E | 09-0.11567E | 14 | 0.82696E | 13-0.27580E | 14 | 0.33017E | 14-0.87742E | 13 | 0.23036E | 13 |
| 10.58823 | 0.63463E | 10-0.66726E | 09-0.31573E | 10-0.70297E | 09-0.11453E | 14 | 0.89220E | 13-0.28551E | 14 | 0.33669E | 14-0.82575E | 13 | 0.26737E | 13 |
| | | | | | | | | | | | | | | |
| 10.58823 | 0.63464E | 10-0.66727E | 09-0.31573E | 10-0.70347E | 09-0.11398E | 14 | 0.88690E | 13-0.28531E | 14 | 0.33657E | 14-0.82659E | 13 | 0.26741E | 13 |
| 10.82353 | 0.65110E | 10-0.66845E | 09-0.32415E | 10-0.69616E | 09-0.11030E | 14 | 0.91640E | 13-0.29481E | 14 | 0.34127E | 14-0.77832E | 13 | 0.30373E | 13 |
| 11.05882 | 0.66744E | 10-0.66938E | 09-0.33250E | 10-0.69357E | 09-0.10209E | 14 | 0.89089E | 13-0.30303E | 14 | 0.34413E | 14-0.73706E | 13 | 0.34141E | 13 |
| 11.29412 | 0.68382E | 10-0.67010E | 09-0.34081E | 10-0.70156E | 09-0.86896E | 13 | 0.78636E | 13-0.30773E | 14 | 0.34613E | 14-0.70759E | 13 | 0.38614E | 13 |
| | | | | | | | | | | | | | | |
| 11.29412 | 0.68383E | 10-0.67009E | 09-0.34081E | 10-0.70210E | 09-0.86294E | 13 | 0.78058E | 13-0.30751E | 14 | 0.34600E | 14-0.70851E | 13 | 0.38618E | 13 |
| 11.52941 | 0.70062E | 10-0.67069E | 09-0.34906E | 10-0.72927E | 09-0.62223E | 13 | 0.57853E | 13-0.30501E | 14 | 0.35013E | 14-0.69515E | 13 | 0.44904E | 13 |
| 11.76471 | 0.71836E | 10-0.67132E | 09-0.35725E | 10-0.78350E | 09-0.30986E | 13 | 0.29484E | 13-0.29103E | 14 | 0.36228E | 14-0.69672E | 13 | 0.54548E | 13 |
| 12.00000 | 0.73774E | 10-0.67228E | 09-0.36534E | 10-0.86921E | 09-0.64970E | 11 | 0.62289E | 11-0.26142E | 14 | 0.39218E | 14-0.70175E | 13 | 0.69531E | 13 |

STABILITY AND FREE VIBRATION OF SHELLS WITH AXISYMMETRIC PRESTRESS, BY A. KALNINS, LEHIGH UNIV, BETHLEHEM, PA
WRIGHT PATTERSON AIR FORCE BASE FLIGHT DYNAMICS LABORATORY VERSION, 22 JULY 1968

SOLUTION AT POINTS ALONG MERIDIAN AT EIGENVALUE PARAMETER OMEGA= 0.1796245E 03 FOR WAVE NUMBER NX= 2

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      S          W           Q          UPHI        NPHI        BPHI        MPHI       UTHETA         N          NTHETA       MTHETA

   MAIN SHELL PART NO  1
 OMEGA= 0.17962E 03  OMSQ= 0.12738E 07  XMR= 0.10000E 01  LC= -0  PRESTRESS=   0.                0.

  0.      -0.          0.32701E 12-0.          -0.64751E 13-0.          -0.11238E 12-0.          -0.90191E 12-0.19425E 13-0.33715E 11
  0.23529 0.37836E 08 0.20983E 12-0.30095E 08-0.63531E 13-0.28438E 09-0.50189E 11-0.15480E 08-0.11340E 13-0.18263E 13-0.14596E 11
  0.47059 0.12027E 09 0.11374E 12-0.60020E 08-0.62052E 13-0.39519E 09-0.14522E 11-0.37441E 08-0.13247E 13-0.13357E 13-0.28005E 10
  0.70588 0.21682E 09 0.49332E 11-0.89946E 08-0.60381E 13-0.41590E 09 0.19323E 10-0.65118E 08-0.14519E 13-0.80828E 12 0.34031E 10

  0.70588 0.21683E 09 0.49332E 11-0.89946E 08-0.60381E 13-0.41594E 09 0.19266E 10-0.65118E 08-0.14519E 13-0.80819E 12 0.34016E 10
  0.94118 0.31335E 09 0.13400E 11-0.11972E 09-0.58592E 13-0.40192E 09 0.67405E 10-0.97669E 08-0.15216E 13-0.39047E 12 0.60754E 10
  1.17647 0.40569E 09-0.21698E 10-0.14909E 09-0.56745E 13-0.38378E 09 0.57893E 10-0.13439E 09-0.15498E 13-0.11618E 12 0.69236E 10
  1.41176 0.49470E 09-0.55488E 10-0.17782E 09-0.54880E 13-0.37479E 09 0.28536E 10-0.17478E 09-0.15528E 13 0.35184E 11 0.70974E 10

  1.41176 0.49471E 09-0.55515E 10-0.17782E 09-0.54880E 13-0.37481E 09 0.28502E 10-0.17478E 09-0.15528E 13 0.35226E 11 0.70965E 10
  1.64706 0.58304E 09-0.29879E 10-0.20576E 09-0.53019E 13-0.37797E 09-0.46932E 08-0.21850E 09-0.15429E 13 0.10148E 12 0.72459E 10
  1.88235 0.67338E 09 0.15411E 10-0.23280E 09-0.51173E 13-0.39134E 09-0.20810E 10-0.26533E 09-0.15282E 13 0.11839E 12 0.76611E 10
  2.11765 0.76773E 09 0.59357E 10-0.25890E 09-0.49343E 13-0.41146E 09-0.31157E 10-0.31513E 09-0.15128E 13 0.11240E 12 0.84149E 10

  2.11765 0.76773E 09 0.59242E 10-0.25890E 09-0.49343E 13-0.41148E 09-0.31227E 10-0.31513E 09-0.15128E 13 0.11244E 12 0.84129E 10
  2.35294 0.86729E 09 0.93551E 10-0.28404E 09-0.47530E 13-0.43510E 09-0.33377E 10-0.36779E 09-0.14984E 13 0.99965E 11 0.94704E 10
  2.58824 0.97257E 09 0.11705E 11-0.30823E 09-0.45732E 13-0.45978E 09-0.29929E 10-0.42322E 09-0.14851E 13 0.89249E 11 0.10762E 11
  2.82353 0.10836E 10 0.13177E 11-0.33149E 09-0.43948E 13-0.48403E 09-0.23242E 10-0.48132E 09-0.14726E 13 0.83293E 11 0.12220E 11

  2.82353 0.10836E 10 0.13155E 11-0.33149E 09-0.43948E 13-0.48408E 09-0.23378E 10-0.48132E 09-0.14726E 13 0.83374E 11 0.12216E 11
  3.05882 0.12003E 10 0.14039E 11-0.35382E 09-0.42178E 13-0.50718E 09-0.15194E 10-0.54197E 09-0.14602E 13 0.82482E 11 0.13784E 11
  3.29412 0.13222E 10 0.14585E 11-0.37525E 09-0.40422E 13-0.52883E 09-0.64656E 09-0.60507E 09-0.14473E 13 0.85492E 11 0.15430E 11
  3.52941 0.14490E 10 0.14968E 11-0.39578E 09-0.38681E 13-0.54899E 09 0.23091E 09-0.67050E 09-0.14336E 13 0.91030E 11 0.17136E 11

  3.52941 0.14490E 10 0.14932E 11-0.39578E 09-0.38681E 13-0.54907E 09 0.20956E 09-0.67050E 09-0.14337E 13 0.91143E 11 0.17130E 11
  3.76471 0.15805E 10 0.15263E 11-0.41543E 09-0.36956E 13-0.56793E 09 0.10656E 10-0.73815E 09-0.14188E 13 0.98321E 11 0.18882E 11
  4.00000 0.17162E 10 0.15589E 11-0.43421E 09-0.35248E 13-0.58554E 09 0.19097E 10-0.80788E 09-0.14028E 13 0.10623E 12 0.20681E 11
  4.23529 0.18559E 10 0.15923E 11-0.45212E 09-0.33560E 13-0.60197E 09 0.27489E 10-0.87958E 09-0.13853E 13 0.11433E 12 0.22524E 11

  4.23529 0.18559E 10 0.15872E 11-0.45212E 09-0.33559E 13-0.60208E 09 0.27189E 10-0.87958E 09-0.13854E 13 0.11448E 12 0.22516E 11
  4.47059 0.19995E 10 0.16220E 11-0.46918E 09-0.31892E 13-0.61754E 09 0.35481E 10-0.95313E 09-0.13667E 13 0.12287E 12 0.24400E 11
  4.70588 0.21465E 10 0.16569E 11-0.48540E 09-0.30246E 13-0.63196E 09 0.43869E 10-0.10284E 10-0.13465E 13 0.13126E 12 0.26328E 11
  4.94118 0.22968E 10 0.16905E 11-0.50078E 09-0.28625E 13-0.64536E 09 0.52391E 10-0.11053E 10-0.13250E 13 0.13954E 12 0.28297E 11

  4.94118 0.22968E 10 0.16837E 11-0.50078E 09-0.28625E 13-0.64550E 09 0.51994E 10-0.11053E 10-0.13251E 13 0.13972E 12 0.28286E 11
  5.17647 0.24502E 10 0.17163E 11-0.51536E 09-0.27029E 13-0.65802E 09 0.60496E 10-0.11837E 10-0.13022E 13 0.14830E 12 0.30290E 11
  5.41176 0.26064E 10 0.17470E 11-0.52913E 09-0.25461E 13-0.66955E 09 0.69158E 10-0.12634E 10-0.12778E 13 0.15693E 12 0.32333E 11
  5.64706 0.27652E 10 0.17749E 11-0.54211E 09-0.23923E 13-0.68007E 09 0.77977E 10-0.13445E 10-0.12520E 13 0.16549E 12 0.34410E 11

  5.64706 0.27652E 10 0.17662E 11-0.54210E 09-0.23922E 13-0.68024E 09 0.77474E 10-0.13445E 10-0.12522E 13 0.16570E 12 0.34395E 11
  5.88235 0.29264E 10 0.17924E 11-0.55432E 09-0.22415E 13-0.68995E 09 0.86219E 10-0.14266E 10-0.12249E 13 0.17466E 12 0.36498E 11
  6.11765 0.30898E 10 0.18164E 11-0.56577E 09-0.20940E 13-0.69868E 09 0.95105E 10-0.15099E 10-0.11961E 13 0.18370E 12 0.38631E 11
  6.35294 0.32551E 10 0.18376E 11-0.57649E 09-0.19500E 13-0.70641E 09 0.10412E 11-0.15940E 10-0.11659E 13 0.19259E 12 0.40790E 11

  6.35294 0.32552E 10 0.18268E 11-0.57648E 09-0.19499E 13-0.70662E 09 0.10351E 11-0.15940E 10-0.11661E 13 0.19284E 12 0.40772E 11
  6.58823 0.34223E 10 0.18463E 11-0.58648E 09-0.18096E 13-0.71361E 09 0.11238E 11-0.16790E 10-0.11343E 13 0.20218E 12 0.42948E 11
  6.82353 0.35909E 10 0.18636E 11-0.59578E 09-0.16731E 13-0.71966E 09 0.12136E 11-0.17647E 10-0.11009E 13 0.21153E 12 0.45145E 11
  7.05882 0.37608E 10 0.18784E 11-0.60440E 09-0.15406E 13-0.72475E 09 0.13046E 11-0.18510E 10-0.10660E 13 0.22066E 12 0.47361E 11

  7.05882 0.37609E 10 0.18654E 11-0.60440E 09-0.15405E 13-0.72501E 09 0.12972E 11-0.18510E 10-0.10663E 13 0.22095E 12 0.47339E 11
  7.29412 0.39320E 10 0.18785E 11-0.61236E 09-0.14122E 13-0.72945E 09 0.13859E 11-0.19378E 10-0.10298E 13 0.23053E 12 0.49562E 11
  7.52941 0.41041E 10 0.18897E 11-0.61968E 09-0.12884E 13-0.73300E 09 0.14754E 11-0.20250E 10-0.99176E 12 0.24011E 12 0.51799E 11
  7.76471 0.42769E 10 0.18979E 11-0.62640E 09-0.11691E 13-0.73566E 09 0.15657E 11-0.21125E 10-0.95213E 12 0.24941E 12 0.54047E 11
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7.76471 0.42769E 10 0.18827E 11-0.62639E 09-0.11690E 13-0.73597E 09 0.15571E 11-0.21126E 10-0.95244E 12 0.24973E 12 0.54022E 11
8.00000 0.44504E 10 0.18901E 11-0.63252E 09-0.10545E 13-0.73809E 09 0.16443E 11-0.22003E 10-0.91119E 12 0.25963E 12 0.56269E 11
8.23529 0.46242E 10 0.18971E 11-0.63808E 09-0.94505E 12-0.73943E 09 0.17322E 11-0.22882E 10-0.86832E 12 0.26964E 12 0.58523E 11
8.47059 0.47983E 10 0.19037E 11-0.64311E 09-0.84076E 12-0.73994E 09 0.18213E 11-0.23762E 10-0.82381E 12 0.27953E 12 0.60783E 11

8.47059 0.47983E 10 0.18861E 11-0.64310E 09-0.84067E 12-0.74028E 09 0.18115E 11-0.23762E 10-0.82416E 12 0.27988E 12 0.60755E 11
8.70588 0.49725E 10 0.18960E 11-0.64762E 09-0.74173E 12-0.74036E 09 0.18979E 11-0.24641E 10-0.77794E 12 0.29066E 12 0.63010E 11
8.94118 0.51467E 10 0.19111E 11-0.65166E 09-0.64841E 12-0.73964E 09 0.19873E 11-0.25519E 10-0.72998E 12 0.30170E 12 0.65274E 11
9.17647 0.53205E 10 0.19316E 11-0.65525E 09-0.56091E 12-0.73799E 09 0.20815E 11-0.26395E 10-0.68026E 12 0.31243E 12 0.67549E 11

9.17647 0.53206E 10 0.19117E 11-0.65525E 09-0.56080E 12-0.73838E 09 0.20704E 11-0.26395E 10-0.68066E 12 0.31281E 12 0.67517E 11
9.41176 0.54941E 10 0.19392E 11-0.65842E 09-0.47928E 12-0.73608E 09 0.21663E 11-0.27269E 10-0.62916E 12 0.32369E 12 0.69793E 11
9.64706 0.56669E 10 0.19678E 11-0.66120E 09-0.40401E 12-0.73254E 09 0.22703E 11-0.28138E 10-0.57602E 12 0.33282E 12 0.72087E 11
9.88235 0.58387E 10 0.19812E 11-0.66363E 09-0.33516E 12-0.72745E 09 0.23819E 11-0.29004E 10-0.52156E 12 0.33786E 12 0.74391E 11

9.88235 0.58387E 10 0.19590E 11-0.66362E 09-0.33504E 12-0.72786E 09 0.23696E 11-0.29004E 10-0.52200E 12 0.33829E 12 0.74355E 11
10.11765 0.60093E 10 0.19304E 11-0.66571E 09-0.27269E 12-0.72158E 09 0.24774E 11-0.29865E 10-0.46677E 12 0.33772E 12 0.76632E 11
10.35294 0.61782E 10 0.18139E 11-0.66748E 09-0.21694E 12-0.71387E 09 0.25729E 11-0.30721E 10-0.41170E 12 0.32782E 12 0.78851E 11
10.58823 0.63452E 10 0.15459E 11-0.66897E 09-0.16770E 12-0.70561E 09 0.26287E 11-0.31572E 10-0.35808E 12 0.30631E 12 0.80925E 11

10.58823 0.63452E 10 0.15215E 11-0.66896E 09-0.16757E 12-0.70609E 09 0.26151E 11-0.31572E 10-0.35856E 12 0.30677E 12 0.80885E 11
10.82353 0.65106E 10 0.10410E 11-0.67016E 09-0.12453E 12-0.70001E 09 0.25817E 11-0.32417E 10-0.30759E 12 0.27688E 12 0.82671E 11
11.05882 0.66751E 10 0.31625E 10-0.67111E 09-0.87380E 11-0.69972E 09 0.24081E 11-0.33257E 10-0.25989E 12 0.24776E 12 0.84028E 11
11.29412 0.68408E 10-0.58939E 10-0.67185E 09-0.55780E 11-0.71134E 09 0.20368E 11-0.34092E 10-0.21416E 12 0.24290E 12 0.84814E 11

11.29412 0.68409E 10-0.61629E 10-0.67184E 09-0.55632E 11-0.71186E 09 0.20220E 11-0.34092E 10-0.21470E 12 0.24339E 12 0.84770E 11
11.52941 0.70116E 10-0.14365E 11-0.67247E 09-0.29537E 11-0.74385E 09 0.14211E 11-0.34921E 10-0.16552E 12 0.30828E 12 0.84949E 11
11.76471 0.71931E 10-0.15747E 11-0.67318E 09-0.10047E 11-0.80315E 09 0.66612E 10-0.35744E 10-0.10086E 12 0.50969E 12 0.84834E 11
12.00000 0.73919E 10 0.29057E 09-0.67429E 09-0.16121E 09-0.89166E 09 0.16005E 09-0.36557E 10 0.58223E 09 0.93299E 12 0.85308E 11
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MAX OF 1ST PRESCRIBED VARIABLE AT FINAL EDGE IS 0.32701E 12 DET= 0.14418E 07  ACC= 0.10000E-01

OMEGA= 0.17962E 03 IS AN EIGENVALUE

STABILITY AND FREE VIBRATION OF SHELLS WITH AXISYMMETRIC PRESTRESS, BY A. KALNINS, LEHIGH UNIV, BETHLEHEM, PA
WRIGHT PATTERSON AIR FORCE BASE FLIGHT DYNAMICS LABORATORY VERSION, 22 JULY 1968

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SOLUTION AT POINTS ALONG MERIDIAN AT EIGENVALUE PARAMETER OMEGA= 0.1796245E 03  FOR WAVE NUMBER NX=  2

   S          W          UPHI        UTHETA       BPHI        SPHI IN     SPHI OUT   STHETA IN   STHETA OUT   SFITH IN    SFITH OUT

 MAIN SHELL PART NO  1
OMEGA= 0.17962E 03  OMSQ= 0.12738E 07  XMR= 0.10000E 01  LC= -0  PRESTRESS=   0.              0.
 0.      -0.        -0.        -0.        -0.         -0.86457E 13-0.94955E 14-0.25937E 13-0.28487E 14-0.71002E 13-0.73292E 13
 0.23529 0.37836E 08-0.30095E 08-0.15480E 08-0.28438E 09-0.31552E 14-0.70098E 14-0.90055E 13-0.20215E 14-0.11441E 14-0.67281E 13
 0.47059 0.12027E 09-0.60020E 08-0.37441E 08-0.39519E 09-0.44065E 14-0.55218E 14-0.96100E 13-0.11761E 14-0.13900E 14-0.73292E 13
 0.70588 0.21682E 09-0.89946E 08-0.65118E 08-0.41590E 09-0.49047E 14-0.47563E 14-0.77730E 13-0.51594E 13-0.15053E 14-0.82131E 13

 0.70588 0.21683E 09-0.89946E 08-0.65118E 08-0.41594E 09-0.49044E 14-0.47565E 14-0.77717E 13-0.51593E 13-0.15054E 14-0.82128E 13
 0.94118 0.31335E 09-0.11972E 09-0.97669E 08-0.40192E 09-0.49462E 14-0.44285E 14-0.54567E 13-0.79078E 12-0.15443E 14-0.89372E 13
 1.17647 0.40569E 09-0.14909E 09-0.13439E 09-0.38378E 09-0.47619E 14-0.43173E 14-0.35881E 13 0.17292E 13-0.15469E 14-0.93601E 13
 1.41176 0.49470E 09-0.17782E 09-0.17478E 09-0.37479E 09-0.45000E 14-0.42809E 14-0.24439E 13 0.30069E 13-0.15379E 14-0.94960E 13

 1.41176 0.49471E 09-0.17782E 09-0.17478E 09-0.37481E 09-0.44999E 14-0.42810E 14-0.24432E 13 0.30069E 13-0.15379E 14-0.94959E 13
 1.64706 0.58304E 09-0.20576E 09-0.21850E 09-0.37797E 09-0.42397E 14-0.42434E 14-0.19706E 13 0.35943E 13-0.15298E 14-0.94191E 13
 1.88235 0.67338E 09-0.23280E 09-0.26533E 09-0.39134E 09-0.40139E 14-0.41737E 14-0.19947E 13 0.38890E 13-0.15271E 14-0.92113E 13
 2.11765 0.76773E 09-0.25890E 09-0.31513E 09-0.41146E 09-0.38278E 14-0.40671E 14-0.23321E 13 0.41306E 13-0.15301E 14-0.89373E 13

 2.11765 0.76773E 09-0.25890E 09-0.31513E 09-0.41148E 09-0.38275E 14-0.40673E 14-0.23310E 13 0.41301E 13-0.15301E 14-0.89372E 13
 2.35294 0.86729E 09-0.28404E 09-0.36779E 09-0.43510E 09-0.36742E 14-0.39305E 14-0.28369E 13 0.44364E 13-0.15371E 14-0.86390E 13
 2.58824 0.97257E 09-0.30823E 09-0.42322E 09-0.45978E 09-0.35436E 14-0.37735E 14-0.34188E 13 0.48468E 13-0.15460E 14-0.83398E 13
 2.82353 0.10836E 10-0.33149E 09-0.48132E 09-0.48403E 09-0.34266E 14-0.36051E 14-0.40260E 13 0.53587E 13-0.15552E 14-0.80493E 13

 2.82353 0.10836E 10-0.33149E 09-0.48132E 09-0.48408E 09-0.34261E 14-0.36056E 14-0.40238E 13 0.53578E 13-0.15553E 14-0.80492E 13
 3.05882 0.12003E 10-0.35382E 09-0.54197E 09-0.50718E 09-0.33159E 14-0.34326E 14-0.46332E 13 0.59529E 13-0.15637E 14-0.77685E 13
 3.29412 0.13222E 10-0.37525E 09-0.60507E 09-0.52883E 09-0.32089E 14-0.32586E 14-0.52414E 13 0.66092E 13-0.15705E 14-0.74961E 13
 3.52941 0.14490E 10-0.39578E 09-0.67050E 09-0.54899E 09-0.31033E 14-0.30856E 14-0.58519E 13 0.73084E 13-0.15754E 14-0.72290E 13

 3.52941 0.14490E 10-0.39578E 09-0.67050E 09-0.54907E 09-0.31025E 14-0.30864E 14-0.58486E 13 0.73069E 13-0.15755E 14-0.72289E 13
 3.76471 0.15805E 10-0.41543E 09-0.73815E 09-0.56793E 09-0.29974E 14-0.29155E 14-0.64642E 13 0.80373E 13-0.15785E 14-0.69632E 13
 4.00000 0.17162E 10-0.43421E 09-0.80788E 09-0.58554E 09-0.28932E 14-0.27465E 14-0.70916E 13 0.87913E 13-0.15795E 14-0.66975E 13
 4.23529 0.18559E 10-0.45212E 09-0.87958E 09-0.60197E 09-0.27903E 14-0.25792E 14-0.77347E 13 0.95641E 13-0.15785E 14-0.64305E 13
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4.23529 0.18559E 10-0.45212E 09-0.87958E 09-0.60206E 09-0.27892E 14-0.25603E 14-0.77302E 13 0.95619E 13-0.15786E 14-0.64304E 13
4.47059 0.19995E 10-0.46916E 09-0.95313E 09-0.61754E 09-0.26876E 14-0.24151E 14-0.83066E 13 0.10353E 14-0.15758E 14-0.61600E 13
4.70588 0.21465E 10-0.48540E 09-0.10284E 10-0.63196E 09-0.25882E 14-0.22512E 14-0.90597E 13 0.11160E 14-0.15710E 14-0.58866E 13
4.94116 0.22968E 10-0.50078E 09-0.11053E 10-0.64536E 09-0.24912E 14-0.20888E 14-0.97498E 13 0.11982E 14-0.15643E 14-0.56103E 13

4.94118 0.22968E 10-0.50078E 09-0.11053E 10-0.64550E 09-0.24896E 14-0.20903E 14-0.97439E 13 0.11979E 14-0.15645E 14-0.56101E 13
5.17647 0.24502E 10-0.51536E 09-0.11837E 10-0.65802E 09-0.23946E 14-0.19300F 14-0.10445E 14 0.12808E 14-0.15560E 14-0.53290E 13
5.41176 0.26064E 10-0.52913E 09-0.12634E 10-0.66955E 09-0.23025E 14-0.17713E 14-0.11160E 14 0.13671E 14-0.15456E 14-0.50443E 13
5.64706 0.27652E 10-0.54211E 09-0.13445E 10-0.68007E 09-0.22133E 14-0.16144E 14-0.11889E 14 0.14537E 14-0.15332E 14-0.47960E 13

5.64706 0.27652E 10-0.54210E 09-0.13445E 10-0.68024E 09-0.22113E 14-0.16163E 14-0.11882E 14 0.14533E 14-0.15335E 14-0.47558E 13
5.88235 0.29264E 10-0.55432E 09-0.14266E 10-0.68995E 09-0.21243E 14-0.14621E 14-0.12618E 14 0.15413E 14-0.15193E 14-0.44618E 13
6.11765 0.30896E 10-0.56577E 09-0.15099E 10-0.69868E 09-0.20404E 14-0.13100E 14-0.13365E 14 0.16304E 14-0.15032E 14-0.41636E 13
6.35294 0.32551E 10-0.57649E 09-0.15940E 10-0.70641F 09-0.19598E 14-0.11602E 14-0.14123E 14 0.17204E 14-0.14851E 14-0.38611E 13

6.35294 0.32552E 10-0.57648E 09-0.15940E 10-0.70662E 09-0.19574E 14-0.11625E 14-0.14114E 14 0.17199E 14-0.14854E 14-0.38609E 13
6.58823 0.34223E 10-0.56648E 09-0.16790E 10-0.71361E 09-0.18792E 14-0.10161E 14-0.14874E 14 0.18109E 14-0.14655E 14-0.35518E 13
6.82353 0.35909E 10-0.59578E 09-0.17647E 10-0.71966E 09-0.18045E 14-0.87242E 13-0.15643E 14 0.19028E 14-0.14436E 14-0.32376E 13
7.05882 0.37608E 10-0.60440E 09-0.18510E 10-0.72475E 09-0.17334E 14-0.73151E 13-0.16421E 14 0.19952E 14-0.14197E 14-0.29187E 13

7.05882 0.37609E 10-0.60440E 09-0.18510E 10-0.72501F 09-0.17306E 14-0.73428E 13-0.16411E 14 0.19946E 14-0.14202E 14-0.29185E 13
7.29412 0.39320E 10-0.61236E 09-0.19378E 10-0.72945E 09-0.16620E 14-0.59761E 13-0.17188E 14 0.20876E 14-0.13945E 14-0.25917E 13
7.52941 0.41041E 10-0.61968E 09-0.20250E 10-0.73300E 09-0.15972E 14-0.46415E 13-0.17970E 14 0.21812E 14-0.13669E 14-0.22592E 13
7.76471 0.42769E 10-0.62640E 09-0.21125E 10-0.73566E 09-0.15365E 14-0.33406E 13-0.18759E 14 0.22749E 14-0.13374E 14-0.19212E 13

7.76471 0.42769E 10-0.62639E 09-0.21126E 10-0.73597E 09-0.15332E 14-0.33729E 13-0.18747E 14 0.22742E 14-0.13379E 14-0.19209E 13
8.00000 0.44504E 10-0.63252E 09-0.22003E 10-0.73809E 09-0.14750E 14-0.21223E 13-0.19530E 14 0.23684E 14-0.13066E 14-0.15738E 13
8.23529 0.46242E 10-0.63808E 09-0.22882E 10-0.73943E 09-0.14212E 14-0.90870E 12-0.20316E 14 0.24630E 14-0.12734E 14-0.12199E 13
8.47059 0.47983E 10-0.64311E 09-0.23762E 10-0.73994E 09-0.13720E 14 0.26767E 12-0.21105E 14 0.25577E 14-0.12383E 14-0.85930E 12

8.47059 0.47983E 10-0.64310E 09-0.23762E 10-0.74028E 09-0.13681E 14 0.23084E 12-0.21091E 14 0.25569E 14-0.12388E 14-0.85909E 12
8.70588 0.49725E 10-0.64762E 09-0.24641E 10-0.74036E 09-0.13222E 14 0.13541E 13-0.21870E 14 0.26521E 14-0.12020E 14-0.48791E 12
8.94118 0.51467E 10-0.65166E 09-0.25519E 10-0.73964E 09-0.12819E 14 0.24441E 13-0.22651E 14 0.27479E 14-0.11632E 14-0.10902E 12
9.17647 0.53205E 10-0.65525E 09-0.26395E 10-0.73799E 09-0.12480E 14 0.35057E 13-0.23439E 14 0.28438E 14-0.11221E 14 0.27632E 12

9.17647 0.53206E 10-0.65525E 09-0.26395E 10-0.73838E 09-0.12437E 14 0.34641E 13-0.23424E 14 0.28429E 14-0.11228E 14 0.27654E 12
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 9.41176 0.54941E 10-0.65842E 09-0.27269E 10-0.73608E 09-0.12153E 14 0.44943E 13-0.24211E 14 0.29390E 14-0.10798E 14 0.67100E 12
 9.64706 0.56669E 10-0.66120E 09-0.28138E 10-0.73254E 09-0.11950E 14 0.54859E 13-0.25019E 14 0.30344E 14-0.10345E 14 0.10683E 13
 9.88235 0.58387E 10-0.66363E 09-0.29004E 10-0.72745E 09-0.11828E 14 0.64652E 13-0.25863E 14 0.31269E 14-0.98680E 13 0.14630E 13

 9.88235 0.58387E 10-0.66362E 09-0.29004E 10-0.72788E 09-0.11779E 14 0.64189E 13-0.25846E 14 0.31259E 14-0.98753E 13 0.14633E 13
10.11765 0.60093E 10-0.66571E 09-0.29865E 10-0.72158E 09-0.11695E 14 0.73318E 13-0.26725E 14 0.32128E 14-0.93818E 13 0.18940E 13
10.35294 0.61782E 10-0.66748E 09-0.30721E 10-0.71387E 09-0.11616E 14 0.81444E 13-0.27656E 14 0.32901E 14-0.88772E 13 0.22311E 13
10.58823 0.63452E 10-0.66897E 09-0.31572E 10-0.70561E 09-0.11436E 14 0.87526E 13-0.28625E 14 0.33526E 14-0.83793E 13 0.25920E 13

10.58823 0.63452E 10-0.66896E 09-0.31572E 10-0.70609E 09-0.11383E 14 0.87016E 13-0.28606E 14 0.33514E 14-0.83874E 13 0.25923E 13
10.82353 0.65106E 10-0.67016E 09-0.32417E 10-0.70001E 09-0.10910E 14 0.89174E 13-0.29531E 14 0.33961E 14-0.79303E 13 0.29513E 13
11.05882 0.66751E 10-0.67111E 09-0.33257E 10-0.69972E 09-0.99461E 13 0.85480E 13-0.30285E 14 0.34249E 14-0.75511E 13 0.33353E 13
11.29412 0.68408E 10-0.67185E 09-0.34092E 10-0.71134E 09-0.82677E 13 0.73752E 13-0.30625E 14 0.34512E 14-0.72943E 13 0.38090E 13

11.29412 0.68409E 10-0.67184E 09-0.34092E 10-0.71186E 09-0.82096E 13 0.73195E 13-0.30605E 14 0.34499E 14-0.73032E 13 0.38093E 13
11.52941 0.70116E 10-0.67247E 09-0.34921E 10-0.74385E 09-0.56932E 13 0.52206E 13-0.30154E 14 0.35087E 14-0.72019E 13 0.44918E 13
11.76471 0.71931E 10-0.67318E 09-0.35744E 10-0.80315E 09-0.26383E 13 0.24775E 13-0.28499E 14 0.36654E 14-0.72234E 13 0.55421E 13
12.00000 0.73919E 10-0.67429E 09-0.36557E 10-0.89166E 09-0.62746E 11 0.60169E 11-0.25294E 14 0.40222E 14-0.72164E 13 0. 1498E 13
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STABILITY AND FREE VIBRATION OF SHELLS WITH AXISYMMETRIC PRESTRESS, BY A. KALNINS, LEHIGH UNIV, BETHLEHEM, PA
WRIGHT PATTERSON AIR FORCE BASE FLIGHT DYNAMICS LABORATORY VERSION, 22 JULY 1968

SUMMARY OF RESULTS FOR N= 2

| | OMEGA | ACTUAL DET | ADJUSTED DET | NORMAL DET | XMC | XME |
|---|---|---|---|---|---|---|
| | 0.1750000E 03 | 0.2891946E 11 | 0.2891946E 11 | 0.8928480E-01 | 1.0 | 1.0 |
| | 0.1770000E 03 | 0.1651560E 11 | 0.1651560E 11 | 0.5078134E-01 | 1.0 | 1.0 |
| | 0.1790000E 03 | 0.3955098E 10 | 0.3955098E 10 | 0.1211054E-01 | 1.0 | 1.0 |
| | 0.1810000E 03 | -0.8766489E 10 | -0.8766489E 10 | -0.2673032E-01 | 1.0 | 1.0 |
| EIGENVALUE AT | 0.1796245E 03 | -0.1441792E 07 | -0.1441792E 07 | -0.4408996E-05 | 1.0 | 1.0 |

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STABILITY AND FREE VIBRATION OF SHELLS WITH AXISYMMETRIC PRESTRESS, BY A. KALNINS, LEHIGH UNIV, BETHLEHEM, PA
         WRIGHT PATTERSON AIR FORCE BASE FLIGHT DYNAMICS LABORATORY VERSION, 22 JULY 1968

          STABILITY ANALYSIS  PARTS=  2  BRANCHES=  0  NUMBER OF SUBCASES=  1

ANGLES OF ROTATION OF BOUNDARY CONDITIONS ARE ALFL=-0.          ALFRS=-0.

          PART NO  1

SI= 0.34091E-01   SX= 0.12616E 01   IPAR= 10   ING=  2   SHELL TYPE 5   NTP= 0   LAYERS MLY= 1

ELLIPSOIDAL SHELL NO  5     H= 0.63000E-01    A= 0.10000E 01    B= 0.30000E 01  DIRECTN= 1.

LAYER NO  1  FROM Z=-0.31500E-01  TO Z= 0.31500E-01
     CONSISTS OF ISOTROPIC MATERIAL, YOUNGS MODULUS E= 0.10000E 01  POISSONS RATIO NU= 0.40000E-00
     COEFFICIENTS OF THERMAL EXPANSION  AFI=-0.          ATHETA=-0.          MASS DENSITY RHO=-0.

PRESTRESS PART 1  POINTS= 26  LC= 1        S              NPHI            NTHETA

                                     0.34091000E-01 -0.17330917E-00 -0.69323669E-01
                                     0.83191359E-01 -0.12340079E-00 -0.11218233E-00
                                     0.13229170E-00 -0.11780769E-00 -0.12356821E-00
                                     0.18139207E-00 -0.11841565E-00 -0.13461012E-00
                                     0.23049243E-00 -0.12148704E-00 -0.14675558E-00
                                     0.27959279E-00 -0.12598061E-00 -0.15989803E-00
                                     0.32869314E-00 -0.13145041E-00 -0.17381085E-00
                                     0.37779351E-00 -0.13765848E-00 -0.18831594E-00
                                     0.42689385E-00 -0.14446341E-00 -0.20330206E-00
                                     0.47599421E-00 -0.15177891E-00 -0.21872125E-00
                                     0.52509458E 00 -0.15955573E-00 -0.23458390E-00
                                     0.57419492E 00 -0.16777322E-00 -0.25095647E-00
                                     0.62329528E 00 -0.17643540E-00 -0.26796138E-00
                                     0.67239564E 00 -0.18556996E-00 -0.28577881E-00
                                     0.72149599E 00 -0.19522838E-00 -0.30464903E-00
                                     0.77059635E 00 -0.20548749E-00 -0.32487351E-00
                                     0.81969669E 00 -0.21645091E-00 -0.34681325E-00
                                     0.86879706E 00 -0.22825070E-00 -0.37088130E-00
                                     0.91789740E 00 -0.24104732E-00 -0.39752666E-00
                                     0.96699777E 00 -0.25502740E-00 -0.42720736E-00
                                     0.10160980E 01 -0.27039684E-00 -0.46035320E-00
                                     0.10651984E 01 -0.28736734E-00 -0.49732129E-00
                                     0.11142998E 01 -0.30613334E-00 -0.53835284E 00
                                     0.11633991E 01 -0.32683595E-00 -0.58353489E 00
                                     0.12124995E 01 -0.34950922E-00 -0.63274185E 00
                                     0.12616000E 01 -0.37398630E-00 -0.68549491E 00

          PART NO  2

SI= 0.12616E 01   SX= 0.15706E 01   IPAR= 10   ING=  2   SHELL TYPE 5   NTP= 0   LAYERS MLY= 1

ELLIPSOIDAL SHELL NO  5     H= 0.61000E-01    A= 0.10000E 01    B= 0.30000E 01  DIRECTN= 1.

LAYER NO  1  FROM Z=-0.31500E-01  TO Z= 0.31500E-01
     CONSISTS OF ISOTROPIC MATERIAL, YOUNGS MODULUS E= 0.10000E 01  POISSONS RATIO NU= 0.40000E-00
     COEFFICIENTS OF THERMAL EXPANSION  AFI=-0.          ATHETA=-0.          MASS DENSITY RHO=-0.

PRESTRESS PART 2  POINTS= 26  LC= 1        S              NPHI            NTHETA

                                     0.12616000E 01 -0.37398630E-00 -0.68549491E 00
                                     0.12739679E 01 -0.38039020E-00 -0.69922971E 00
                                     0.12863359E 01 -0.38686968E-00 -0.71310013E 00
                                     0.12987039E 01 -0.39341102E-00 -0.72707722E 00
                                     0.13110719E 01 -0.39999837E-00 -0.74112779E 00
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| | | |
|---|---|---|
| 0.13234394E 01 | -0.40661347E-00 | -0.75521371E 00 |
| 0.13358079E 01 | -0.41323543E-00 | -0.76429165E 00 |
| 0.13481759E 01 | -0.41984094E-00 | -0.78331295E 00 |
| 0.13605434E 01 | -0.42640371E-00 | -0.79722317E 00 |
| 0.13729119E 01 | -0.43269474E-00 | -0.81096204E 00 |
| 0.13852799E 01 | -0.43928245E-00 | -0.82446393E 00 |
| 0.13976479E 01 | -0.44553234E-00 | -0.83765753E 00 |
| 0.14100159E 01 | -0.45160764E-00 | -0.85046694E 00 |
| 0.14223839E 01 | -0.45746931E-00 | -0.86261182E 00 |
| 0.14347518E 01 | -0.46307663E-00 | -0.87460845E 00 |
| 0.14471199E 01 | -0.46838756E-00 | -0.88577066E 00 |
| 0.14594878E 01 | -0.47335947E-00 | -0.89621089E 00 |
| 0.14718559E 01 | -0.47794978E-00 | -0.90584194E 00 |
| 0.14842238E 01 | -0.48211694E-00 | -0.91457823E 00 |
| 0.14965918E 01 | -0.48582111E-00 | -0.92233810E 00 |
| 0.15089598E 01 | -0.48902526E-00 | -0.92904618E 00 |
| 0.15213278E 01 | -0.49169610E-00 | -0.93463475E 00 |
| 0.15336958E 01 | -0.49380495E-00 | -0.93904556E 00 |
| 0.15460638E 01 | -0.49532846E-00 | -0.94223177E 00 |
| 0.15584318E 01 | -0.49624953E-00 | -0.94415797E 00 |
| 0.15707999E 01 | -0.49656000E-00 | -0.94480000E 00 |

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STABILITY AND FREE VIBRATION OF SHELLS WITH AXISYMMETRIC PRESTRESS, BY A. KALNINS, LEHIGH UNIV, BETHLEHEM, PA
WRIGHT PATTERSON AIR FORCE BASE FLIGHT DYNAMICS LABORATORY VERSION, 22 JULY 1968

SUBCASE NO  1  WITH WAVE NUMBER  3

STARTING OMEGA= 0.39000E-03  INCREMENT= 0.20000E-04  FINAL OMEGA= 0.49000E-03    1 EIGENVALUES

BOUNDARY CONDITIONS AT STARTING EDGE       1-0.          3-0.          5-0.          7-0.

BOUNDARY CONDITIONS AT FINAL    EDGE       2-0.          3-0.          5-0.          8-0.
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              STABILITY AND FREE VIBRATION OF SHELLS WITH AXISYMMETRIC PRESTRESS, BY A. KALNINS, LEHIGH UNIV, BETHLEHEM, PA
                   WRIGHT PATTERSON AIR FORCE BASE FLIGHT DYNAMICS LABORATORY VERSION, 22 JULY 1968

SOLUTION AT POINTS ALONG MERIDIAN AT EIGENVALUE PARAMETER OMEGA= 0.3900000E-03  FOR WAVE NUMBER NX=  1

    S          W          Q          UPHI       NPHI       BPHI       MPHI       UTHETA     N          NTHETA     MTHETA

  MAIN SHELL PART NO  1
OMEGA= 0.39000E-03  OPSG= 0.          XMR= 0.39000E-03  LC=  1  PRESTRESS=  -0.1733092E-00  -0.6932367E-01

  0.03409-0.          0.12240E 01-0.          0.19225E-01-0.          -0.46373E-02-0.          -0.56242E-02 0.76899E-02-0.18549E-02
  0.09547 0.21431E-01 0.21203E-00 0.52084E-02 0.16880E-01-0.21132E 01-0.20034E-02-0.62827E-02-0.10594E-01-0.16196E-01 0.17368E-02
  0.15684 0.10069E-00 0.19248E-00 0.12185E-01 0.27536E-01-0.57151E 01-0.31827E-02-0.20385E-01-0.18670E-01-0.28951E-01 0.32348E-02

  0.15684 0.10058E-00 0.19236E-00 0.12178E-01 0.27501E-01-0.57089E 01-0.31762E-02-0.20356E-01-0.18646E-01-0.28888E-01 0.32328E-02
  0.21822 0.27997E-00 0.19528E-00 0.17373E-01 0.39210E-01-0.11234E 02-0.43817E-02-0.48878E-01-0.27974E-01-0.43340E-01 0.46878E-02
  0.27959 0.61246E 00 0.20228E-00 0.14341E-01 0.52083E-01-0.18885E 02-0.55648E-02-0.10209E-00-0.39396E-01-0.60826E-01 0.62637E-02

  0.27959 0.61239E 00 0.20229E-00 0.14346E-01 0.52083E-01-0.18885E 02-0.55644E-02-0.10210E-00-0.39409E-01-0.60840E-01 0.62637E-02
  0.34097 0.11662E 01 0.21176E-00-0.64095E-02 0.66646E-01-0.28951E 02-0.67079E-02-0.19583E-00-0.53799E-01-0.82531E-01 0.80320E-02
  0.40234 0.20333E 01 0.22350E-00-0.58315E-01 0.83672E-01-0.41832E 02-0.77934E-02-0.35390E-00-0.72236E-01-0.10971E-00 0.10064E-01

  0.40234 0.20333E 01 0.22349E-00-0.58312E-01 0.83673E-01-0.41833E 02-0.77938E-02-0.35390E-00-0.72241E-01-0.10971E-00 0.10064E-01
  0.46372 0.33415E 01 0.23757E-00-0.16006E-00 0.10427E-00-0.58072E 02-0.87963E-02-0.61196E 00-0.96122E-01-0.14391E-00 0.12645E-01
  0.52509 0.52730E 01 0.25417E-00-0.33761E-00 0.13006E-00-0.78391E 02-0.96799E-02-0.10235E 01-0.12729E-00-0.18693E-00 0.15287E-01

  0.52509 0.52730E 01 0.25419E-00-0.33756E-00 0.13008E-00-0.78377E 02-0.96688E-02-0.10253E 01-0.12816E-00-0.18870E-00 0.15291E-01
  0.58647 0.80918E 01 0.27339E-00-0.62645E 00 0.16368E-00-0.10373E 03-0.10379E-01-0.16729E 01-0.16986E-00-0.24405E-00 0.18738E-01
  0.64784 0.12190E 02 0.29552E-00-0.10771E 01 0.20865E-00-0.13540E 03-0.10846E-01-0.26800E 01-0.22516E-00-0.31381E-00 0.22984E-01

  0.64784 0.12190E 02 0.29556E-00-0.10770E 01 0.20871E-00-0.13540E 03-0.10846E-01-0.26801E 01-0.22512E-00-0.31380E-00 0.22984E-01
  0.70922 0.18162E 02 0.32089E-00-0.17602E 01 0.27061E-00-0.17514E 03-0.10962E-01-0.42390E 01-0.29892E-00-0.40138E-00 0.28295E-01
  0.77059 0.26922E 02 0.34974E-00-0.27766E 01 0.35809E-00-0.22527E 03-0.10570E-01-0.66528E 01-0.39802E-00-0.51107E 00 0.35039E-01

  0.77059 0.26922E 02 0.34973E-00-0.27766E 01 0.35809E-00-0.22527E 03-0.10571E-01-0.66528E 01-0.39803E-00-0.51107E 00 0.35039E-01
  0.83197 0.39906E 02 0.38244E-00-0.42699E 01 0.48469E-00-0.28891E 03-0.94399E-02-0.10405E 02-0.53198E 00-0.64773E 00 0.43730E-01
  0.89334 0.59415E 02 0.41930E-00-0.64469E 01 0.67190E 00-0.37030E 03-0.72228E-02-0.16283E 02-0.71418E 00-0.81668E 00 0.55104E-01

  0.89334 0.59415E 02 0.41930E-00-0.64468E 01 0.67191E 00-0.37030E 03-0.72232E-02-0.16283E 02-0.71419E 00-0.81669E 00 0.55104E-01
  0.95472 0.89197E 02 0.46055E-00-0.96037E 01 0.95445E 00-0.47502E 03-0.33998E-02-0.25582E 02-0.96330E 00-0.10230E 01 0.70213E-01
  1.01609 0.13546E 03 0.50618E 00-0.14160E 02 0.13889E 01-0.61030E 03 0.28131E-02-0.40473E 02-0.13048E 01-0.12700E 01 0.90581E-01

  1.01609 0.13546E 03 0.50618E 00-0.14160E 02 0.13889E 01-0.61030E 03 0.28128E-02-0.40473E 02-0.13048E 01-0.12700E 01 0.90581E-01
  1.07747 0.20858E 03 0.55569E 00-0.20692E 02 0.20678E 01-0.78488E 03 0.12590E-01-0.64611E 02-0.17722E 01-0.15556E 01 0.11841E-00
  1.13884 0.32588E 03 0.60760E 00-0.29924E 02 0.31421E 01-0.10079E 04 0.27677E-01-0.10416E 03-0.24054E 01-0.18661E 01 0.15685E-00

  1.13884 0.32588E 03 0.60748E 00-0.29923E 02 0.31423E 01-0.10079E 04 0.27661E-01-0.10416E 03-0.24058E 01-0.18664E 01 0.15684E-00
  1.20021 0.51576E 03 0.65817E 00-0.42595E 02 0.48522E 01-0.12849E 04 0.50511E-01-0.16925E 03-0.32421E 01-0.21643E 01 0.21018E-00
  1.26159 0.82197E 03 0.70048E 00-0.58971E 02 0.75566E 01-0.16081E 04 0.84341E-01-0.27561E 03-0.42875E 01-0.23691E 01 0.28305E-00

  MAIN SHELL PART NO  2
OMEGA= 0.39000E-03  OPSG= 0.          XMR= 0.39000E-03  LC=  1  PRESTRESS=  -0.3739811E-00  -0.6854838E 00

  1.26159 0.82198E 03 0.70138E 00-0.56979E 02 0.75553E 01-0.16079E 04 0.84454E-01-0.27560E 03-0.42848E 01-0.23656E 01 0.28391E-00
  1.27705 0.92419E 03 0.70967E 00-0.63586E 02 0.84467E 01-0.16926E 04 0.95187E-01-0.31132E 03-0.45729E 01-0.23871E 01 0.30637E-00
  1.29251 0.10385E 04 0.71639E 00-0.68303E 02 0.94386E 01-0.17766E 04 0.10691E-00-0.35136E 03-0.48663E 01-0.23913E 01 0.33056E-00

  1.29251 0.10385E 04 0.71637E 00-0.68303E 02 0.94386E 01-0.17766E 04 0.10691E-00-0.35136E 03-0.48664E 01-0.23914E 01 0.33055E-00
  1.30797 0.11658E 04 0.72121E 00-0.73060E 02 0.10539E 02-0.18585E 04 0.11967E-00-0.39607E 03-0.51611E 01-0.23756E 01 0.35652E-00
  1.32343 0.13071E 04 0.72379E 00-0.77762E 02 0.11753E 02-0.19362E 04 0.13350E-00-0.44578E 03-0.54519E 01-0.23369E 01 0.38431E-00

  1.32343 0.13071E 04 0.72376E 00-0.77762E 02 0.11753E 02-0.19362E 04 0.13350E-00-0.44578E 03-0.54520E 01-0.23369E 01 0.38431E-00
  1.33889 0.14630E 04 0.72370E 00-0.82292E 02 0.13089E 02-0.20077E 04 0.14841E-00-0.50072E 03-0.57328E 01-0.22724E 01 0.41390E-00
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1.35435 0.16339E 04 0.72059E 00-0.86505E 02 0.14548E 02-0.20703E 04 0.16440E-00-0.56108E 03-0.59961E 01-0.21797E 01 0.44527E-00

1.35435 0.16339E 04 0.72061E 00-0.66505E 02 0.14548E 02-0.20703E 04 0.16441E-00-0.56108E 03-0.59960E 01-0.21796E 01 0.44527E-00
1.36981 0.18200E 04 0.71402E 00-0.90227E 02 0.16133E 02-0.21209E 04 0.18145E-00-0.62686E 03-0.62330E 01-0.20568E 01 0.47831E-00
1.38527 0.20206E 04 0.70348E 00-0.93255E 02 0.17841E 02-0.21562E 04 0.19947E-00-0.69792E 03-0.64342E 01-0.19034E 01 0.51287E 00

1.38527 0.20206E 04 0.70346E 00-0.93254E 02 0.17841E 02-0.21562E 04 0.19946E-00-0.69792E 03-0.64343E 01-0.19033E 01 0.51286E 00
1.40073 0.22348E 04 0.68851E 00-0.95359E 02 0.19666E 02-0.21726E 04 0.21836E-00-0.77389E 03-0.65892E 01-0.17199E 01 0.54871E 00
1.41619 0.24606E 04 0.66857E 00-0.96289E 02 0.21598E 02-0.21660E 04 0.23799E-00-0.85411E 03-0.66878E 01-0.15104E 01 0.58555E 00

1.41619 0.24606E 04 0.66834E 00-0.96287E 02 0.21598E 02-0.21661E 04 0.23794E-00-0.85412E 03-0.66885E 01-0.15106E 01 0.58553E 00
1.43166 0.26953E 04 0.64265E 00-0.95771E 02 0.23620E 02-0.21328E 04 0.25806E-00-0.93764E 03-0.67216E 01-0.12813E 01 0.62292E 00
1.44712 0.29351E 04 0.61066E 00-0.93531E 02 0.25711E 02-0.20691E 04 0.27838E-00-0.10232E 04-0.66828E 01-0.10443E 01 0.66036E 00

1.44712 0.29350E 04 0.60908E 00-0.93523E 02 0.25712E 02-0.20693E 04 0.27826E-00-0.10232E 04-0.66858E 01-0.10481E 01 0.66030E 00
1.46258 0.31753E 04 0.56734E 00-0.89273E 02 0.27850E 02-0.19732E 04 0.29810E-00-0.11092E 04-0.65752E 01-0.81917E 00 0.69706E 00
1.47804 0.34108E 04 0.51500E 00-0.82746E 02 0.30008E 02-0.18447E 04 0.31670E-00-0.11937E 04-0.63949E 01-0.60545E 00 0.73235E 00

1.47804 0.34106E 04 0.51554E 00-0.82751E 02 0.30007E 02-0.18447E 04 0.31676E-00-0.11937E 04-0.63933E 01-0.60255E 00 0.73238E 00
1.49350 0.36357E 04 0.45363E-00-0.73720E 02 0.32160E 02-0.16881E 04 0.33303E-00-0.12747E 04-0.61442E 01-0.38330E-00 0.76532E 00
1.50896 0.38451E 04 0.39226E-00-0.62055E 02 0.34279E 02-0.15141E 04 0.34602E-00-0.13496E 04-0.58081E 01-0.59867E-01 0.79529E 00

1.50896 0.38452E 04 0.39297E-00-0.62061E 02 0.34278E 02-0.15140E 04 0.34611E-00-0.13496E 04-0.58059E 01-0.56531E-01 0.79533E 00
1.52442 0.40356E 04 0.36810E-00-0.47900E 02 0.36314E 02-0.13334E 04 0.35748E-00-0.14159E 04-0.52924E 01 0.62032E 00 0.82306E 00
1.53988 0.42040E 04 0.45917E-00-0.31892E 02 0.38152E 02-0.11321E 04 0.37649E-00-0.14700E 04-0.43755E 01 0.20394E 01 0.85241E 00

1.53988 0.42041E 04 0.46298E-00-0.31929E 02 0.38147E 02-0.11309E 04 0.37707E-00-0.14699E 04-0.43635E 01 0.20541E 01 0.85266E 00
1.55534 0.43396E 04 0.78766E 00-0.15465E 02 0.39534E 02-0.79520E 03 0.43017E-00-0.15068E 04-0.26667E 01 0.43531E 01 0.89300E 00
1.57080 0.44031E 04 0.13697E 01 0.65801E-01 0.40077E 02-0.16987E 01 0.56331E 00-0.15205E 04-0.21671E-01 0.60619E 01 0.95602E 00
```

STABILITY AND FREE VIBRATION OF SHELLS WITH AXISYMMETRIC PRESTRESS, BY A. KALNINS, LEHIGH UNIV, BETHLEHEM, PA
WRIGHT PATTERSON AIR FORCE BASE FLIGHT DYNAMICS LABORATORY VERSION, 22 JULY 1968

SOLUTION AT POINTS ALONG MERIDIAN AT EIGENVALUE PARAMETER OMEGA= 0.4100000E-03 FOR WAVE NUMBER NX= 3

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     S          W          V           UPHI         NPHI          APHI          MPHI        UTHETA         N          NTHETA       MTHETA

  MAIN SHELL PART NO  1
OMEGA= 0.41000E-03  OMSU= 0.           XMA= 0.41000E-03  LC=  1  PRESTRESS=  -0.1733092E-00  -0.6932367E-01

  0.03409-0.            0.13487E 01-0.            0.23041E-01-0.            -0.51699E-02-0.            -0.80535E-02 0.92165E-02-0.20439E-02
  0.09547 0.23624E-01 0.23343E-00 0.69041E-02 0.22127E-01-0.23259E 01-0.22347E-02-0.61204E-02-0.14995E-01-0.21238E-01 0.19112E-02
  0.15684 0.11099E-00 0.21189E-00 0.16760E-01 0.36026E-01-0.62894E 01-0.35016E-02-0.25732E-01-0.26182E-01-0.37480E-01 0.35605E-02

  0.15684 0.11098E-00 0.21175E-00 0.16699E-01 0.35987E-01-0.62926E 01-0.34945E-02-0.25698E-01-0.26153E-01-0.37405E-01 0.35582E-02
  0.21822 0.30857E-00 0.21499E-00 0.25665E-01 0.51240E-01-0.12362E 02-0.48191E-02-0.60279E-01-0.38745E-01-0.55551E-01 0.51682E-02
  0.27959 0.67487E 00 0.22273E-00 0.26862E-01 0.67953E-01-0.20777E 02-0.61173E-02-0.12324E-00-0.53744E-01-0.77085E-01 0.68954E-02

  0.27959 0.67486E 00 0.22273E-00 0.26867E-01 0.67953E-01-0.20777E 02-0.61170E-02-0.12325E-00-0.53756E-01-0.77098E-01 0.68954E-02
  0.34097 0.12849E 01 0.23319E-00 0.10071E-01 0.86762E-01-0.31845E 02-0.73692E-02-0.23216E-00-0.72162E-01-0.10332E-00 0.88427E-02
  0.40234 0.22396E 01 0.24613E-00-0.34163E-01 0.10659E-00-0.46003E 02-0.85545E-02-0.41342E-00-0.95208E-01-0.13569E-00 0.11080E-01

  0.40234 0.22398E 01 0.24612E-00-0.34159E-01 0.10659E-00-0.46003E 02-0.85548E-02-0.41345E-00-0.95232E-01-0.13569E-00 0.11080E-01
  0.46372 0.36801E 01 0.26163E-00-0.14092E-00 0.13475E-00-0.63641E 02-0.96649E-02-0.70657E 00-0.12453E-00-0.17587E-00 0.13703E-01
  0.52509 0.58056E 01 0.27987E-00-0.32306E-00 0.16716E-00-0.86146E 02-0.10599E-01-0.11706E 01-0.16215E-00-0.22590E-00 0.16832E-01

  0.52509 0.58056E 01 0.27986E-00-0.32307E-00 0.16716E-00-0.86136E 02-0.10592E-01-0.11721E 01-0.16289E-00-0.22736E-00 0.16835E-01
  0.58647 0.89062E 01 0.30096E-00-0.62385E 00 0.20881E-00-0.11395E 03-0.11351E-01-0.18980E 01-0.21229E-00-0.29077E-00 0.20628E-01
  0.64784 0.13412E 02 0.32522E-00-0.10972E 01 0.26383E-00-0.14867E 03-0.11836E-01-0.30217E 01-0.27705E-00-0.37002E-00 0.25301E-01

  0.64784 0.13412E 02 0.32524E-00-0.10972E 01 0.26387E-00-0.14867E 03-0.11836E-01-0.30219E 01-0.27704E-00-0.37009E-00 0.25300E-01
  0.70922 0.19973E 02 0.35295E-00-0.18188E 01 0.33867E-00-0.19218E 03-0.11926E-01-0.47551E 01-0.36262E-00-0.46891E-00 0.31141E-01
  0.77059 0.29550E 02 0.38444E-00-0.28961E 01 0.44310E-00-0.24700E 03-0.11448E-01-0.74305E 01-0.47653E-00-0.59105E 00 0.38552E-01

  0.77059 0.29590E 02 0.38444E-00-0.28961E 01 0.44310E-00-0.24700E 03-0.11448E-01-0.74309E 01-0.47654E-00-0.59106E 00 0.38552E-01
  0.83197 0.43830E 02 0.42002E-00-0.44818E 01 0.57251E 00-0.31650E 03-0.10143E-01-0.11578E 02-0.62931E 00-0.74404E 00 0.48096E-01
  0.89334 0.65202E 02 0.45999E-00-0.67949E 01 0.81115E 00-0.40518E 03-0.76273E-02-0.18057E 02-0.83561E 00-0.93093E 00 0.60569E-01

  0.89334 0.65202E 02 0.45998E-00-0.67960E 01 0.81116E 00-0.40518E 03-0.76276E-02-0.18058E 02-0.83562E 00-0.93094E 00 0.60569E-01
  0.95472 0.97784E 02 0.50448E 00-0.10147E 02 0.11379E 01-0.51961E 03-0.33277E-02-0.28282E 02-0.11157E 01-0.11579E 01 0.77115E-01
  1.01609 0.14831E 03 0.55337E 00-0.14977E 02 0.16356E 01-0.66556E 03 0.36174E-02-0.44608E 02-0.14960E 01-0.14261E 01 0.99378E-01

  1.01609 0.14831E 03 0.55337E 00-0.14977E 02 0.16358E 01-0.66556E 03 0.36170E-02-0.44608E 02-0.14969E 01-0.14261E 01 0.99377E-01
  1.07747 0.22797E 03 0.60590E 00-0.21878E 02 0.24071E 01-0.85383E 03 0.14492E-01-0.70991E 02-0.20143E 01-0.17326E 01 0.12972E-00
  1.13884 0.35541E 03 0.65908E 00-0.31580E 02 0.36169E 01-0.10927E 04 0.31190E-01-0.11405E 03-0.27082E 01-0.20585E 01 0.17148E-00

  1.13884 0.35541E 03 0.65995E 00-0.31579E 02 0.36171E 01-0.10927E 04 0.31173E-01-0.11405E 03-0.27085E 01-0.20589E 01 0.17147E-00
  1.20021 0.56003E 03 0.71120E 00-0.44782E 02 0.55242E 01-0.13864E 04 0.56323E-01-0.18458E 03-0.36123E 01-0.23579E 01 0.22916E-00
  1.26159 0.89016E 03 0.75117E 00-0.61605E 02 0.85061E 01-0.17231E 04 0.93292E-01-0.29907E 03-0.47163E 01-0.25315E 01 0.30828E-00

  MAIN SHELL PART NO  2
OMEGA= 0.41000E-03  OMSU= 0.           XMA= 0.41000E-03  LC=  1  PRESTRESS=  -0.3739811E-00  -0.6894838E 00

  1.26159 0.89016E 03 0.75208E 00-0.61613E 02 0.85067E 01-0.17228E 04 0.93421E-01-0.29905E 03-0.47136E 01-0.25286E 01 0.30833E-00
  1.27705 0.99956E 03 0.75901E 00-0.66204E 02 0.94795E 01-0.18094E 04 0.10500E-00-0.33731E 03-0.50113E 01-0.25349E 01 0.33232E-00
  1.29251 0.11216E 04 0.76390E 00-0.71031E 02 0.10566E 02-0.18944E 04 0.11779E-00-0.38008E 03-0.53105E 01-0.25202E 01 0.35007E-00

  1.29251 0.11216E 04 0.76396E 00-0.71031E 02 0.10566E 02-0.18944E 04 0.11779E-00-0.38008E 03-0.53106E 01-0.25203E 01 0.35007E-00
  1.30797 0.12572E 04 0.76659E 00-0.75775E 02 0.11752E 02-0.19760E 04 0.13157E-00-0.42770E 03-0.56062E 01-0.24813E 01 0.38563E-00
  1.32343 0.14073E 04 0.76640E 00-0.80411E 02 0.13062E 02-0.20521E 04 0.14646E-00-0.48046E 03-0.58917E 01-0.24147E 01 0.41500E-00
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1.32343 0.14073E 04 0.76645E 00-0.80410E 02 0.13062E 02-0.20521E 04 0.14646E-00-0.48046E 03-0.58918E 01-0.24148E 01 0.41500E-00
1.33889 0.15722E 04 0.76313E 00-0.84811E 02 0.14494E 02-0.21202E 04 0.16244E-00-0.53858E 03-0.61595E 01-0.23176E 01 0.44615E-00
1.35435 0.17524E 04 0.75614E 00-0.88821E 02 0.16049E 02-0.21772E 04 0.17949E-00-0.60215E 03-0.64001E 01-0.21870E 01 0.47901E-00

1.35435 0.17524E 04 0.75613E 00-0.88821E 02 0.16049E 02-0.21772E 04 0.17949E-00-0.60215E 03-0.64002E 01-0.21870E 01 0.47901E-00
1.36981 0.19476E 04 0.74497E 00-0.92259E 02 0.17724E 02-0.22200E 04 0.19756E-00-0.67112E 03-0.66032E 01-0.20211E 01 0.51342E 00
1.38527 0.21571E 04 0.72915E 00-0.94917E 02 0.19515E 02-0.22448E 04 0.21654E-00-0.74523E 03-0.67568E 01-0.18193E 01 0.54918E 00

1.38527 0.21571E 04 0.72914E 00-0.94917E 02 0.19515E 02-0.22448E 04 0.21654E-00-0.74523E 03-0.67569E 01-0.18193E 01 0.54918E 00
1.40073 0.23794E 04 0.70816E 00-0.96566E 02 0.21408E 02-0.22479E 04 0.23628E-00-0.82397E 03-0.68487E 01-0.15824E 01 0.58600E 00
1.41619 0.26122E 04 0.68151E 00-0.96962E 02 0.23386E 02-0.22251E 04 0.25660E-00-0.90652E 03-0.68662E 01-0.13139E 01 0.62350E 00

1.41619 0.26122E 04 0.68134E 00-0.96961E 02 0.23387E 02-0.22251E 04 0.25656E-00-0.90652E 03-0.68667E 01-0.13142E 01 0.62348E 00
1.43166 0.28524E 04 0.64842E 00-0.95855E 02 0.25427E 02-0.21728E 04 0.27717E-00-0.99176E 03-0.67981E 01-0.10203E 01 0.66116E 00
1.44712 0.30955E 04 0.60852E 00-0.93008E 02 0.27500E 02-0.20871E 04 0.29776E-00-0.10782E 04-0.66340E 01-0.71176E 00 0.69845E 00

1.44712 0.30955E 04 0.60748E 00-0.92999E 02 0.27501E 02-0.20872E 04 0.29762E-00-0.10782E 04-0.66373E 01-0.71596E 00 0.69839E 00
1.46258 0.33365E 04 0.55905E 00-0.88183E 02 0.29572E 02-0.19666E 04 0.31754E-00-0.11640E 04-0.63741E 01-0.40726E-00 0.73452E 00
1.47804 0.35694E 04 0.50161E 00-0.81221E 02 0.31601E 02-0.18103E 04 0.33621E-00-0.12472E 04-0.60098E 01-0.10753E-00 0.76871E 00

1.47804 0.35694E 04 0.50221E 00-0.81226E 02 0.31601E 02-0.18102E 04 0.33629E-00-0.12471E 04-0.60079E 01-0.10465E-00 0.76875E 00
1.49350 0.37879E 04 0.43658E-00-0.72003E 02 0.33547E 02-0.16202E 04 0.35295E-00-0.13252E 04-0.55424E 01 0.18849E-00 0.80017E 00
1.50896 0.39857E 04 0.36789E-00-0.60498E 02 0.35368E 02-0.14035E 04 0.36682E-00-0.13957E 04-0.49706E 01 0.51531E 00 0.82806E 00

1.50896 0.39858E 04 0.36863E-00-0.60505E 02 0.35367E 02-0.14033E 04 0.36693E-00-0.13957E 04-0.49682E 01 0.51826E 00 0.82811E 00
1.52442 0.41577E 04 0.31523E-00-0.46892E 02 0.37011E 02-0.11669E 04 0.37864E-00-0.14561E 04-0.42518E 01 0.99394E 00 0.85247E 00
1.53988 0.42992E 04 0.31680E-00-0.31646E 02 0.38397E 02-0.90750E 03 0.39249E-00-0.15035E 04-0.32895E 01 0.18055E 01 0.87486E 00

1.53988 0.42993E 04 0.32033E-00-0.31680E 02 0.38392E 02-0.90632E 03 0.39303E-00-0.15034E 04-0.32782E 01 0.18190E 01 0.87509E 00
1.55534 0.44024E 04 0.43598E-00-0.15711E 02 0.39374E 02-0.56981E 03 0.42219E-00-0.15344E 04-0.18932E 01 0.30271E 01 0.89995E 00
1.57080 0.44461E 04 0.67573E 00 0.64025E-01 0.39745E 02-0.18556E 01 0.48693E-00-0.15455E 04-0.21078E-01 0.38967E 01 0.93276E 00
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        STABILITY AND FREE VIBRATION OF SHELLS WITH AXISYMMETRIC PRESTRESS, BY A. KALNINS, LEHIGH UNIV, BETHLEHEM, PA
                 WRIGHT PATTERSON AIR FORCE BASE FLIGHT DYNAMICS LABORATORY VERSION, 22 JULY 1968

SOLUTION AT POINTS ALONG MERIDIAN AT EIGENVALUE PARAMETER OMEGA= 0.4300000E-03   FOR WAVE NUMBER NX=  3

    S          W          Q          UPHI        NPHI        BPHI        MPHI       UTHETA        N         NTHETA      MTHETA

 MAIN SHELL PART NO  1
OMEGA= 0.43000E-03  CMSQ= 0.            XMR= 0.43000E-03  LC=  1  PRESTRESS=  -0.1733092E-00  -0.6932367E-01

 0.03409-0.          0.14923E 01-0.             0.27287E-01-0.              -0.56542E-02-0.              -0.10703E-01 0.10915E-01-0.22617E-02
 0.09547 0.26147E-01 0.25812E-00 0.87653E-02 0.27890E-01-0.25710E 01-0.24364E-02-0.10147E-01-0.19815E-01-0.26805E-01 0.21123E-02
 0.15684 0.12282E-00 0.23432E-00 0.21670E-01 0.45371E-01-0.69505E 01-0.38674E-02-0.31677E-01-0.34472E-01-0.47007E-01 0.39362E-02

 0.15684 0.12269E-00 0.23416E-00 0.21660E-01 0.45325E-01-0.69430E 01-0.38595E-02-0.31637E-01-0.34436E-01-0.46918E-01 0.39337E-02
 0.21822 0.34143E-00 0.23779E-00 0.34762E-01 0.64515E-01-0.13657E 02-0.53179E-02-0.73092E-01-0.50770E-01-0.69421E-01 0.57055E-02
 0.27959 0.74656E 00 0.24642E-00 0.40606E-01 0.85541E-01-0.22946E 02-0.67427E-02-0.14731E-00-0.70018E-01-0.95984E-01 0.76250E-02

 0.27959 0.74655E 00 0.24642E-00 0.40611E-01 0.85541E-01-0.22945E 02-0.67422E-02-0.14732E-00-0.70033E-01-0.96002E-01 0.76250E-02
 0.34097 0.14209E 01 0.25802E-00 0.28216E-01 0.10919E-00-0.35151E 02-0.81103E-02-0.27409E-00-0.93442E-01-0.12824E-00 0.97791E-02
 0.40234 0.24760E 01 0.27230E-00-0.17892E-01 0.13660E-00-0.50748E 02-0.93974E-02-0.48312E-00-0.12256E-00-0.16798E-00 0.12254E-01

 0.40234 0.24760E 01 0.27230E-00-0.17886E-01 0.13660E-00-0.50749E 02-0.93978E-02-0.48313E-00-0.12257E-00-0.16800E-00 0.12254E-01
 0.46372 0.40662E 01 0.28929E-00-0.11914E-00 0.16943E-00-0.70377E 02-0.10572E-01-0.81893E 00-0.15946E-00-0.21755E-00 0.15152E-01
 0.52509 0.64109E 01 0.30906E-00-0.30506E-00 0.21006E-00-0.94888E 02-0.11591E-01-0.13482E 01-0.20686E-00-0.27977E-00 0.18606E-01

 0.52509 0.64109E 01 0.30904E-00-0.30510E-00 0.21003E-00-0.94948E 02-0.11638E-01-0.13403E 01-0.20299E-00-0.27194E-00 0.18589E-01
 0.58647 0.98315E 01 0.33227E-00-0.61885E 00 0.26097E-00-0.12555E 03-0.12453E-01-0.21555E 01-0.26128E-00-0.34458E-00 0.22777E-01
 0.64784 0.14800E 02 0.35895E-00-0.11174E 01 0.32758E-00-0.16373E 03-0.12957E-01-0.34122E 01-0.33692E-00-0.43467E-00 0.27933E-01

 0.64784 0.14800E 02 0.35896E-00-0.11173E 01 0.32759E-00-0.16373E 03-0.12957E-01-0.34123E 01-0.33694E-00-0.43471E-00 0.27933E-01
 0.70922 0.22030E 02 0.38938E-00-0.18818E 01 0.41725E-00-0.21153E 03-0.13017E-01-0.53442E 01-0.43600E-00-0.54637E 00 0.34376E-01
 0.77059 0.32619E 02 0.42385E-00-0.30269E 01 0.54118E 00-0.27166E 03-0.12437E-01-0.83172E 01-0.56685E 00-0.68440E 00 0.42544E-01

 0.77059 0.32619E 02 0.42385E-00-0.30269E 01 0.54118E 00-0.27166E 03-0.12438E-01-0.83172E 01-0.56686E 00-0.68440E 00 0.42544E-01
 0.83197 0.48285E 02 0.46269E-00-0.47159E 01 0.71679E 00-0.34778E 03-0.10932E-01-0.12914E 02-0.74112E 00-0.85421E 00 0.53055E-01
 0.89334 0.71771E 02 0.50616E 00-0.71812E 01 0.97147E 00-0.44473E 03-0.80747E-02-0.20077E 02-0.97486E 00-0.10614E 01 0.66776E-01

 0.89334 0.71771E 02 0.50616E 00-0.71812E 01 0.97148E 00-0.44474E 03-0.80751E-02-0.20077E 02-0.97487E 00-0.10614E 01 0.66776E-01
 0.95472 0.10753E 03 0.55431E 00-0.10752E 02 0.13489E 01-0.56886E 03-0.32301E-02-0.31352E 02-0.12901E 01-0.13107E 01 0.84952E-01
 1.01609 0.16287E 03 0.60686E 00-0.15888E 02 0.19193E 01-0.72813E 03 0.45512E-02-0.49305E 02-0.17162E 01-0.16035E 01 0.10936E-00

 1.01609 0.16287E 03 0.60686E 00-0.15888E 02 0.19193E 01-0.72812E 03 0.45509E-02-0.49306E 02-0.17163E 01-0.16036E 01 0.10936E-00
 1.07747 0.24996E 03 0.66274E 00-0.23200E 02 0.27960E 01-0.93179E 03 0.16678E-01-0.78229E 02-0.22902E 01-0.19332E 01 0.14255E-00
 1.13884 0.38883E 03 0.71941E 00-0.33424E 02 0.41597E 01-0.11884E 04 0.35213E-01-0.12526E 03-0.30522E 01-0.22756E 01 0.18807E-00

 1.13884 0.38883E 03 0.71926E 00-0.33423E 02 0.41600E 01-0.11885E 04 0.35192E-01-0.12526E 03-0.30527E 01-0.22761E 01 0.18806E-00
 1.20021 0.61178E 03 0.77117E 00-0.47212E 02 0.62905E 01-0.15007E 04 0.62958E-01-0.20192E 03-0.40313E 01-0.25739E 01 0.25064E-00
 1.26159 0.96707E 03 0.80799E 00-0.64519E 02 0.95851E 01-0.18519E 04 0.10349E-00-0.32553E 03-0.51991E 01-0.27120E 01 0.33587E-00

 MAIN SHELL PART NO  2
OMEGA= 0.43000E-03  OMSQ= 0.            XMR= 0.43000E-03  LC=  1  PRESTRESS=  -0.3739811E-00  -0.6854838E 00

 1.26159 0.96708E 03 0.80892E 00-0.64528E 02 0.95837E 01-0.18516E 04 0.10362E-00-0.32552E 03-0.51963E 01-0.27087E 01 0.33593E-00
 1.27705 0.10846E 04 0.81423E 00-0.69265E 02 0.10652E 02-0.19402E 04 0.11633E-00-0.36661E 03-0.55038E 01-0.26973E 01 0.36162E-00
 1.29251 0.12153E 04 0.81712E 00-0.74040E 02 0.11831E 02-0.20261E 04 0.13014E-00-0.41243E 03-0.58085E 01-0.26606E 01 0.38912E-00

 1.29251 0.12153E 04 0.81709E 00-0.74039E 02 0.11831E 02-0.20261E 04 0.13013E-00-0.41243E 03-0.58086E 01-0.26608E 01 0.38911E-00
 1.30797 0.13602E 04 0.81713E 00-0.78761E 02 0.13127E 02-0.21072E 04 0.14507E-00-0.46330E 03-0.61039E 01-0.25950E 01 0.41844E-00
 1.32343 0.15199E 04 0.81384E 00-0.83315E 02 0.14543E 02-0.21811E 04 0.16113E-00-0.51948E 03-0.63820E 01-0.24965E 01 0.44957E-00

 1.32343 0.15199E 04 0.81382E 00-0.83315E 02 0.14543E 02-0.21812E 04 0.16112E-00-0.51948E 03-0.63821E 01-0.24965E 01 0.44957E-00
 1.33889 0.16950E 04 0.80670E 00-0.87563E 02 0.16082E 02-0.22450E 04 0.17830E-00-0.58112E 03-0.66333E 01-0.23618E 01 0.48245E-00
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1.35435 0.18854E 04 0.79521E 00-0.91339E 02 0.17741E 02-0.22956E 04 0.19653E-00-0.64826E 03-0.68467E 01-0.21882E 01 0.51694E 00

1.35435 0.18854E 04 0.79523E 00-0.91339E 02 0.17741E 02-0.22956E 04 0.19653E-00-0.64826E 03-0.68467E 01-0.21881E 01 0.51694E 00
1.36981 0.20907E 04 0.77883E 00-0.94452E 02 0.19516E 02-0.23292E 04 0.21574E-00-0.72075E 03-0.70095E 01-0.19736E 01 0.55285E 00
1.38527 0.23099E 04 0.75695E 00-0.96689E 02 0.21395E 02-0.23419E 04 0.23577E-00-0.79820E 03-0.71079E 01-0.17176E 01 0.58992E 00

1.38527 0.23099E 04 0.75690E 00-0.96689E 02 0.21395E 02-0.23419E 04 0.23576E-00-0.79820E 03-0.71080E 01-0.17177E 01 0.58991E 00
1.40073 0.25412E 04 0.72902E 00-0.97822E 02 0.23361E 02-0.23297E 04 0.25643E-00-0.87995E 03-0.71272E 01-0.14214E 01 0.62776E 00
1.41619 0.27816E 04 0.69471E 00-0.97619E 02 0.25388E 02-0.22884E 04 0.27747E-00-0.96501E 03-0.70521E 01-0.10882E 01 0.66592E 00

1.41619 0.27816E 04 0.69460E 00-0.97618E 02 0.25388E 02-0.22884E 04 0.27744E-00-0.96502E 03-0.70524E 01-0.10884E 01 0.66591E 00
1.43166 0.30275E 04 0.65349E 00-0.95853E 02 0.27445E 02-0.22143E 04 0.29855E-00-0.10521E 04-0.68688E 01-0.72407E 00 0.70383E 00
1.44712 0.32740E 04 0.60532E 00-0.92329E 02 0.29492E 02-0.21040E 04 0.31936E-00-0.11394E 04-0.65639E 01-0.33807E-00 0.74085E 00

1.44712 0.32740E 04 0.60426E 00-0.92320E 02 0.29493E 02-0.21041E 04 0.31923E-00-0.11394E 04-0.65672E 01-0.34227E-00 0.74079E 00
1.46258 0.35154E 04 0.54838E 00-0.86878E 02 0.31483E 02-0.19560E 04 0.33917E-00-0.12249E 04-0.61337E 01 0.52167E-01 0.77612E 00
1.47804 0.37451E 04 0.48524E-00-0.79437E 02 0.33362E 02-0.17685E 04 0.35785E-00-0.13063E 04-0.55654E 01 0.44311E-00 0.80900E 00

1.47804 0.37452E 04 0.48585E-00-0.79443E 02 0.33362E 02-0.17685E 04 0.35793E-00-0.13063E 04-0.55634E 01 0.44642E-00 0.80904E 00
1.49350 0.39560E 04 0.41614E-00-0.70015E 02 0.35073E 02-0.15417E 04 0.37493E-00-0.13810E 04-0.48596E 01 0.81649E 00 0.83865E 00
1.50896 0.41406E 04 0.33925E-00-0.58698E 02 0.36556E 02-0.12779E 04 0.38969E-00-0.14465E 04-0.40305E 01 0.11405E 01 0.86414E 00

1.50896 0.41406E 04 0.33990E-00-0.58704E 02 0.36557E 02-0.12778E 04 0.38976E-00-0.14465E 04-0.40285E 01 0.11438E 01 0.86418E 00
1.52442 0.42918E 04 0.25504E-00-0.45708E 02 0.37764E 02-0.98084E 03 0.40170E-00-0.15002E 04-0.30931E 01 0.13925E 01 0.88470E 00
1.53988 0.44035E 04 0.15891E-00-0.31318E 02 0.38648E 02-0.65900E 03 0.40969E-00-0.15402E 04-0.20868E 01 0.15331E 01 0.89930E 00

1.53988 0.44035E 04 0.16221E-00-0.31351E 02 0.38644E 02-0.65784E 03 0.41021E-00-0.15401E 04-0.20762E 01 0.15454E 01 0.89952E 00
1.55534 0.44709E 04 0.51903E-01-0.15943E 02 0.39177E 02-0.32232E 03 0.41310E-00-0.15646E 04-0.10398E 01 0.15585E 01 0.90738E 00
1.57080 0.44929E 04-0.73371E-01 0.61108E-01 0.39359E 02-0.17790E 01 0.40784E-00-0.15729E 04-0.20099E-01 0.15125E 01 0.90738E 00

MAX OF 1ST PRESCRIBED VARIABLE AT FINAL EDGE IS 0.14923E 01 DET= 0.67143E-01  ACC= 0.10000E-01

THIS WAS ITERATION NO  1  ACCURACY NOT ATTAINED
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STABILITY AND FREE VIBRATION OF SHELLS WITH AXISYMMETRIC PRESTRESS, BY A. KALNINS, LEHIGH UNIV, BETHLEHEM, PA
WRIGHT PATTERSON AIR FORCE BASE FLIGHT DYNAMICS LABORATORY VERSION, 22 JULY 1968

SOLUTION AT POINTS ALONG MERIDIAN AT EIGENVALUE PARAMETER OMEGA= 0.4282683E-03  FOR WAVE NUMBER NX=  3

   S         W          Q          UPHI        NPHI        UPHI        MPHI       UTHETA        N         NTHETA      MTHETA

 MAIN SHELL PART NO  1
OMEGA= 0.42827E-03  OMSQ= 0.            XMR= 0.42827E-03  LC=  1  PRESTRESS=  -0.1733092E-00  -0.6932367E-01

 0.03409-0.          0.14786E 01-0.         0.26909E-01-0.          -0.56021E-02-0.          -0.10477E-01 0.10763E-01-0.22409E-02
 0.09547 0.25906E-01 0.25575E-00 0.66043E-02 0.27391E-01-0.25475E 01-0.24143E-02-0.99699E-02-0.19401E-01-0.26318E-01 0.20930E-02
 0.15684 0.12169E-00 0.23215E-00 0.21242E-01 0.44556E-01-0.68874E 01-0.38329E-02-0.31148E-01-0.33746E-01-0.46151E-01 0.39001E-02

 0.15684 0.12156E-00 0.23200E-00 0.21232E-01 0.44513E-01-0.68799E 01-0.38250E-02-0.31108E-01-0.33710E-01-0.46062E-01 0.38976E-02
 0.21822 0.33830E-00 0.23559E-00 0.33976E-01 0.63354E-01-0.13534E 02-0.52715E-02-0.71923E-01-0.49687E-01-0.68124E-01 0.56530E-02
 0.27959 0.73976E 00 0.24412E-00 0.39415E-01 0.83987E-01-0.22741E 02-0.66856E-02-0.14506E-00-0.68500E-01-0.94129E-01 0.75547E-02

 0.27959 0.73975E 00 0.24412E-00 0.39420E-01 0.83967E-01-0.22741E 02-0.66852E-02-0.14507E-00-0.68515E-01-0.94147E-01 0.75547E-02
 0.34097 0.14081E 01 0.25560E-00 0.26629E-01 0.10718E-00-0.34842E 02-0.80446E-02-0.27005E-00-0.91366E-01-0.12564E-00 0.96888E-02
 0.40234 0.24538E 01 0.26976E-00-0.19799E-01 0.13404E-00-0.50309E 02-0.93254E-02-0.47620E-00-0.11975E-00-0.16438E-00 0.12141E-01

 0.40234 0.24538E 01 0.26975E-00-0.19793E-01 0.13405E-00-0.50310E 02-0.93259E-02-0.47622E-00-0.11976E-00-0.16440E-00 0.12140E-01
 0.46372 0.40302E 01 0.28663E-00-0.12122E-00 0.16618E-00-0.69779E 02-0.10497E-01-0.80746E 00-0.15564E-00-0.21253E-00 0.15013E-01
 0.52509 0.63550E 01 0.30632E-00-0.30712E-00 0.20590E-00-0.94101E 02-0.11515E-01-0.13295E 01-0.20161E-00-0.27271E-00 0.18436E-01

 0.52509 0.63551E 01 0.30633E-00-0.30709E-00 0.20592E-00-0.94137E 02-0.11542E-01-0.13246E 01-0.19927E-00-0.26777E-00 0.18427E-01
 0.58647 0.97462E 01 0.32936E-00-0.61972E 00 0.25600E-00-0.12449E 03-0.12352E-01-0.21317E 01-0.25675E-00-0.33968E-00 0.22578E-01
 0.64784 0.14672E 02 0.35580E-00-0.11160E 01 0.32156E-00-0.16235E 03-0.12856E-01-0.33764E 01-0.33145E-00-0.42894E-00 0.27689E-01

 0.64784 0.14672E 02 0.35589E-00-0.11156E 01 0.32167E-00-0.16235E 03-0.12856E-01-0.33762E 01-0.33142E-00-0.42871E-00 0.27689E-01
 0.70922 0.21841E 02 0.38605E-00-0.16762E 01 0.40996E-00-0.20975E 03-0.12918E-01-0.52899E 01-0.42922E-00-0.53922E 00 0.34077E-01
 0.77059 0.32340E 02 0.42024E-00-0.30152E 01 0.53209E 00-0.26940E 03-0.12347E-01-0.82356E 01-0.55851E 00-0.67587E 00 0.42177E-01

 0.77059 0.32340E 02 0.42024E-00-0.30151E 01 0.53210E 00-0.26940E 03-0.12347E-01-0.82356E 01-0.55852E 00-0.6751  E 00 0.42177E-01
 0.83197 0.47876E 02 0.45878E-00-0.46947E 01 0.70529E 00-0.34491E 03-0.10860E-01-0.12791E 02-0.73080E 00-0.84407E 00 0.52598E-01
 0.89334 0.71168E 02 0.50192E 00-0.71461E 01 0.95666E 00-0.44111E 03-0.80340E-02-0.19891E 02-0.96203E 00-0.10494E 01 0.66206E-01

 0.89334 0.71168E 02 0.50192E 00-0.71461E 01 0.95666E 00-0.44111E 03-0.80345E-02-0.19891E 02-0.96204E 00-0.10494E 01 0.66206E-01
 0.95472 0.10663E 03 0.54974E 00-0.10697E 02 0.13294E 01-0.56428E 03-0.32397E-02-0.31070E 02-0.12741E 01-0.12966E 01 0.84232E-01
 1.01609 0.16154E 03 0.60195E 00-0.15805E 02 0.18932E 01-0.72239E 03 0.44645E-02-0.48874E 02-0.16960E 01-0.15872E 01 0.10844E-00

 1.01609 0.16154E 03 0.60195E 00-0.15805E 02 0.18932E 01-0.72239E 03 0.44643E-02-0.48874E 02-0.16961E 01-0.15873E 01 0.10844E-00
 1.07747 0.24794E 03 0.65752E 00-0.23080E 02 0.27601E 01-0.92464E 03 0.16476E-01-0.77565E 02-0.22648E 01-0.19148E 01 0.14137E-00
 1.13884 0.38577E 03 0.71395E 00-0.33256E 02 0.41098E 01-0.11797E 04 0.34640E-01-0.12423E 03-0.30206E 01-0.22558E 01 0.18655E-00

 1.13884 0.38577E 03 0.71381E 00-0.33255E 02 0.41100E 01-0.11797E 04 0.34821E-01-0.12423E 03-0.30211E 01-0.22562E 01 0.18654E-00
 1.20021 0.60711E 03 0.76568E 00-0.46991E 02 0.62200E 01-0.14902E 04 0.62345E-01-0.20033E 03-0.39929E 01-0.25541E 01 0.24867E-00
 1.26159 0.96004E 03 0.80289E 00-0.64255E 02 0.94660E 01-0.18402E 04 0.10255E-00-0.32311E 03-0.51547E 01-0.26948E 01 0.33335E-00

 MAIN SHELL PART NO  2
OMEGA= 0.42827E-03  OMSQ= 0.            XMR= 0.42827E-03  LC=  1  PRESTRESS=  -0.3739811E-00  -0.6854838E 00

 1.26159 0.96003E 03 0.80374E 00-0.64263E 02 0.94846E 01-0.18398E 04 0.10268E-00-0.32309E 03-0.51521E 01-0.26923E 01 0.33340E-00
 1.27705 0.10768E 04 0.80921E 00-0.68994E 02 0.10544E 02-0.19283E 04 0.11530E-00-0.36393E 03-0.54588E 01-0.26825E 01 0.35893E-00
 1.29251 0.12067E 04 0.81230E 00-0.73767E 02 0.11714E 02-0.20141E 04 0.12901E-00-0.40947E 03-0.57630E 01-0.26478E 01 0.38627E-00

 1.29251 0.12067E 04 0.81225E 00-0.73767E 02 0.11714E 02-0.20141E 04 0.12900E-00-0.40947E 03-0.57631E 01-0.26480E 01 0.38627E-00
 1.30797 0.13508E 04 0.81255E 00-0.78491E 02 0.13001E 02-0.20953E 04 0.14383E-00-0.46005E 03-0.60585E 01-0.25847E 01 0.41544E-00
 1.32343 0.15096E 04 0.80957E 00-0.83053E 02 0.14407E 02-0.21694E 04 0.15979E-00-0.51591E 03-0.63372E 01-0.24891E 01 0.44641E-00
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1.32343 0.15096E 04 0.80951E 00-0.83052E 02 0.14407E 02-0.21694E 04 0.15978E-00-0.51591E 03-0.63374E 01-0.24892E 01 0.44641E-00
1.33889 0.16837E 04 0.80275E 00-0.87315E 02 0.15936E 02-0.22337E 04 0.17684E-00-0.57723E 03-0.65902E 01-0.23580E 01 0.47913E-00
1.35435 0.18732E 04 0.79169E 00-0.91112E 02 0.17586E 02-0.22848E 04 0.19497E-00-0.64405E 03-0.68062E 01-0.21883E 01 0.51347E 00

1.35435 0.18732E 04 0.79170E 00-0.91112E 02 0.17586E 02-0.22848E 04 0.19497E-00-0.64405E 03-0.68061E 01-0.21882E 01 0.51347E 00
1.36981 0.20777E 04 0.77580E 00-0.94255E 02 0.19352E 02-0.23193E 04 0.21407E-00-0.71622E 03-0.69727E 01-0.19780E 01 0.54925E 00
1.38527 0.22960E 04 0.75451E 00-0.96531E 02 0.21223E 02-0.23331E 04 0.23402E-00-0.79336E 03-0.70761E 01-0.17268E 01 0.58620E 00

1.38527 0.22960E 04 0.75447E 00-0.96531E 02 0.21223E 02-0.23331E 04 0.23401E-00-0.79336E 03-0.70762E 01-0.17268E 01 0.58620E 00
1.40073 0.25264E 04 0.72729E 00-0.97712E 02 0.23182E 02-0.23223E 04 0.25460E-00-0.87484E 03-0.71019E 01-0.14358E 01 0.62395E 00
1.41619 0.27661E 04 0.69373E 00-0.97565E 02 0.25205E 02-0.22826E 04 0.27559E-00-0.95968E 03-0.70351E 01-0.11085E 01 0.66206E 00

1.41619 0.27662E 04 0.69347E 00-0.97562E 02 0.25206E 02-0.22827E 04 0.27554E-00-0.95969E 03-0.70360E 01-0.11089E 01 0.66204E 00
1.43166 0.30115E 04 0.65314E 00-0.95858E 02 0.27261E 02-0.22106E 04 0.29660E-00-0.10466E 04-0.68628E 01-0.75099E 00 0.69994E 00
1.44712 0.32578E 04 0.60575E 00-0.92396E 02 0.29310E 02-0.21025E 04 0.31741E-00-0.11338E 04-0.65707E 01-0.37217E-00 0.73700E 00

1.44712 0.32577E 04 0.60464E 00-0.92386E 02 0.29312E 02-0.21027E 04 0.31726E-00-0.11338E 04-0.65741E 01-0.37627E-00 0.73693E 00
1.46258 0.34991E 04 0.54945E 00-0.87001E 02 0.31309E 02-0.19570E 04 0.33721E-00-0.12194E 04-0.61562E 01 0.10452E-01 0.77234E 00
1.47804 0.37292E 04 0.48683E-00-0.79604E 02 0.33202E 02-0.17724E 04 0.35589E-00-0.13009E 04-0.56064E 01 0.39307E-00 0.80534E 00

1.47804 0.37292E 04 0.48739E-00-0.79609E 02 0.33202E 02-0.17724E 04 0.35595E-00-0.13009E 04-0.56046E 01 0.39618E-00 0.80537E 00
1.49350 0.39406E 04 0.41800E-00-0.70198E 02 0.34935E 02-0.15490E 04 0.37293E-00-0.13760E 04-0.49224E 01 0.75952E 00 0.83516E 00
1.50896 0.41266E 04 0.34198E-00-0.58864E 02 0.36451E 02-0.12896E 04 0.38759E-00-0.14419E 04-0.41166E 01 0.10847E 01 0.86086E 00

1.50896 0.41266E 04 0.34262E-00-0.58871E 02 0.36450E 02-0.12893E 04 0.38770E-00-0.14419E 04-0.41142E 01 0.10875E 01 0.86091E 00
1.52442 0.42797E 04 0.26063E-00-0.45819E 02 0.37696E 02-0.99777E 03 0.39963E-00-0.14962E 04-0.31985E 01 0.13570E 01 0.88179E 00
1.53988 0.43940E 04 0.17329E-00-0.31351E 02 0.38626E 02-0.68144E 03 0.40817E-00-0.15368E 04-0.21959E 01 0.15581E 01 0.89711E 00

1.53988 0.43941E 04 0.17654E-00-0.31383E 02 0.38622E 02-0.68040E 03 0.40867E-00-0.15368E 04-0.21855E 01 0.15707E 01 0.89732E 00
1.55534 0.44647E 04 0.86532E-01-0.15928E 02 0.39196E 02-0.34474E 03 0.41393E-00-0.15619E 04-0.11173E 01 0.16919E 01 0.90671E 00
1.57080 0.44887E 04-0.61127E-02 0.61620E-01 0.39395E 02-0.18955E 01 0.41514E-00-0.15705E 04-0.20259E-01 0.17289E 01 0.90967E 00

MAX OF 1ST PRESCRIBED VARIABLE AT FINAL EDGE IS 0.14786E 01 DET= 0.15402E-03  ACC= 0.10000E-01

OMEGA= 0.42827E-03 IS AN EIGENVALUE
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STABILITY AND FREE VIBRATION OF SHELLS WITH AXISYMMETRIC PRESTRESS, BY A. KALNINS, LEHIGH UNIV, BETHLEHEM, PA
WRIGHT PATTERSON AIR FORCE BASE FLIGHT DYNAMICS LABORATORY VERSION, 22 JULY 1968

SUMMARY OF RESULTS FOR N= 3

| | OMEGA | ACTUAL DET | ADJUSTED DET | NORMAL DET | XMC | XME |
|---|---|---|---|---|---|---|
| | 0.3900000E-03 | 0.1376526E 01 | 0.1376526E 01 | 0.1124649E 01 | 1.0 | 1.0 |
| | 0.4100000E-03 | 0.6823037E 00 | 0.6823037E 00 | 0.5059071E 00 | 1.0 | 1.0 |
| | 0.4300000E-03 | -0.6714296E-01 | -0.6714296E-01 | -0.4499172E-01 | 1.0 | 1.0 |
| EIGENVALUE AT | 0.4282683E-03 | 0.1540184E-03 | 0.1540184E-03 | 0.1041656E-03 | 1.0 | 1.0 |

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        STABILITY AND FREE VIBRATION OF SHELLS WITH AXISYMMETRIC PRESTRESS, BY A. KALNINS, LEHIGH UNIV, BETHLEHEM, PA
                WRIGHT PATTERSON AIR FORCE BASE FLIGHT DYNAMICS LABORATORY VERSION, 22 JULY 1968

                STABILITY ANALYSIS  PARTS=  1  BRANCHES=  0  NUMBER OF SUBCASES=  3

ANGLES OF ROTATION OF BOUNDARY CONDITIONS ARE ALFL= 0.            ALFRS=-0.

         PART NO  1

SI= 0.          SX= 0.20000E 01   IPAR= 10   ING=  2    SHELL TYPE 2   NTP= 0   LAYERS MLY= 1

CYLINDRICAL SHELL NO  2    H= 0.10950E-01    R= 0.10000E 01  PHI=    90.000  DEGREES

LAYER NO  1  FROM Z=-0.54750E-02  TO Z= 0.54750E-02
     CONSISTS OF ISOTROPIC MATERIAL, YOUNGS MODULUS E= 0.10000E 01  POISSONS RATIO NU= 0.16700E-00
     COEFFICIENTS OF THERMAL EXPANSION  AFI=-0.          ATHETA=-0.          MASS DENSITY RHO=-0.

PRESTRESS PART 1  POINTS=  2  LC=-1       S               NPHI            NTHETA

                                        0.              -0.09999999E 01 -0.
                                        0.20000000E 01 -0.09999999E 01 -0.
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STABILITY AND FREE VIBRATION OF SHELLS WITH AXISYMMETRIC PRESTRESS, BY A. KALNINS, LEHIGH UNIV, BETHLEHEM, PA
WRIGHT PATTERSON AIR FORCE BASE FLIGHT DYNAMICS LABORATORY VERSION, 22 JULY 1968

SUBCASE NO 1 WITH WAVE NUMBER 5

STARTING OMEGA= 0.50000E-04 INCREMENT= 0.10000E-04 FINAL OMEGA= 0.90000E-04 1 EIGENVALUES

| | | | | |
|---|---|---|---|---|
| BOUNDARY CONDITIONS AT STARTING EDGE | 1-0. | 4-0. | 6-0. | 7-0. |
| BOUNDARY CONDITIONS AT FINAL EDGE | 1-0. | 4-0. | 6-0. | 7-0. |

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             STABILITY AND FREE VIBRATION OF SHELLS WITH AXISYMMETRIC PRESTRESS, BY A. KALNINS, LEHIGH UNIV, BETHLEHEM, PA
                          WRIGHT PATTERSON AIR FORCE BASE FLIGHT DYNAMICS LABORATORY VERSION, 22 JULY 1968

  SOLUTION AT POINTS ALONG MERIDIAN AT EIGENVALUE PARAMETER OMEGA= 0.5000000E-04   FOR WAVE NUMBER NX=  5

      S           W           Q          UPHI        NPHI        BPHI        MPHI       UTHETA        N         NTHETA      MTHETA

   MAIN SHELL PART NO  1
  OMEGA= 0.50000E-04  OMSQ= 0.          XMR= 0.50000E-04  LC= -1  PRESTRESS=  -0.1000000E 01  -0.

    0.       0.          -0.35273E-06 0.23873E-02 0.          0.67942E-01 0.          0.          0.83586E-05-0.          -0.
    0.10000-0.67792E-02-0.35123E-06 0.23681E-02-0.41958E-05 0.67490E-01-0.40875E-08 0.13693E-02 0.83640E-05 0.36046E-07-0.18477E-07
    0.20000-0.13466E-01-0.34532E-06 0.23105E-02-0.83951E-05 0.66072E-01-0.82547E-08 0.27196E-02 0.83707E-05 0.42791E-07-0.36724E-07

    0.20000-0.13463E-01-0.34374E-06 0.23105E-02-0.83935E-05 0.66052E-01-0.81803E-08 0.27195E-02 0.83737E-05 0.73799E-07-0.36703E-07
    0.30000-0.19959E-01-0.33220E-06 0.22145E-02-0.12600E-04 0.63706E-01-0.12176E-07 0.40317E-02 0.83921E-05 0.84524E-07-0.54420E-07
    0.40000-0.26172E-01-0.31652E-06 0.20801E-02-0.16812E-04 0.60386E-01-0.16115E-07 0.52865E-02 0.83963E-05 0.51079E-07-0.71385E-07

    0.40000-0.26169E-01-0.31498E-06 0.20801E-02-0.16811E-04 0.60354E-01-0.16074E-07 0.52864E-02 0.83960E-05 0.75627E-07-0.71372E-07
    0.50000-0.31999E-01-0.29415E-06 0.19073E-02-0.21021E-04 0.56081E-01-0.19783E-07 0.64644E-02 0.83851E-05 0.28427E-07-0.87293E-07
    0.60000-0.37352E-01-0.26931E-06 0.16962E-02-0.25213E-04 0.50796E-01-0.23416E-07 0.75456E-02 0.83248E-05-0.90538E-07-0.10195E-06

    0.60000-0.37350E-01-0.26790E-06 0.16962E-02-0.25211E-04 0.50753E-01-0.23413E-07 0.75455E-02 0.83212E-05-0.73939E-07-0.10194E-06
    0.70000-0.42116E-01-0.23404E-06 0.14472E-02-0.29356E-04 0.44400E-01-0.26688E-07 0.85096E-02 0.82058E-05-0.16834E-06-0.11500E-06
    0.80000-0.46200E-01-0.19558E-06 0.11608E-02-0.33428E-04 0.37155E-01-0.29406E-07 0.93367E-02 0.80359E-05-0.28473E-06-0.12617E-06

    0.80000-0.46199E-01-0.19440E-06 0.11608E-02-0.33426E-04 0.37104E-01-0.29441E-07 0.93367E-02 0.80291E-05-0.27741E-06-0.12618E-06
    0.90000-0.49515E-01-0.16036E-06 0.83793E-03-0.37389E-04 0.29095E-01-0.31848E-07 0.10007E-01 0.77679E-05-0.56630E-06-0.13528E-06
    1.00000-0.51976E-01-0.12917E-06 0.48036E-03-0.41167E-04 0.19784E-01-0.35152E-07 0.10499E-01 0.72637E-05-0.12004E-05-0.14229E-06

    1.00000-0.51976E-01-0.12833E-06 0.48039E-03-0.41165E-04 0.19728E-01-0.35222E-07 0.10499E-01 0.72542E-05-0.12032E-05-0.14231E-06
    1.10000-0.53384E-01-0.43354E-07 0.90478E-04-0.44611E-04 0.81058E-02-0.38019E-07 0.10790E-01 0.65097E-05-0.12739E-05-0.14647E-06
    1.20000-0.53598E-01 0.87231E-07-0.33024E-03-0.47731E-04-0.32935E-02-0.34595E-07 0.10864E-01 0.60747E-05-0.72064E-07-0.14644E-06

    1.20000-0.53599E-01 0.87700E-07-0.33021E-03-0.47730E-04-0.33533E-02-0.34703E-07 0.10864E-01 0.60623E-05-0.85607E-07-0.14646E-06
    1.30000-0.52902E-01 0.33387E-07-0.77975E-03-0.50746E-04-0.97275E-02-0.29032E-07 0.10716E-01 0.59383E-05-0.10450E-05-0.14369E-06
    1.40000-0.51560E-01-0.25926E-06-0.12490E-02-0.53305E-04-0.19822E-01-0.46774E-07 0.10319E-01 0.36694E-05-0.85429E-05-0.14320E-06

    1.40000-0.51562E-01-0.25961E-06-0.12490E-02-0.53305E-04-0.19863E-01-0.46895E-07 0.10318E-01 0.36578E-05-0.85685E-05-0.14322E-06
    1.50000-0.47956E-01 0.21414E-06-0.17242E-02-0.53808E-04-0.57472E-01-0.78301E-07 0.95968E-02-0.16676E-05-0.86810E-05-0.13900E-06
    1.60000-0.40077E-01 0.19647E-05-0.22168E-02-0.52817E-04-0.90379E-01-0.80474E-08 0.85708E-02 0.36721E-06 0.21593E-04-0.10628E-06

    1.60000-0.40079E-01 0.19649E-05-0.22167E-02-0.52814E-04-0.90475E-01-0.82723E-08 0.85707E-02 0.34463E-06 0.21559E-04-0.10632E-06
    1.70000-0.34012E-01 0.78735E-06-0.27607E-02-0.56840E-04-0.56964E-02 0.15525E-06 0.75001E-02 0.16633E-04 0.28712E-04-0.63001E-07
    1.80000-0.40073E-01-0.68147E-05-0.32821E-02-0.64819E-04 0.91986E-01-0.91069E-07 0.63242E-02 0.39960E-05-0.10338E-03-0.12136E-06

    1.80000-0.40078E-01-0.68178E-05-0.32822E-02-0.64824E-04 0.92069E-01-0.91089E-07 0.63244E-02 0.40011E-05-0.10342E-03-0.12138E-06
    1.90000-0.34686E-01-0.35818E-05-0.35704E-02-0.47448E-04-0.32020E-00-0.86181E-06 0.38587E-02-0.80614E-04-0.17648E-03-0.23669E-06
    2.00000 0.34184E-01 0.30859E-04-0.38395E-02-0.79378E-10-0.94083E 00-0.99600E-10-0.37792E-08-0.61983E-04 0.37432E-03 0.93486E-07
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STABILITY AND FREE VIBRATION OF SHELLS WITH AXISYMMETRIC PRESTRESS, BY A. KALNINS, LEHIGH UNIV, BETHLEHEM, PA
WRIGHT PATTERSON AIR FORCE BASE FLIGHT DYNAMICS LABORATORY VERSION, 22 JULY 1968

SOLUTION AT POINTS ALONG MERIDIAN AT EIGENVALUE PARAMETER OMEGA= 0.60000000E-04 FOR WAVE NUMBER NX= 5

MAIN SHELL PART NO 1
OMEGA= 0.60000E-04 OMSQ= 0. XMR= 0.60000E-04 LC= -1 PRESTRESS= -0.1000000E 01 -0.

| S | W | G | UPHI | VPHI | BPHI | MPHI | UTHETA | N | NTHETA | MTHETA |
|---|---|---|---|---|---|---|---|---|---|---|
| 0. | 0. | -0.71940E-06 | 0.46595E-02 | 0. | 0.13735E-00 | 0. | 0. | 0.20625E-04 | -0. | -0. |
| 0.10000 | -0.13695E-01 | -0.71629E-06 | 0.46127E-02 | -0.10324E-04 | 0.13615E-00 | -0.89402E-08 | 0.27621E-02 | 0.20503E-04 | -0.46367E-06 | -0.37442E-07 |
| 0.20000 | -0.27141E-01 | -0.70036E-06 | 0.44728E-02 | -0.20519E-04 | 0.13231E-00 | -0.18175E-07 | 0.54719E-02 | 0.20103E-04 | -0.10375E-05 | -0.74281E-07 |
| 0.20000 | -0.27136E-01 | -0.69885E-06 | 0.44728E-02 | -0.20517E-04 | 0.13231E-00 | -0.17971E-07 | 0.54718E-02 | 0.20111E-04 | -0.98154E-06 | -0.74232E-07 |
| 0.30000 | -0.40065E-01 | -0.66525E-06 | 0.42418E-02 | -0.30444E-04 | 0.12579E-00 | -0.26999E-07 | 0.80774E-02 | 0.19429E-04 | -0.15531E-05 | -0.10968E-06 |
| 0.40000 | -0.52205E-01 | -0.60782E-06 | 0.39227E-02 | -0.39959E-04 | 0.11663E-00 | -0.35040E-07 | 0.10528E-01 | 0.18485E-04 | -0.19200E-05 | -0.14289E-06 |
| 0.40000 | -0.52201E-01 | -0.61237E-06 | 0.39226E-02 | -0.39962E-04 | 0.11684E-00 | -0.34699E-07 | 0.10528E-01 | 0.18524E-04 | -0.18694E-05 | -0.14282E-06 |
| 0.50000 | -0.63354E-01 | -0.56199E-06 | 0.35193E-02 | -0.48977E-04 | 0.10587E-00 | -0.42181E-07 | 0.12776E-01 | 0.17385E-04 | -0.24147E-05 | -0.17335E-06 |
| 0.60000 | -0.73291E-01 | -0.50105E-06 | 0.30375E-02 | -0.57336E-04 | 0.92384E-01 | -0.49918E-07 | 0.14775E-01 | 0.15893E-04 | -0.31905E-05 | -0.20072E-06 |
| 0.60000 | -0.73289E-01 | -0.51135E-06 | 0.30374E-02 | -0.57344E-04 | 0.92779E-01 | -0.49467E-07 | 0.14775E-01 | 0.15958E-04 | -0.31496E-05 | -0.20064E-06 |
| 0.70000 | -0.81771E-01 | -0.41416E-06 | 0.24841E-02 | -0.64880E-04 | 0.76233E-01 | -0.57485E-07 | 0.16480E-01 | 0.14035E-04 | -0.39717E-05 | -0.22425E-06 |
| 0.80000 | -0.88457E-01 | -0.25478E-06 | 0.18669E-02 | -0.71396E-04 | 0.57510E-01 | -0.60297E-07 | 0.17848E-01 | 0.12020E-04 | -0.33684E-05 | -0.24226E-06 |
| 0.80000 | -0.88457E-01 | -0.26994E-06 | 0.18667E-02 | -0.71409E-04 | 0.58060E-01 | -0.59770E-07 | 0.17848E-01 | 0.12107E-04 | -0.33393E-05 | -0.24217E-06 |
| 0.90000 | -0.93417E-01 | -0.21775E-06 | 0.11930E-02 | -0.77043E-04 | 0.41460E-01 | -0.60117E-07 | 0.18855E-01 | 0.10377E-04 | -0.34493E-05 | -0.25525E-06 |
| 1.00000 | -0.96705E-01 | -0.25099E-06 | 0.47460E-03 | -0.81677E-04 | 0.23230E-01 | -0.68841E-07 | 0.19471E-01 | 0.78677E-05 | -0.65024E-05 | -0.26536E-06 |
| 1.00000 | -0.96708E-01 | -0.27008E-06 | 0.47437E-03 | -0.81694E-04 | 0.23900E-01 | -0.68278E-07 | 0.19472E-01 | 0.79709E-05 | -0.64897E-05 | -0.26527E-06 |
| 1.10000 | -0.97766E-01 | -0.31824E-07 | -0.27444E-03 | -0.84676E-04 | -0.49379E-02 | -0.82871E-07 | 0.19650E-01 | 0.37350E-05 | -0.88243E-05 | -0.27051E-06 |
| 1.20000 | -0.95616E-01 | 0.55941E-06 | -0.10442E-02 | -0.85609E-04 | -0.35726E-01 | -0.66063E-07 | 0.19361E-01 | 0.66590E-06 | -0.12618E-05 | -0.26198E-06 |
| 1.20000 | -0.95619E-01 | 0.53863E-06 | -0.10445E-02 | -0.85628E-04 | -0.35013E-01 | -0.65524E-07 | 0.19362E-01 | 0.77301E-06 | -0.12628E-05 | -0.26190E-06 |
| 1.30000 | -0.91707E-01 | 0.32357E-06 | -0.18324E-02 | -0.86182E-04 | -0.37742E-01 | -0.28662E-07 | 0.18667E-01 | 0.19200E-05 | 0.34336E-05 | -0.24542E-06 |
| 1.40000 | -0.88291E-01 | -0.85143E-06 | -0.26157E-02 | -0.86909E-04 | -0.37129E-01 | -0.71498E-07 | 0.17585E-01 | -0.85710E-06 | -0.18530E-04 | -0.24382E-06 |
| 1.40000 | -0.88297E-01 | -0.87412E-06 | -0.26159E-02 | -0.86930E-04 | -0.36373E-01 | -0.71014E-07 | 0.17586E-01 | -0.74658E-06 | -0.18559E-04 | -0.24376E-06 |
| 1.50000 | -0.81552E-01 | 0.13560E-06 | -0.33510E-02 | -0.83025E-04 | -0.11494E-00 | -0.17297E-06 | 0.15925E-01 | -0.16316E-04 | -0.34977E-04 | -0.24324E-06 |
| 1.60000 | -0.64516E-01 | 0.40574E-05 | -0.40451E-02 | -0.72120E-04 | -0.20755E-00 | -0.41388E-07 | 0.13611E-01 | -0.21886E-04 | 0.26690E-04 | -0.17594E-06 |
| 1.60000 | -0.64521E-01 | 0.40405E-05 | -0.40453E-02 | -0.72134E-04 | -0.20700E-00 | -0.41059E-07 | 0.13611E-01 | -0.21807E-04 | 0.26670E-04 | -0.17589E-06 |
| 1.70000 | -0.48231E-01 | 0.17999E-05 | -0.47672E-02 | -0.67853E-04 | -0.70946E-01 | 0.27281E-06 | 0.11227E-01 | 0.92798E-05 | 0.75233E-04 | -0.80225E-07 |
| 1.80000 | -0.53084E-01 | -0.90113E-05 | -0.54404E-02 | -0.77242E-04 | 0.11946E-00 | -0.60904E-07 | 0.90683E-02 | 0.13501E-04 | -0.97676E-04 | -0.15041E-06 |
| 1.80000 | -0.53097E-01 | -0.90316E-05 | -0.54406E-02 | -0.77264E-04 | 0.12008E-00 | -0.60740E-07 | 0.90692E-02 | 0.13584E-04 | -0.97772E-04 | -0.15042E-06 |
| 1.90000 | -0.49414E-01 | -0.27871E-05 | -0.56070E-02 | -0.62022E-04 | -0.33205E-00 | -0.94435E-06 | 0.55917E-02 | -0.88004E-04 | -0.24529E-03 | -0.28981E-06 |
| 2.00000 | 0.22078E-01 | 0.28761E-04 | -0.60128E-02 | 0.76587E-08 | -0.96469E 00 | -0.39626E-09 | -0.33673E-06 | -0.11800E-03 | 0.24174E-03 | 0.60324E-07 |

STABILITY AND FREE VIBRATION OF SHELLS WITH AXISYMMETRIC PRESTRESS, BY A. KALNINS, LEHIGH UNIV, BETHLEHEM, PA
WRIGHT PATTERSON AIR FORCE BASE FLIGHT DYNAMICS LABORATORY VERSION, 22 JULY 1968

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SOLUTION AT POINTS ALONG MERIDIAN AT EIGENVALUE PARAMETER OMEGA= 0.7000000E-04  FOR WAVE NUMBER NX=  5

    S          W          Q          UPHI        NPHI        BPHI        MPHI       UTHETA        N         NTHETA      MTHETA

  MAIN SHELL PART NC  1
OMEGA= 0.70000E-04  OMSQ= 0.           XMN= 0.70000E-04  LC= -1  PRESTRESS=  -0.1000000E 01  -0.

  0.       0.          -0.40827E-04 0.10838E-01 0.           0.15737E 01 0.           0.           0.29560E-03-0.           -0.
  0.10000-0.11415E-00-0.34361E-05 0.10943E-01-0.10707E-03 0.37868E-00-0.22014E-05 0.10005E-01 0.69210E-04-0.71999E-03-0.67439E-06
  0.20000-0.71812E-01 0.37939E-04 0.10797E-01-0.66482E-04-0.92143E 00-0.20791E-06 0.13739E-01-0.17258E-03-0.45205E-04-0.22363E-06

  0.20000-0.71749E-01 0.37934E-04 0.10797E-01-0.66440E-04-0.92138E 00-0.20557E-06 0.13738E-01-0.17256E-03-0.44609E-04-0.22307E-06
  0.30000-0.17274E-01 0.30634E-05 0.96452E-02-0.16994E-04 0.19570E-00 0.21767E-05 0.16822E-01 0.42590E-04 0.72899E-03 0.32546E-06
  0.40000-0.12105E-00-0.43287E-04 0.85665E-02-0.11625E-03 0.15841E 01 0.20594E-06 0.25651E-01 0.29543E-03 0.59464E-04-0.28268E-06

  0.40000-0.12108E-00-0.43296E-04 0.85665E-02-0.11627E-03 0.15842E 01 0.20483E-06 0.25651E-01 0.29541E-03 0.59205E-04-0.28294E-06
  0.50000-0.24141E-00-0.88470E-05 0.76134E-02-0.22557E-03 0.41106E-00-0.25188E-05 0.34342E-01 0.65970E-04-0.80085E-03-0.10622E-05
  0.60000-0.18619E-00 0.44367E-04 0.65871E-02-0.17035E-03-0.12190E 01-0.56466E-06 0.35560E-01-0.22736E-03-0.12034E-03-0.58412E-06

  0.60000-0.18614E-00 0.44352E-04 0.65870E-02-0.17032E-03-0.12187E 01-0.56256E-06 0.35558E-01-0.22732E-03-0.11983E-03-0.58363E-06
  0.70000-0.97821E-01 0.93180E-05 0.47913E-02-0.91633E-04-0.83235E-01 0.24872E-05 0.35081E-01-0.34094E-05 0.83426E-03 0.16699E-06
  0.80000-0.19184E-00-0.50890E-04 0.27319E-02-0.18227E-03 0.16475E 01 0.38619E-06 0.40811E-01 0.30154E-03 0.10335E-03-0.43791E-06

  0.80000-0.19192E-00-0.50938E-04 0.27316E-02-0.18234E-03 0.16486E 01 0.38351E-06 0.40814E-01 0.30166E-03 0.10257E-03-0.43858E-06
  0.90000-0.31586E-00-0.12236E-04 0.12794E-02-0.29017E-03 0.30142E-00-0.30972E-05 0.46385E-01 0.35375E-04-0.96753E-03-0.13558E-05
  1.00000-0.22536E-00 0.57073E-04-0.35942E-05-0.19986E-03-0.17594E 01-0.76427E-06 0.42872E-01-0.32529E-03-0.15385E-03-0.72061E-06

  1.00000-0.22531E-00 0.57053E-04-0.37402E-05-0.19983E-03-0.17590E 01-0.76202E-06 0.42871E-01-0.32524E-03-0.15332E-03-0.72008E-06
  1.10000-0.90051E-01 0.13656E-04-0.20230E-02-0.80479E-04-0.34736E-00 0.31745E-05 0.37308E-01-0.46694E-04 0.10431E-02 0.30423E-06
  1.20000-0.18552E-00-0.64367E-04-0.41776E-02-0.17206E-03 0.18506E 01 0.48716E-06 0.39637E-01 0.33125E-03 0.10997E-03-0.40440E-06

  1.20000-0.18563E-00-0.64427E-04-0.41774E-02-0.17215E-03 0.18519E 01 0.48353E-06 0.39641E-01 0.33140E-03 0.10894E-03-0.40531E-06
  1.30000-0.31651E-00-0.12640E-04-0.53349E-02-0.28103E-03 0.91948E-01-0.39342E-05 0.41860E-01-0.12951E-04-0.12209E-02-0.14998E-05
  1.40000-0.17398E-00 0.75880E-04-0.61111E-02-0.14142E-03-0.24587E 01-0.71481E-06 0.33064E-01-0.44855E-03-0.11790E-03-0.57704E-06

  1.40000-0.17386E-00 0.75868E-04-0.61112E-02-0.14137E-03-0.24584E 01-0.71186E-06 0.33062E-01-0.44852E-03-0.11717E-03-0.57634E-06
  1.50000 0.96794E-02 0.11462E-04-0.77058E-02 0.10262E-04-0.45471E-00 0.42573E-05 0.22038E-01-0.59260E-04 0.13594E-02 0.74994E-06
  1.60000-0.11136E-00-0.84585E-04-0.92904E-02-0.96661E-04 0.22924E 01 0.40910E-06 0.23447E-01 0.40405E-03 0.47976E-04-0.22351E-06

  1.60000-0.11145E-00-0.84605E-04-0.92910E-02-0.96701E-04 0.22926E 01 0.40618E-06 0.23449E-01 0.40402E-03 0.47286E-04-0.22419E-06
  1.70000-0.25861E-00-0.87149E-05-0.94565E-02-0.21543E-03-0.10494E-00-0.56591E-05 0.23764E-01-0.71291E-04-0.15667E-02-0.15392E-05
  1.80000-0.51547E-01 0.10072E-03-0.91695E-02-0.17391E-04-0.32427E 01-0.32922E-06 0.10692E-01-0.50130E-03 0.10007E-04-0.19013E-06

  1.80000-0.51430E-01 0.10373E-03-0.91694E-02-0.17307E-04-0.32428E 01-0.32409E-06 0.10688E-01-0.50139E-03 0.19134E-04-0.18908E-06
  1.90000 0.17044E-00 0.70136E-05-0.94676E-02 0.17094E-03-0.27136E-00 0.57245E-05-0.20804E-02-0.15571E-04 0.17810E-02 0.14211E-05
  2.00000-0.11391E-01-0.11217E-03-0.10637E-01 0.32120E-07 0.30395E 01 0.33467E-06-0.13554E-05 0.53219E-03-0.12480E-03-0.30591E-07

MAX OF 1ST PRESCRIBED VARIABLE AT FINAL EDGE IS 0.31651E-00 DET= 0.11463E-01  ACC= 0.10000E-01

THIS WAS ITERATION NO  1  ACCURACY NOT ATTAINED
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          STABILITY AND FREE VIBRATION OF SHELLS WITH AXISYMMETRIC PRESTRESS, BY A. KALNINS, LEHIGH UNIV, BETHLEHEM, PA
                        WRIGHT PATTERSON AIR FORCE BASE FLIGHT DYNAMICS LABORATORY VERSION, 22 JULY 1968

SOLUTION AT POINTS ALONG MERIDIAN AT EIGENVALUE PARAMETER OMEGA= 0.6730140E-04   FOR WAVE NUMBER NX=  5
   S          W            Q            UPHI         NPHI         BPHI         MPHI       UTHETA        N          NTHETA       MTHETA

  MAIN SHELL PART NO  1
OMEGA= 0.67301E-04  OMSQ= 0.               XMR= 0.67301E-04  LC= -1  PRESTRESS=  -0.1000000E 01  -0.

  0.       0.           -0.13579E-05 0.85269E-02 0.          0.25989E-00 0.          0.          0.45915E-04-0.          -0.
  0.10000-0.25916E-01-0.13665E-05 0.84230E-02-0.22957E-04 0.25764E-00-0.17084E-07 0.52248E-02 0.45541E-04-0.15588E-05-0.70883E-07
  0.20000-0.51329E-01-0.13545E-05 0.81142E-02-0.45480E-04 0.24937E-00-0.36891E-07 0.10328E-01 0.44090E-04-0.42055E-05-0.14091E-06

  0.20000-0.51322E-01-0.13550E-05 0.81142E-02-0.45478E-04 0.24942E-00-0.36600E-07 0.10328E-01 0.44103E-04-0.41411E-05-0.14084E-06
  0.30000-0.75520E-01-0.12575E-05 0.76085E-02-0.66919E-04 0.23309E-00-0.57101E-07 0.15180E-01 0.41194E-04-0.69977E-05-0.20780E-06
  0.40000-0.97709E-01-0.10719E-05 0.69178E-02-0.86595E-04 0.20999E-00-0.71850E-07 0.19658E-01 0.37212E-04-0.80955E-05-0.26851E-06

  0.40000-0.97708E-01-0.10813E-05 0.69177E-02-0.86599E-04 0.21024E-00-0.71621E-07 0.19658E-01 0.37250E-04-0.80725E-05-0.26846E-06
  0.50000-0.11749E-00-0.95827E-06 0.60574E-02-0.10421E-03 0.18518E-00-0.81880E-07 0.23660E-01 0.32982E-04-0.85326E-05-0.32209E-06
  0.60000-0.13466E-00-0.89995E-06 0.50482E-02-0.11956E-03 0.15728E-00-0.96083E-07 0.27096E-01 0.28039E-04-0.10975E-04-0.36955E-06

  0.60000-0.13466E-00-0.91762E-06 0.50481E-02-0.11957E-03 0.15772E-00-0.95944E-07 0.27096E-01 0.28100E-04-0.10998E-04-0.36954E-06
  0.70000-0.14868E-00-0.69235E-06 0.39163E-02-0.13206E-03 0.12069E-00-0.11459E-06 0.29875E-01 0.21447E-04-0.14435E-04-0.40946E-06
  0.80000-0.15858E-00-0.27744E-06 0.26885E-02-0.14095E-03 0.77639E-01-0.11745E-06 0.31908E-01 0.14124E-04-0.13038E-04-0.43592E-06

  0.80000-0.15859E-00-0.30119E-06 0.26884E-02-0.14096E-03 0.78222E-01-0.11741E-06 0.31909E-01 0.14200E-04-0.13100E-04-0.43594E-06
  0.90000-0.16454E-00-0.15008E-06 0.13908E-02-0.14653E-03 0.42508E-01-0.10915E-06 0.33168E-01 0.82629E-05-0.10243E-04-0.45015E-06
  1.00000-0.16724E-00-0.26406E-06 0.55362E-04-0.14926E-03 0.10307E-01-0.11776E-06 0.33647E-01 0.23137E-05-0.14004E-04-0.45870E-06

  1.00000-0.16725E-00-0.29237E-06 0.55169E-04-0.14928E-03 0.10989E-01-0.11783E-06 0.33647E-01 0.23997E-05-0.14108E-04-0.45874E-06
  1.10000-0.16608E-00 0.78132E-07-0.12802E-02-0.14833E-03-0.37994E-01-0.14075E-06 0.33300E-01-0.67060E-05-0.20176E-04-0.45957E-06
  1.20000-0.15939E-00 0.90483E-06-0.25850E-02-0.14254E-03-0.93255E-01-0.12091E-06 0.32085E-01-0.15754E-04-0.12495E-04-0.43862E-06

  1.20000-0.15941E-00 0.87613E-06-0.25852E-02-0.14256E-03-0.92584E-01-0.12106E-06 0.32086E-01-0.15672E-04-0.12608E-04-0.43869E-06
  1.30000-0.14863E-00 0.80838E-06-0.38379E-02-0.13362E-03-0.11661E-00-0.75009E-07 0.30096E-01-0.19043E-04-0.20136E-05-0.40260E-06
  1.40000-0.13661E-00-0.12692E-07-0.50046E-02-0.12356E-03-0.12724E-00-0.92786E-07 0.27452E-01-0.22081E-04-0.13501E-04-0.37414E-06

  1.40000-0.13663E-00-0.41440E-07-0.50048E-02-0.12358E-03-0.12657E-00-0.93056E-07 0.27453E-01-0.22000E-04-0.13652E-04-0.37424E-06
  1.50000-0.12150E-00 0.70294E-06-0.60399E-02-0.10991E-03-0.18582E-00-0.14938E-06 0.24103E-01-0.33884E-04-0.29134E-04-0.34410E-06
  1.60000-0.98979E-01 0.26179E-05-0.69253E-02-0.89825E-04-0.25577E-00-0.80319E-07 0.20000E-01-0.44094E-04-0.38400E-05-0.27321E-06

  1.60000-0.98993E-01 0.25960E-05-0.69255E-02-0.89841E-04-0.25527E-00-0.80576E-07 0.20000E-01-0.44038E-04-0.39619E-05-0.27329E-06
  1.70000-0.73896E-01 0.16312E-05-0.76692E-02-0.68876E-04-0.22815E-00 0.49926E-07 0.15452E-01-0.37005E-04 0.25315E-04-0.18534E-06
  1.80000-0.54446E-01-0.12646E-05-0.82382E-02-0.52556E-04-0.17478E-00-0.33424E-07 0.10770E-01-0.31583E-04-0.15281E-04-0.14861E-06

  1.80000-0.54464E-01-0.12844E-05-0.82383E-02-0.52573E-04-0.17433E-00-0.33848E-07 0.10771E-01-0.31534E-04-0.15444E-04-0.14873E-06
  1.90000-0.33336E-01 0.88996E-06-0.85671E-02-0.32210E-04-0.27715E-00-0.20956E-06 0.56910E-02-0.53573E-04-0.58822E-04-0.12307E-06
  2.00000 0.19257E-02 0.60120E-05-0.86759E-02 0.14099E-06-0.39880E-00 0.15903E-09-0.21959E-07-0.68078E-04 0.21086E-04 0.52939E-08

MAX OF 1ST PRESCRIBED VARIABLE AT FINAL EDGE IS 0.16725E-00 DET= 0.19221E-02  ACC= 0.10000E-01

THIS WAS ITERATION NO  2  ACCURACY NOT ATTAINED
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STABILITY AND FREE VIBRATION OF SHELLS WITH AXISYMMETRIC PRESTRESS, BY A. KALNINS, LEHIGH UNIV, BETHLEHEM, PA
WRIGHT PATTERSON AIR FORCE BASE FLIGHT DYNAMICS LABORATORY VERSION, 22 JULY 1968

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SOLUTION AT POINTS ALONG MERIDIAN AT EIGENVALUE PARAMETER OMEGA= 0.67728062E-04  FOR WAVE NUMBER N= 5

    S          W          G         UPHI        VPHI        BPHI        MPHI      UTHETA         N        NTHETA      MTHETA

  MAIN SHELL PART NO  1
OMEGA= 0.67729E-04  OMSQ= 0.             XMR= 0.67729E-04  LC= -1  PRESTRESS=  -0.10000000E 01  -0.

 0.      0.          -0.14468E-05 0.88826E-02 0.           0.27235E-00 0.          0.          0.40600E-04-0.           -0.
 0.10000-0.27127E-01-0.14327E-05 0.87733E-02-0.24260E-04 0.26909E-00-0.19620E-07 0.54577E-02 0.40022E-04-0.22015E-05-0.76409E-07
 0.20000-0.53595E-01-0.13821E-05 0.84481E-02-0.47946E-04 0.25916E-00-0.39132E-07 0.10781E-01 0.46245E-04-0.45977E-05-0.14723E-06

 0.20000-0.53589E-01-0.13828E-05 0.84481E-02-0.47944E-04 0.25921E-00-0.38835E-07 0.10781E-01 0.46250E-04-0.45325E-05-0.14717E-06
 0.30000-0.78746E-01-0.12966E-05 0.79151E-02-0.70446E-04 0.24282E-00-0.57677E-07 0.15840E-01 0.43325E-04-0.60075E-05-0.21636E-06
 0.40000-0.10194E-00-0.11650E-05 0.71874E-02-0.91197E-04 0.22018E-00-0.74401E-07 0.20508E-01 0.39301E-04-0.87076E-05-0.28000E-06

 0.40000-0.10194E-00-0.11752E-05 0.71873E-02-0.91201E-04 0.22045E-00-0.74265E-07 0.20508E-01 0.39340E-04-0.86803E-05-0.28002E-06
 0.50000-0.12764E-00-0.10242E-05 0.62826E-02-0.10972E-03 0.19264E-00-0.89432E-07 0.24672E-01 0.34308E-04-0.10452E-04-0.33609E-06
 0.60000-0.14031E-00-0.84995E-06 0.52232E-02-0.12554E-03 0.16017E-00-0.10206E-06 0.28228E-01 0.28607E-04-0.11902E-04-0.38940E-06

 0.60000-0.14032E-00-0.86860E-06 0.52231E-02-0.12555E-03 0.16064E-00-0.10195E-06 0.28229E-01 0.28669E-04-0.11931E-04-0.38939E-06
 0.70000-0.15458E-00-0.66451E-06 0.40352E-02-0.13830E-03 0.12380E-00-0.11365E-06 0.31092E-01 0.22087E-04-0.13470E-04-0.42479E-06
 0.80000-0.16497E-00-0.42341E-06 0.27481E-02-0.14760E-03 0.83594E-01-0.12031E-06 0.33108E-01 0.14955E-04-0.13996E-04-0.45316E-06

 0.80000-0.16497E-00-0.44867E-06 0.27479E-02-0.14761E-03 0.84203E-01-0.12031E-06 0.33108E-01 0.15033E-04-0.14069E-04-0.45319E-06
 0.90000-0.17133E-00-0.22123E-06 0.13924E-02-0.15331E-03 0.42621E-01-0.12465E-06 0.34468E-01 0.76435E-05-0.14494E-04-0.47059E-06
 1.00000-0.17348E-00-0.44179E-08 0.32800E-05-0.15525E-03 0.36406E-03-0.12591E-06 0.34901E-01 0.11105E-06-0.14680E-04-0.47644E-06

 1.00000-0.17349E-00-0.33883E-07 0.30850E-05-0.15527E-03 0.10610E-02-0.12602E-06 0.34902E-01 0.19775E-06-0.14792E-04-0.47650E-06
 1.10000-0.17144E-00 0.21686E-06-0.13864E-02-0.15343E-03-0.42497E-01-0.12706E-06 0.34477E-01-0.76003E-05-0.15248E-04-0.47128E-06
 1.20000-0.16501E-00 0.49924E-06-0.27418E-02-0.14768E-03-0.85523E-01-0.12039E-06 0.33199E-01-0.15199E-04-0.13913E-04-0.45320E-06

 1.20000-0.16502E-00 0.46850E-06-0.27420E-02-0.14771E-03-0.84806E-01-0.12061E-06 0.33198E-01-0.15111E-04-0.14050E-04-0.45336E-06
 1.30000-0.15456E-00 0.67260E-06-0.40306E-02-0.13838E-03-0.12370E-00-0.11093E-06 0.31104E-01-0.21985E-04-0.12591E-04-0.42427E-06
 1.40000-0.14042E-00 0.81043E-06-0.52206E-02-0.12576E-03-0.15969E-00-0.10154E-06 0.28249E-01-0.28276E-04-0.11956E-04-0.38559E-06

 1.40000-0.14044E-00 0.78134E-06-0.52208E-02-0.12578E-03-0.15802E-00-0.10194E-06 0.28250E-01-0.28198E-04-0.12110E-04-0.38568E-06
 1.50000-0.12290E-00 0.10082E-05-0.62815E-02-0.11007E-03-0.19278E-00-0.94298E-07 0.24698E-01-0.34479E-04-0.11914E-04-0.33840E-06
 1.60000-0.10200E-00 0.12872E-05-0.71975E-02-0.91366E-04-0.22370E-00-0.74603E-07 0.20527E-01-0.39834E-04-0.83165E-05-0.28023E-06

 1.60000-0.10202E-00 0.12629E-05-0.71977E-02-0.91386E-04-0.22316E-00-0.74951E-07 0.20528E-01-0.39772E-04-0.84661E-05-0.28033E-06
 1.70000-0.78628E-01 0.13190E-05-0.79186E-02-0.70534E-04-0.24258E-00-0.50943E-07 0.15854E-01-0.43095E-04-0.47282E-05-0.21490E-06
 1.80000-0.53762E-01 0.12232E-05-0.84553E-02-0.48346E-04-0.25458E-00-0.38496E-07 0.10807E-01-0.45462E-04-0.51023E-05-0.14757E-06

 1.80000-0.53777E-01 0.12056E-05-0.84554E-02-0.48360E-04-0.25420E-00-0.38871E-07 0.10807E-01-0.45421E-04-0.52390E-05-0.14767E-06
 1.90000-0.27594E-01 0.14069E-05-0.87416E-02-0.24836E-04-0.27028E-00-0.32008E-07 0.54861E-02-0.48501E-04-0.59413E-05-0.77822E-07
 2.00000 0.11970E-03 0.17407E-05-0.86911E-02 0.75456E-04-0.28101E-00 0.33411E-09-0.17307E-06-0.49994E-04 0.13025E-05 0.38312E-09

MAX OF 1ST PRESCRIBED VARIABLE AT FINAL EDGE IS 0.17349E-00 DET= 0.10990E-03  ACC= 0.10000E-01

OMEGA= 0.67729E-04 IS AN EIGENVALUE
```

STABILITY AND FREE VIBRATION OF SHELLS WITH AXISYMMETRIC PRESTRESS, BY A. KALNINS, LEHIGH UNIV, BETHLEHEM, PA
WRIGHT PATTERSON AIR FORCE BASE FLIGHT DYNAMICS LABORATORY VERSION, 22 JULY 1968

SUMMARY OF RESULTS FOR N= 5

| | OMEGA | ACTUAL DET | ADJUSTED DET | NORMAL DET | XMC | XME |
|---|---|---|---|---|---|---|
| | 0.5000000E-04 | 0.3418640E-01 | 0.3418640E-01 | -0.9691896E 05 | 1.0 | 1.0 |
| | 0.6000000E-04 | 0.2208006E-01 | -0.2208006E-01 | -0.3069234E 05 | 1.0 | -1.0 |
| | 0.7000000E-04 | -0.1146311E-01 | 0.1146311E-01 | 0.2807757E 03 | 1.0 | -1.0 |
| | 0.6730140E-04 | 0.1922064E-02 | -0.1922064E-02 | -0.1415475E 04 | 1.0 | -1.0 |
| EIGENVALUE AT | 0.6772862E-04 | 0.1090020E-03 | -0.1090020E-03 | -0.7533914E 02 | 1.0 | -1.0 |

3. Nonsymmetric Eigenvalue Program

G. Stability of Cylindrical Shell Under Bending

This case seeks the buckling load for a circular cylindrical shell subject to pure bending by couples applied to the ends of the shell. Since few analyses of nonsymmetric prestress problems exist, this example was selected for comparison with the results obtained by Seide and Weingarten* for a simply supported cylindrical shell.

The prestress state used as a model for this problem is an axial stress resultant, constant along a meridian but varying as COS θ at each cross section. Thus a convenient eigenvalue parameter for this problem is the theoretical buckling stress resultant for a long, axially loaded cylinder,

$$N_c = \frac{Et^2}{\sqrt{3(1-\nu^2)}R}$$

For a prestress = 1.0, radius to thickness ratio = 100, Poisson's ratio = 0.3, and E = 1653, the eigenvalue for this case represents the ratio of the critical stress in bending to that of a long cylinder in axial compression.

*P. Seide and V. Weingarten, "On the Buckling of Circular Shells Under Pure Bending", Journal of Applied Mechanics, March 1961, pp. 112-116.

A four component solution of wave numbers 4, 5, 6, 7 was used. These terms were selected on the basis of the form of deflection function used by Flugge for a similar problem. At the mid-span cross section, this solution predicted a mode shape similar to that of Figure 24, page 456 in Flugge, Stresses in Shells, Springer Verlag, 1962 Edition.

For a length to radius ratio = 1, this four term solution yielded an eigenvalue ALFA = 1.61. Seide and Weingarten, using a twelve term solution which neglects end effects, obtained a corresponding value of 1.015. As in the case of an axially loaded cylindrical shell, the actual collapse load is much less than the theoretical value.

TEST CASE FOR NONSYMMETRIC EIGENVALUE PROGRAM

1. CYLINDRICAL SHELL - BUCKLING WITH NONSYMMETRIC PRESTRESS

```
    1     0     0     1     1     0     1
0.0        10.0             6     9     2     0     1
0.10      10.0       90.0
1653.0     0.3
-0.05     0.05
    1     2    -1
 0.0                  1.0
10.0                 1.0
1.60       0.3       1.90             1     4     4
    1     4     6     7     2     3     5     8
    2     3     5     8     1     4     6     7
```

```
        NONSYMMETRIC (PRESTRESS) EIGENVALUE PROGRAM JAN 1969 VERSION

        PART NO.  1

SI= 0.          SF= 0.10000E 02    6 SEGMENTS  9 POINTS  SHELL NO.  2    1 LAYERS

CYLINDRICAL SHELL NO  2  H/L=  0.100000  R/L= 10.000000  PHI= 90.0

YOUNG S MODULUS/EREF=  0.1653C000E 04  POISSONSS RATIO=  0.300000

LAYER  1 Z= -0.050000 TO Z=  C.050000
 B11= 0.182E 04 B12= 0.545E 03 B22= 0.182E 04 B66= 0.636E 03


                  PRESTRESS VARIABLES ARE

       X              NPHI           NTHETA         NTHETA PHI
 0.                0.09999999E 01 -0.               -0.
 0.09999999E 02    0.09999999E 01 -0.               -0.
```

```
          NONSYMMETRIC (PRESTRESS) EIGENVALUE PROGRAM JAN 1969 VERSION
STARTING OMEGA=-0.00000E-19  INCREMENT=-0.00000E-19  FINAL OMEGA=-0.00000E-1934572 EIGENVALUES
          EIGENVALUE ANALYSIS    1 PARTS,    4  WAVE NUMBERS    4    5    6    7

AT STARTING EDGE, PRESCRIBED VARIABLES ARE    1    4    6    7    UNPRESCRIBED VARIABLES ARE    2    3    5    8
AT FINAL EDGE, UNPRESCRIBED VARIABLES ARE     2    3    5    8      PRESCRIBED VARIABLES ARE    1    4    6    7
```

NONSYMMETRIC (PRESTRESS) EIGENVALUE PROGRAM JAN 1969 VERSION

```
STABILITY ANALYSIS WITH OMEGA= 0.16000E 01

UNDRFLOW AT 30034 IN MQ

UNDRFLOW AT 30426 IN MQ

UNDRFLOW AT 30411 IN MQ

UNDRFLOW AT 30415 IN AC

UNDRFLOW AT 30426 IN MC
```



```
DETERMINANT OF CK MATRIX AT OMEGA=  0.16000E 01 IS -0.00000E-19
                         SOLUTION FOR  4 FOURRIER COMPONENTS FOR EACHVARIABLE FOLLOWS
         2          3          5          8          1          4          6          7
X= 0.10000E 02

 0.100000E 01-0.173880E-01-0.129124E 01-0.906146E 01-0.277865E 01 0.319481E-04 0.152737E-06-0.405312E-05
-0.253734E 01 0.100329E-00 0.411124E 01 0.131928E 02-0.233650E-04-0.238419E-04-0.171363E-06 0.333786E-05
 0.291340E 01-0.180740E-00-0.523504E 01-0.178470E 02 0.200272E-04 0.476837E-05 0.108965E-06-0.214577E-05
-0.199301E 01 0.132368E-00 0.367832E 01 0.120517E 02-0.429153E-05 0.953674E-06-0.745058E-07 0.864267E-06

-0.876183E 00 0.100006E 01-0.173876E-01 0.184029E-05-0.129112E 01-0.208332E-04-0.124989E-05-0.906155E 01
 0.370648E-04-0.253715E 01 0.100330E-00 0.679977E-04 0.411133E 01 0.146322E-03-0.476255E-06 0.131926E 02
-0.114225E-03 0.291349E 01-0.180740E-00-0.438243E-04-0.523489E 01-0.337660E-04-0.477768E-06-0.178471E 02
 0.276342E-04-0.199290E 01 0.132369E-00 0.929348E-04 0.367834E 01 0.788588E-04-0.768108E-06 0.120515E 02
         1          2          3          4          5          6          7          8
              MODE SHAPE  W AT TOP OF SHELL=-0.87623E 00

-0.67122E 00-0.15212E-00 0.43815E-00 0.82337E 00 0.82330E 00 0.43801E-00-0.15218E-00-0.67113E 00-0.87607E 00

-0.67114E 00-0.15223E-00 0.43794E-00 0.82327E 00 0.82343E 00 0.43827E-00-0.15205E-00-0.67128E 00-0.87636E 00

X= 0.83333E 01
```

```
-0.472533E-00-0.720571E 00-0.406089E-01-0.241324E 01 0.797196E 00 0.386872E-00 0.118288E-00-0.239599E-01
 0.140454E 01 0.125485E 01 0.104890E-00 0.579011E 01-0.889701E 00-0.498813E-00-0.272187E-00 0.104187E 01
-0.276160E 01-0.127242E 01-0.164955E-00-0.970763E 01 0.553145E 00 0.372574E-00 0.435279E-00-0.142506E 01
 0.228831E 01 0.776565E 00 0.111881E-00 0.736184E 01-0.184528E-00-0.158805E-00-0.314833E-00 0.367004E-00

-0.472460E-00-0.720590E 00-0.406085E-01-0.241323E 01 0.796867E 00 0.386818E-00 0.118288E-00-0.240389E-01
 0.140459E 01 0.125433E 01 0.104888E-00 0.579006E 01-0.889700E 00-0.498946E-00-0.272186E-00 0.104238E 01
-0.276149E 01-0.127240E 01-0.164955E-00-0.970770E 01 0.552694E 00 0.372515E-00 0.435281E-00-0.142502E 01
 0.228835E 01 0.776292E 00 0.111879E-00 0.736170E 01-0.184497E-00-0.158863E-00-0.314831E-00 0.367583E-00

                   MODE SHAPE  W AT TOP OF SHELL= 0.45898E-00

-0.57158E-01-0.69817E 00-0.20046E-00 0.90220E 00 0.83641E 00-0.67879E 00-0.15505E 01-0.55560E-01 0.22890E 01

 0.20932E 01-0.13751E 01-0.43717E 01-0.27100E 01 0.27472E 01 0.61959E 01 0.32957E 01-0.34281E 01-0.69269E 01


X= 0.66667E 01

-0.898135E 00-0.732078E 00-0.296316E-01-0.200490E 01 0.111781E 01-0.473755E-00 0.123120E-00 0.282258E 01
 0.219612E 01 0.122011E 01 0.679375E-01 0.519130E 01-0.180161E 01 0.759869E 00-0.320939E-00-0.527604E 01
-0.359628E 01-0.126120E 01-0.925282E-01-0.824105E 01 0.181608E 01-0.768874E 00 0.520803E 00 0.690226E 01
 0.279300E 01 0.816343E 00 0.562348E-01 0.543411E 01-0.114801E 01 0.465871E-00-0.365648E-00-0.519350E 01

-0.897902E 00-0.732649E 00-0.296343E-01-0.200492E 01 0.111791E 01-0.473676E-00 0.123117E-00 0.282261E 01
 0.219619E 01 0.121991E 01 0.679344E-01 0.519105E 01-0.180168E 01 0.759702E 00-0.320927E-00-0.527531E 01
-0.359598E 01-0.126217E 01-0.925288E-01-0.824101E 01 0.181677E 01-0.768945E 00 0.520806E 00 0.690210E 01
 0.279304E 01 0.816203E 00 0.562339E-01 0.543399E 01-0.114806E 01 0.486045E-00-0.365630E-00-0.519298E 01

                   MODE SHAPE  W AT TOP OF SHELL= 0.49535E-00

-0.11886E-00-0.87888E 00-0.27587E-00 0.10630E 01 0.10452E 01-0.65241E 00-0.15864E 01 0.16792E-00 0.26981E 01

 0.20524E 01-0.23214E 01-0.56416E 01-0.29538E 01 0.42204E 01 0.83657E 01 0.41631E 01-0.48527E 01-0.94831E 01


X= 0.50000E 01

 0.307182E-01 0.141467E 01 0.552079E-02-0.112060E 01-0.151987E 01 0.100168E-00 0.989066E-01-0.223107E 01
 0.751025E 00-0.229399E 01-0.212674E-02 0.325874E 01 0.248501E 01-0.165669E-00-0.292021E-00 0.466975E 01
-0.216354E 01 0.229614E 01-0.156929E-02-0.503454E 01-0.247143E 01 0.152878E-00 0.487659E-00-0.512825E 01
 0.190388E 01-0.143147E 01 0.369412E-02 0.257872E 01 0.152615E 01-0.846281E-01-0.337941E-00 0.342318E 01

 0.306158E-01 0.141478E 01 0.552098E-02-0.112062E 01-0.151972E 01 0.100156E-00 0.989073E-01-0.223112E 01
 0.751062E 00-0.229373E 01-0.212595E-02 0.325882E 01 0.248517E 01-0.165486E-00-0.292023E-00 0.466953E 01
-0.216371E 01 0.229628E 01-0.156841E-02-0.503455E 01-0.247120E 01 0.152854E-00 0.487660E-00-0.512848E 01
 0.190390E 01-0.143134E 01 0.369522E-02 0.257889E 01 0.152622E 01-0.845301E-01-0.337944E-00 0.342285E 01

                   MODE SHAPE  W AT TOP OF SHELL= 0.52187E 00

 0.75544E-01-0.50172E 00-0.15086E-00 0.67793E 00 0.50748E 00-0.85153E 00-0.15607E 01-0.10844E-00 0.21943E 01

 0.23191E 01-0.59235E 00-0.35065E 01-0.27288E 01 0.14282E 01 0.44477E 01 0.26761E 01-0.21923E 01-0.47881E 01


X= 0.33333E 01
```

```
-0.242540E-00-0.629266E 00 0.229234E-01-0.245959E 01 0.182999E-00 0.429787E-00 0.143027E-00-0.192601E 01
 0.140645E 01 0.960104E 00-0.527110E-01 0.685822E 01-0.183727E-00-0.642152E 00-0.385050E-00 0.406798E 01
-0.299159E 01-0.885838E 00 0.763872E-01-0.101405E 02 0.110643E-00 0.556659E 00 0.588200E 00-0.496721E 01
 0.255070E 01 0.496225E-00-0.457246E-01 0.683840E 01 0.101457E-01-0.290059E-00-0.413107E-00 0.374788E 01

-0.242427E-00-0.629337E 00 0.229235E-01-0.245960E 01 0.182510E-00 0.429692E-00 0.143027E-00-0.192605E 01
 0.140657E 01 0.959336E 00-0.527138E-01 0.685815E 01-0.163296E-00-0.642345E 00-0.385048E-00 0.406877E 01
-0.299144E 01-0.885880E 00 0.763862E-01-0.101406E 02 0.109978E-00 0.556542E 00 0.588203E 00-0.496707E 01
 0.255077E 01 0.497813E-00-0.457268E-01 0.683823E 01 0.101443E-01-0.290152E-00-0.413104E-00 0.374868E 01

                   MODE SHAPE  * AT TOP OF SHELL= 0.72346E 00

 0.95107E-01-0.74462E 00-0.31450E-00 0.84472E 00 0.76303E 00-0.89155E 00-0.17922E 01-0.94538E-02 0.27490E 01

 0.26295E 01-0.12834E 01-0.48489E 01-0.32989E 01 0.26023E 01 0.65398E 01 0.36520E 01-0.34579E 01-0.71912E 01


X= 0.16667E 01

-0.157006E 01-0.698949E 00 0.329397E-01-0.326059E 01 0.915890E 00-0.570342E 00 0.151509E-00 0.282931E 01
 0.334905E 01 0.116096E 01-0.101818E-00 0.871275E 01-0.176994E 01 0.963687E 00-0.371031E-00-0.604972E 01
-0.453877E 01-0.122560E 01 0.165117E-00-0.126796E 02 0.222756E 01-0.102878E 01 0.525738E 00 0.725822E 01
 0.336602E 01 0.811721E 00-0.114754E-00 0.938592E 01-0.157442E 01 0.679438E 00-0.372350E-00-0.437360E 01

-0.156994E 01-0.699342E 00 0.329382E-01-0.326059E 01 0.915855E 00-0.570369E 00 0.151509E-00 0.282963E 01
 0.334908E 01 0.116082E 01-0.101819E-00 0.871203E 01-0.177053E 01 0.963448E 00-0.371027E-00-0.604965E 01
-0.453854E 01-0.122626E 01 0.165114E-00-0.126798E 02 0.222739E 01-0.102886E 01 0.525739E 00 0.725908E 01
 0.336607E 01 0.811603E 00-0.114755E-00 0.938577E 01-0.157484E 01 0.679274E 00-0.372347E-00-0.437338E 01


      1          4          6          7          2          3          5          8

X= 0.
 0.         0.         0.         0.          0.127487E 01 0.418481E-01-0.504237E 00 0.458942E 01
 0.         0.         0.         0.         -0.194447E 01-0.132561E-00 0.156192E-00-0.101289E 02
 0.         0.         0.         0.          0.174191E 01 0.215807E-00 0.781032E 00 0.125808E 02
 0.         0.         0.         0.         -0.934925E 00-0.154992E-00-0.943044E 00-0.799817E 01
```

NONSYMMETRIC (PRESTRESS) EIGENVALUE PROGRAM JAN 1969 VERSION

```
STABILITY ANALYSIS WITH OMEGA= 0.19000E 01

DETERMINANT OF CM MATRIX AT OMEGA=  0.19000E 01 IS -0.00000E-19
                              SOLUTION FOR  4 FOURRIER COMPONENTS FOR EACHVARIABLE FOLLOWS
      2           3           5           8           1           4           6           7
X= 0.10000E 02

 0.100000E 01-0.102677E 09-0.161430E 11 0.138520E 12 0.112114E 03-0.305176E-04-0.190735E-05-0.114441E-04
 0.852378E 10-0.640826E 09-0.260421E 11-0.151708E 12 0.152588E-04 0.305176E-04-0.953674E-06 0.152588E-04
-0.189493E 11 0.202506E 10 0.554115E 11 0.282478E 12 0.762939E-04 0.915527E-04 0.476837E-05-0.953674E-05
 0.166758E 11-0.184736E 10-0.471234E 11-0.213871E 12 0.109673E-04-0.457764E-04-0.953674E-06 0.405312E-05

-0.699489E 10 0.196167E 08-0.102638E 09-0.592352E 11-0.161290E 11-0.426503E 10 0.694634E 10 0.138512E 12
-0.567648E 11 0.855655E 10-0.640747E 09-0.990372E 11-0.260196E 11-0.563435E 09 0.110875E 11-0.151729E 12
 0.954880E 11-0.189176E 11 0.202516E 10 0.148749E 12 0.554360E 11 0.364299E 10-0.152831E 11 0.282450E 12
-0.599431E 11 0.166962E 11-0.184729E 10-0.957079E 11-0.471080E 11-0.330580E 10 0.815957E 10-0.213895E 12
      1           2           3           4           5           6           7           8
                  MODE SHAPE  W AT TOP OF SHELL=-0.28215E 11

-0.14604E 11 0.68175E 10 0.90814E 10 0.17615E 10 0.14699E 11 0.40631E 11 0.29158E 11-0.40258E 11-0.10248E 12

-0.65947E 11 0.63900E 11 0.15734E 12 0.93936E 11-0.84101E 11-0.19306E 12-0.10474E 12 0.99372E 11 0.20520E 12

X= 0.83333E 01

-0.150589E 11 0.716423E 09 0.658276E 09-0.102253E 11 0.192727E 10-0.215803E 10 0.476603E 10 0.359591E 11
-0.737762E 11-0.428467E 10 0.385287E 09-0.176766E 12 0.149896E 10 0.583275E 09 0.136785E 11-0.243800E 11
 0.132083E 12 0.654766E 10 0.261887E 09 0.308965E 12-0.434322E 10 0.222398E 10-0.207415E 11 0.239546E 11
-0.931149E 11-0.486638E 10-0.510591E 09-0.230990E 12 0.256123E 10-0.289570E 10 0.126955E 11-0.932524E 10

-0.150630E 11 0.731985E 09 0.658312E 09-0.102244E 11 0.194152E 10-0.215220E 10 0.476600E 10 0.359518E 11
-0.737828E 11-0.425288E 10 0.385364E 09-0.176763E 12 0.151719E 10 0.592941E 09 0.136784E 11-0.244004E 11
 0.132076E 12 0.657258E 10 0.261963E 09 0.308969E 12-0.431981E 10 0.223359E 10-0.207416E 11 0.239334E 11
```

```
-0.931192E 11-0.484695E 10-0.510527E 09-0.230985E 12 0.257274E 10-0.286967E 10 0.126954E 11-0.934644E 10

                    MODE SHAPE  W AT TOP OF SHELL=-0.49869E 11

-0.24776E 11 0.15492E 11 0.19997E 11 0.12798E 10 0.13723E 11 0.56156E 11 0.50617E 11-0.46593E 11-0.14714E 12

-0.10856E 12 0.76226E 11 0.22306E 12 0.14666E 12-0.10504E 12-0.26909E 12-0.15280E 12 0.13377E 12 0.28391E 12


X= 0.66667E 01

-0.111863E 11 0.731726E 10 0.984485E 09-0.530223E 10-0.786490E 10 0.120509E 10 0.403036E 10-0.268779E 11
-0.637372E 11-0.126028E 11 0.157481E 10-0.140275E 12 0.249882E 11-0.313140E 10 0.116811E 11 0.117022E 12
 0.110123E 12 0.138020E 11-0.205248E 10 0.232466E 12-0.344605E 11 0.458455E 10-0.177880E 11-0.168993E 12
-0.755189E 11-0.959536E 10 0.126726E 10-0.161774E 12 0.251600E 11-0.361234E 10 0.106440E 11 0.118693E 12

-0.111896E 11 0.733074E 10 0.984519E 09-0.530140E 10-0.765532E 10 0.120938E 10 0.403034E 10-0.268849E 11
-0.637421E 11-0.125815E 11 0.157487E 10-0.140272E 12 0.250042E 11-0.312387E 10 0.116810E 11 0.117008E 12
 0.110118E 12 0.138239E 11-0.205242E 10 0.232470E 12-0.344449E 11 0.459167E 10-0.177881E 11-0.169011E 12
-0.755220E 11-0.958212E 10 0.126730E 10-0.161771E 12 0.251700E 11-0.360755E 10 0.105440E 11 0.118680E 12
                    MODE SHAPE  W AT TOP OF SHELL=-0.40336E 11

-0.20315E 11 0.11920E 11 0.16083E 11 0.22395E 10 0.13000E 11 0.46081E 11 0.38887E 11-0.41492E 11-0.12131E 12

-0.85770E 11 0.67344E 11 0.18534E 12 0.11815E 12-0.91325E 11-0.22513E 12-0.12593E 12 0.11329E 12 0.23819E 12


X= 0.50000E 01

-0.159555E 11-0.155689E 10 0.119983E 10-0.288070E 11-0.147262E 10 0.167027E 10 0.423004E 10-0.370924E 11
-0.319205E 11 0.102838E 10 0.236381E 10-0.285183E 11 0.181984E 11-0.352217E 10 0.646773E 10 0.136201E 12
 0.586323E 11 0.125789E 10-0.340466E 10 0.446512E 11-0.311615E 11 0.395321E 10-0.100108E 11-0.186758E 12
-0.354628E 11-0.241127E 10 0.213041E 10-0.124757E 11 0.244963E 11-0.247858E 10 0.517781E 10 0.124905E 12

-0.159552E 11-0.155779E 10 0.119983E 10-0.288069E 11-0.147459E 10 0.186965E 10 0.423004E 10-0.370926E 11
-0.319199E 11 0.102405E 10 0.236360E 10-0.285184E 11 0.181975E 11-0.352306E 10 0.646773E 10 0.136203E 12
 0.586330E 11 0.125684E 10-0.340467E 10 0.448506E 11-0.311645E 11 0.395233E 10-0.100108E 11-0.186757E 12
-0.354623E 11-0.241362E 10 0.213040E 10-0.124768E 11 0.244958E 11-0.247906E 10 0.517783E 10 0.124907E 12
                    MODE SHAPE  W AT TOP OF SHELL=-0.24704E 11

-0.15552E 11 0.62146E 09 0.76994E 10 0.95134E 10 0.20303E 11 0.32919E 11 0.17906E 11-0.32667E 11-0.74588E 11

-0.50411E 11 0.35185E 11 0.10030E 12 0.68317E 11-0.38159E 11-0.10901E 12-0.64797E 11 0.49739E 11 0.11006E 12


X= 0.33333E 01

-0.195921E 11-0.690536E 10 0.143508E 10-0.456930E 11 0.674145E 10-0.102213E 10 0.400720E 10-0.128551E 10
-0.930472E 09 0.101633E 11 0.244634E 10 0.605434E 11 0.296977E 10 0.178340E 10 0.163701E 10 0.513343E 11
 0.886214E 10-0.877202E 10-0.337245E 10-0.100152E 12-0.119218E 11-0.225672E 10-0.274422E 10-0.710328E 11
 0.174097E 10 0.463385E 10 0.182600E 10 0.981330E 11 0.952273E 10 0.193148E 10 0.305906E 09 0.427647E 11

-0.195895E 11-0.691520E 10 0.143505E 10-0.456935E 11 0.673418E 10-0.102502E 10 0.400722E 10-0.128061E 10
```

```
-0.927226E 09 0.101471E 11 0.244630E 10 0.605415E 11 0.295811E 10 0.177725E 10 0.163706E 10 0.513449E 11
 0.886653E 10-0.878788E 10-0.337250E 10-0.100155E 12-0.119338E 11-0.226150E 10-0.274416E 10-0.710195E 11
 0.174302E 10 0.462389E 10 0.182597E 10 0.981302E 11 0.951547E 10 0.192765E 10 0.305961E 09 0.427752E 11

                    MODE SHAPE  W AT TOP OF SHELL=-0.99072E 10

-0.10573E 11-0.90092E 10 0.22158E 09 0.15149E 11 0.24875E 11 0.19069E 11-0.10018E 10-0.21788E 11-0.28456E 11

-0.17092E 11 0.30649E 10 0.18253E 11 0.20808E 11 0.12801E 11 0.16349E 10-0.66606E 10-0.10573E 11-0.11539E 11


X= 0.16667E 01

-0.145466E 11 0.226094E 09 0.168243E 10-0.346766E 11 0.575305E 10-0.279608E 10 0.252336E 10 0.379505E 11
 0.998119E 10-0.575903E 09 0.209815E 10 0.656336E 11 0.602493E 08 0.470661E 10-0.411214E 09-0.455026E 11
-0.104877E 11 0.475135E 09-0.262399E 10-0.107397E 12-0.175560E 10-0.517967E 10 0.441495E 09 0.628480E 11
 0.130275E 11 0.164085E 09 0.100439E 10 0.956402E 11-0.153578E 10 0.366130E 10-0.128149E 10-0.510389E 11

-0.145457E 11 0.219559E 09 0.168242E 10-0.346767E 11 0.575220E 10-0.279721E 10 0.252336E 10 0.379535E 11
 0.998245E 10-0.579046E 09 0.209815E 10 0.656329E 11 0.525919E 08 0.470601E 10-0.411194E 09-0.455013E 11
-0.104857E 11 0.464417E 09-0.262402E 10-0.107398E 12-0.175824E 10-0.518180E 10 0.441516E 09 0.628570E 11
 0.130286E 11 0.161942E 09 0.100437E 10 0.956388E 11-0.154095E 10 0.365953E 10-0.128146E 10-0.510356E 11


     1           4           6           7           2           3           5           8

X= 0.
 0.          0.          0.          0.          0.641944E 10 0.179310E 10 0.293543E 10 0.536473E 11
 0.          0.          0.          0.         -0.102113E 11 0.188751E 10 0.243107E 10-0.851518E 11
 0.          0.          0.          0.          0.939910E 10-0.220888E 10-0.702211E 09 0.118211E 12
 0.          0.          0.          0.         -0.478232E 10 0.588593E 09-0.439135E 10-0.896310E 11
```

## 6. Explanation of Output of SP and AEP.

The output of the programs consists of two parts. The first part reproduces first the Control and Rotation Cards, and then the Shell Dimension, Shell Parameter, Elastic Parameter, Z-coordinate, and Prestress cards for each Part in the order in which the shell is constructed. The second part of the output pertains to the subcases which are run from the shell geometry and prestress given before. Each subcase is identified by a number. The output for each subcase consists of the reproduction of the Wave Number or Eigenvalue Card, Boundary Condition Cards, and, for the Static Program, the Load Cards over each of the Parts, again in the order in which the shell is constructed.

The useful output for each subcase begins with the page with the heading: Solution at Points Along Meridian. The solution can be either in terms of stress resultants, stresses, or both. The column headings of the stress resultant output have the following meaning:

X = meridional coordinate. Either arclength s or angle $\phi$, depending on Part.

W = normal deflection, w.

Q = effective transverse shear resultant, Q.

UPHI = meridional deflection, $u_\phi$.

NPHI = meridional membrane stress resultant, $N_\phi$.

BPHI = rotation of normal in meridional direction, $\beta_\phi$.

MPHI = meridional bending moment, $M_\phi$.

UTHETA = circumferential deflection, $u_\theta$.

N = effective membrane shear resultant, N.

NTHETA = circumferential membrane stress resultant, $N_\theta$.

MTHETA = circumferential bending moment, $M_\theta$.

The column headings of the stress output have the following meaning:

SPHI = meridional normal stress, $\sigma_\phi$.

STHETA = circumferential normal stress, $\sigma_\theta$.

SFITH = in-plane shear stress, $\sigma_{\phi\theta}$.

The letters IN and OUT refer to the "inside" and "outside" bounding surfaces of each layer of the shell. For example, with reference to Figure 8, $Z_1$ is the coordinate of the inside and $Z_2$ of the outside bounding surface of layer No. 1. For a multilayered Part, the stresses for each of the layers appear in single-spaced blocks corresponding to one value of X. The first line of the block refers to layer No. 1, the second to layer No. 2, etc.

After a sign change in the value of the characteristic determinant (DET) has been detected, the Eigenvalue Programs print out at the end of the solution the maximum value of the first prescribed variable at the Final Edge and the value of DET. The eigenvalue parameter, for which this solution was obtained, is declared an eigenvalue, if the solution is such that DET is smaller than 0.01 times this maximum value.

If after five iterations such accuracy is still not attained, this sign change in the DET is ignored. The user of the program should then examine the solution and make a decision himself whether or not the eigenvalue parameter is an eigenvalue. It sometimes happens that the DET changes sign when going through infinity, instead of zero, and such points do not represent eigenvalues.

Finally, for each subcase the AEP prints out a table of the eigenvalue parameter (OMEGA) versus three kinds of values of the characteristic determinants. It is always recommended to plot these determinants vs. OMEGA to make sure that they indeed go through zero at the indicated eigenvalue. At present, a normalized determinant, NORMAL DET, is used for all interpolation calculations. After considerable experimentation, it was found that such a normalized determinant makes the inverse interpolation easier.

## 7. Explanation of Output for NEP

The output of the NEP differs somewhat from that of the other programs. The first part reproduces the Shell Dimension, Shell Parameter, Elastic Parameter, Z-coordinate and Prestress data for each Part. The second part begins with the heading Stability Analysis, and records output for each subcase which is run for the given shell geometry. The output for each subcase is a listing of the number of Parts in the shell, the number of wave numbers used, the wave numbers, and a reproduction of the NEP Boundary Condition cards.

The useful output begins with a page indicating the Part number, and number of segments in each Part. For each segment in a Part, the values of the determinants of $E_i$ and $C_i$ are recorded*.

The most valuable parts of the output are the two numbers DETCM and ALFA. These are the values of the determinant of the $C_M$ matrix and the corresponding trial eigenvalue. Note that a value of DETCM such as 0.16825E-07 10, is read $0.16825 \times 10^{+3}$.

CM matrix refers to the individual elements of $C_M$, while COFACTORS OF CM are the cofactors of this matrix. These co-

*The format of this output is read as the floating point number times ten to the power of the algebraic sum of the two digit numbers which follow it.

factors are used to check the computation, the first value in the check should duplicate DETCM while the remaining elements should be several orders of magnitude smaller. The cofactors are normalized with respect to the first element and are output as the values of the nonhomogeneous boundary condition variables at the final edge of the shell.

The column headings which follow refer to the code numbers of the fundamental variables*. At the end of each segment the meridional coordinate at that point is output followed by the values of the fundamental variables computed first by solving the homogeneous matrix equations and then by numerical integration. In addition, at intermediate points the mode shape of the normal deflection is output at 10° intervals beginning at the top, $\theta = 0°$, and ending with $\theta = 180°$. At the starting edge of the shell, the boundary condition code numbers are reproduced along with the matrix equation values for the nonhomogeneous fundamental variables.

The user of the program should note that DETCM may change sign when going through infinity, rather than through zero. The corresponding ALFA is not a true eigenvalue and indicates that additional increments in the value of the trial eigenvalue are required.

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*See Item 6, Section 2, Chapter I, Part II.

## SECTION II

## DESCRIPTION OF PROGRAMS AND LISTINGS

### 1. Description of SP and AEP

The two computer programs, SP and AEP, have similar features and will be described together. The NEP was prepared by a different programmer, and it will be described separately.

#### 1. MAIN of SP and AEP

The main flow in SP and AEP is controlled from MAIN, and for anyone who is interested in following the logic of these programs, it is recommended to become familiar with the main loops of the flow. The main loops of MAIN are as follows:

1. Shell Geometry Loop (DO 881 loop). Within this loop, the geometry of the shell is read in over all parts. The Shell Dimension card is read at Format #505, Shell Parameter card through INPUT and Elastic Parameter and Z-Co ordinate cards through ORTHO.

2. Initial Value Integration Loop (between statements #82 and #501). Within this loop the initial value integration is performed over all Segments of all Parts. The initial value solutions are stored in array D.

3. The initial value solutions, in array D, are brought into TRIANG near statement #351, where the solution matrix is triangularized, and the solutions at beginnings of segments calculated and stored in array DM.

4. Final Integration Loop (DO 641 loop). Within this loop the solutions at beginnings of Segments are taken from the DM array, integrated, and printed out at any requested intermediate points within each segment. The solution of all fundamental variables is stored in array Z and printed out near statement #51 by Format #39.

2. <u>ORTHO</u>

ORTHO reads in the Elastic Parameter and Z-coordinate Cards and calculates the shell stiffnesses. ORTHO calls FGEN to store or retrieve elastic parameters and Z-coordinates which are variable along the meridian.

3. <u>DIFFEQ</u>

DIFFEQ calculates the derivatives of fundamental variables (stored in YD) when the fundamental variables themselves (stored in Y) and the elastic and geometric properties of the shell are given. DIFFEQ calls INPUT to supply the geometrical parameters and ORTHO for the elastic parameters. For Eigenvalue Programs, it calls PGEN for the prestress variables.

4. <u>TRIANG</u>

TRIANG receives the initial value solutions over all segments (stored in D) and calculates the determinant of the frequency matrix, and performs inverse interpolation for the eigenvalue parameter once a sign change in the determinant is detected.

5. <u>BCOND</u>

BCOND calculates the elements of transformation matrices TLI and TR which are needed for rotated boundary conditions.

6. INPUT

INPUT reads in the Shell Parameter and Load cards. It calculates the geometrical and load parameters of the shell, which are defined in INPUT as follows:

$$R1 = 1/R_\phi$$

$$R2 = 1/\sin\phi \; R_\theta$$

$$SXN = \sin\phi$$

$$CXS = \cos\phi$$

$$P1 = p_1$$

$$P2 = p_2$$

$$PN = p$$

$$PL = p_\phi$$

$$PC = p$$

$$TU = T_u$$

$$TL = T_L$$

INPUT calls FGEN to read in or retrieve shell parameters and loads which vary along the meridian.

7. INVERT

INVERT is a typical matrix inversion subroutine.

8. FGEN

FGEN reads in parameters whcih vary along the meridian and are given at discrete points along the meridian. The meridional coordinate is stored in XP and the parameters in YP. Then for a given value of the meridional coordinate, S, FGEN calculates the variable parameter.

9. PGEN

PGEN reads in and stores the prestress variables at a number of points along the meridian of a Part. Then for a given value of the meridional coordinate, S, PGEN calculates the prestress variables.

10. RUNGE

RUNGE provides the entry to the direct numerical integration subroutines:

RUNKUT, ASJSTP, STEP, INTPOL. RUNGE calls RUNKUT to begin integration.

11. RUNKUT

RUNKUT advances the numerical integration from step to step. RUNKUT calls ADJSTP to aojust the step size, and INTPOL to see if the end point is not closer than one step size.

12. ADJSTP

ADJSTP adjusts the integration step size. Initially, the step size is input as one hundredth of the length of a segment. ADJSTP tests this initial step size by comparing solutions obtained after taking two separate steps of step size h and then one step with step size 2h. The initial step size is adjusted until the difference in the solution lies between prescribed tolerances. Only then the integration is begun. After every three steps, the step size is checked again and adjusted up or down, depending on whether the difference in the solutions is too small or too large. ADJSTP calls INTPOL to check if the end point is not closer than one step size.

### 13. STEP

STEP calculates the fundamental variables at a point if the variables at the preceding point are known. The standard fourth-order Runge-Kutta formulas are used. STEP calls DIFFEQ to receive the derivatives of the fundamental variables.

### 14. INTPOL

INTPOL checks if the distance to the end of the segment is smaller than the step size. If it is, the step size is set equal to the difference between the end and current points and integration is terminated. If it is not, integration is continued.

## 2. Description of NEP

MAIN for this program controls the flow in three distinct areas. In the first portion the shell dimensions, shell parameters, elastic parameters, and prestress parameters for each Part are read into the machine using the INPUT, ORTHO, and PGEN subroutines. For each case, a set of eigenvalue parameters, wave numbers, and boundary conditions is read. The remaining two loops are then activated for each eigenvalue. First, the initial value integrations are performed over the length of the shell via the SEGM subroutine. Then the TRIA subroutine is called to find the determinant of the C matrix of the last segment. These two steps are repeated for each incremented eigenvalue within each case.

The SEGM (M) subroutine integrates the (8 times the number of wave numbers) initial value problems over the length of each shell segment using an integration procedure composed of the RUNGE, RUNKUT, DIFFEQ, ADJSTP, STEP, and INTPOL subroutines. These solutions are then written on tape 4 as the DM array for each segment. SEGM also insures that in the first segment the integrations are performed in the order specified by the boundary conditions at the starting end. Similarly, the solutions in the last shell segment are written on the tape in an order determined by the boundary conditions at the final end.

SYLSEG is used to reduce computation time for cylindrical shells. The special geometry for this case is applied to control of SEGM by eliminating integration of the initial value problems for intermediate segments.

DIFFEQ for the NEP program is unique since the derivatives of the fundamental variables are computed in two parts. The first portion is identical to the AEP computation while the second adds terms which are dependent on combinations of the selected wave numbers.

The TRIA subroutine has two distinct functions. A Gaussian elimination is applied to the linear homogeneous matrix equations which define the eigenvalue problem. This procedure is performed on the DM arrays for each segment stored on tape 2, and yields a value for the determinant of the C matrix portion of the last shell segment, $C_M$. The computation of this determinant is then checked using an expansion by cofactors of the original $C_M$ matrix.

The second portion of the subroutine checks the overall accuracy of the computation. At the end of each segment the values of the fundamental variables are obtained by two methods: (a) Solution of the set of homogeneous, linear equations determined by the DM arrays, and (b) Numerical integration over each segment.

TRIA also calls a MODES subroutine at the end of each segment. MODES adds the integrated values for the Fourier components of the normal displacement to produce the shape of the shell cross section at each segment interval.

It should be emphasized that the NEP does not have an automatic root search. The determination of the eigenvalue is a trial and error procedure based on the judgement of the user.

This restriction is, in part, a consequence of the form of the governing equations for the nonaxisymmetric problem. The equations for this case are not separable in terms of individual Fourier components. Thus a solution composed of a finite sum of such components is necessarily approximate in nature. Hence, the best approximation is the minimum eigenvalue obtained from several sets of trial solutions.

At present, the procedure for selecting a range of eigenvalues and Fourier components must be based on the experience of the user in analytic and experimental results for problems having some similarity to the one considered. For example, in the case of a buckling program, the initial range of eigenvalues to be searched should begin with those near the eigenvalue for the corresponding axisymmetric problem.

Another guide for selection of trial values can be found in the procedure used for analysis of free vibration of shells* which applied the same inverse interpolation procedure used in the NEP. Trail eigenvalues are interpreted in terms of the value of the determinant of the $C_M$ matrix. While a zero value

*A. Kalnins, "Free Vibration of Rotationally Symmetric Shells", Journal of the Acoustical Society of America, Volume 36, Number 7, 1964, p. 1362.

of this determinant is sought to determine the eigenvalue, a plot of $C_M$ as a function of trial eigenvalues indicates the range of eigenvalues which should be searched on the succeeding trial.

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# APPENDIX

## COMPUTER PROGRAM LISTINGS

```
C     STATIC PROGRAM FOR STRESS ANALYSIS OF SHELLS OF REVOLUTION
      DIMENSION Y(8),YD(8),DUMM(27)
      DIMENSION MLY(20),ZLY(20,5)
      DIMENSIOND(100,8,9),DM(101,9),GA(4),GB(4,4),ITP(100),IA(8),IB(8,4)
      DIMENSION TR(8,8,4),TLI(8,8),ALFR(4)
      DIMENSION B11(20,4),B12(20,4),B22(20,4),B66(20,4),AL1(20,4)
      DIMENSION AL2(20,4),RHO(20,4)
      COMMON NDE,S,Y,YD,HH,J9,JMAX,M9,XOUT,IFREQ,DUMM,IBR,ISH
      COMMON XN,XLD,OMSQ,HTT,R1,R2,R3,SXN,CXS,INDEX
      COMMON PN,PL,PC,TO,T1,RH1,RH2,RH3,MLY,ZLY,EMFI,EMTH
      COMMON TR,TLI,IBCL,ALFL,ALFR,NBR
      COMMON NPRT,D,DM,GA,GB,ITP,NF,NFP,NPL,NH,IA,IB,OMEGA
      COMMON B11,B12,B22,B66,AL1,AL2,RHO,H11,H12,H21,H22,G12,G21
      COMMON C11,C12,C22,E11,E12,E22,D11,D12,D22,C66,E66,D66
      DIMENSION          SI(20),SX(20),XPR(20), ING(20), IPAR(20),Z(20)
      DIMENSION ISS(20),TR1(8,8),TR2(8,8),NTP(20),DSC(20,8)
   10 READ (5,20)    IBRM,ISTK,NBR,NXT,IVB,NPRT                            1
   20 FORMAT (16I5)
      IF(IBRM)      30,30,50
   30 WRITE (6,40)IBRM                                                     9
   40 FORMAT (1H0,50HIBRM=0 WHICH INDICATES END OF JOB. EXIT IS CALLED )
      CALL EXIT                                                           10
   50 CONTINUE
      ERP=1.0E-05
      XLD=0.0
      IBCL=0
      INXC=1
      WRITE  (6,60)                                                       13
   60 FORMAT (1H1, 10X, 109HSTRESS ANALYSIS PROGRAM OF SHELLS OF REVOLUT
     1ION UNDER STATIC LOADS, BY A. KALNINS, LEHIGH UNIV, BETHLEHEM, PA/
     220X,80HWRIGHT PATTERSON AIR FORCE BASE FLIGHT DYNAMICS LABORATORY
     3VERSION, 22 JULY 1968)
      WRITE (6,70)  IBRM,NBR,NXT                                          14
   70 FORMAT (1H0, 20X, 18HSTATIC ANALYSIS    , 8H  PARTS=,I3,
     111H  BRANCHES=,I3,21H  NUMBER OF SUBCASES=,I3)
      IF(NBR-3) 100,100,80
   80 WRITE (6,90) NBR                                                    16
   90 FORMAT (1H0,I3,38H  BRANCHES EXCEED ALLOWED MAXIMUM OF 3)
      CALL EXIT                                                           17
  100 CONTINUE
      IF(IBRM-20) 130,130,110
  110 WRITE (6,120) IBRM                                                  20
  120 FORMAT (1H0,I3,36H  PARTS EXCEED ALLOWED MAXIMUM OF 20)
      CALL EXIT                                                           21
  130 CONTINUE
      NBR=NBR+1
      READ (5,140) ALFL, (ALFR(I),I=1,NBR)                                23
  140 FORMAT (5F10.5)
      WRITE (6,150) ALFL, (ALFR(I),I=1,NBR)                               30
  150 FORMAT (1H0, 51HANGLES OF ROTATION OF BOUNDARY CONDITIONS ARE ALFLA
     1=,  E12.5, 8H  ALFRS=, 4E12.5)
      IGCT=0
      DO 220 I=1,IBRM
      IBR=I
      WRITE (6,160) I                                                     41
```

```
160 FORMAT (1H0, 10X, 7HPART NO,I3)                                     A 0056
    READ (5,170)                                                         A 0057
   1 SI(I),SX(I),IPAR(I),ING(I),ISS(I),NTP(I),MLY(I)                     A 0058    42
170 FORMAT (2F10.5,5I5)                                                  A 0059
    WRITE (6,180)                                                        A 0060
   1SI(I),SX(I),IPAR(I),ING(I),ISS(I),NTP(I),MLY(I)                      A 0061    50
180 FORMAT (1H0,3HSI=,E12.5,6H   SX=,F12.5, 8H   IPAR=,I3,7H   ING=,I3A 0062
   1,14H    SHELL TYPE,I2, 7H   NTP=,I2, 14H   LAYERS MLY=,I2)           A 0063
    IGCT=IGCT+IPAR(I)                                                    A 0064
    ISH=ISS(I)                                                           A 0065
    INDEX=1                                                              A 0066
    CALL INPUT                                                           A 0067    63
    CALL ORTHO                                                           A 0068    65
    IF(MLY(I)-4) 210,210,190                                             A 0069
190 WRITE (6,200) MLY(I),I                                               A 0070    69
200 FORMAT(1H0,I3,19H  LAYERS IN PART NO,I3,22H  EXCEED ALLOWED MAX 4)A 0071
    CALL EXIT                                                            A 0072    71
210 CONTINUE                                                             A 0073
220 WRITE (6,1830)                                                       A 0074    73
    IF(IGCT-100) 250,250,230                                             A 0075
230 WRITE (6,240) IGCT                                                   A 0076    77
240 FORMAT (1H0, I3, 40H  SEGMENTS EXCEED ALLOWED MAXIMUM OF 100)        A 0077
    CALL EXIT                                                            A 0078    78
250 CONTINUE                                                             A 0079
260 LD=0                                                                 A 0080
    READ (5,20) NX,NPCH                                                  A 0081
C   NPCH=0 MEANS PRESTRESS IS NOT PUNCHED, NPCH=1 MEANS IT IS PUNCHED A 0082    81
    NNN=0                                                                A 0083
    WRITE (6,60)                                                         A 0084    84
    IF(NX) 290,270,270                                                   A 0085
270 WRITE (6,280) INXC,NX                                                A 0086    86
280 FORMAT (1H0,30X, 10HSUBCASE NO,I3,27H   FOR FOURIER HARMONIC COS,  A 0087
   1I2, 6H THETA)                                                        A 0088
    GO TO 310                                                            A 0089
290 I=-NX                                                                A 0090
    WRITE (6,300)  INXC,I                                                A 0091    89
300 FORMAT (1H0,30X, 10HSUBCASE NO,I3,27H   FOR FOURIER HARMONIC SIN,  A 0092
   1I2, 6H THETA)                                                        A 0093
310 CONTINUE                                                             A 0094
    XN=NX                                                                A 0095
    READ (5,320) (IA(I),GA(I),I=1,4)                                     A 0096    91
320 FORMAT (4(I5,F10.5))                                                 A 0097
    WRITE  (6,330)                                                       A 0098
   1(IA(I),GA(I),I=1,4)                                                  A 0099    99
330 FORMAT(1H0,39HBOUNDARY CONDITIONS AT STARTING EDGE    ,4(I5,E12.5))A 0100
    DO 380 K=1,NBR                                                       A 0101
    READ (5,320)                                                         A 0102
   1 (IB(I+4,K),GB(I,K),I=1,4)                                           A 0103   109
    IF(K-1) 340,340,360                                                  A 0104
340 WRITE (6,350)  (IB(I+4,K), GB(I,K),I=1,4)                            A 0105   116
350 FORMAT(1H0,39HBOUNDARY CONDITIONS AT FINAL     EDGE    ,4(I5,E12.5))A 0106
    GO TO 380                                                            A 0107
360 J=K-1                                                                A 0108
    WRITE (6,370) J,                                                     A 0109
   1(IB(I+4,K),GB(I,K),I=1,4)                                            A 0110   124
370 FORMAT(1H0,36HBOUNDARY CONDITION AT BRANCH EDGE NO,I3,4(I5,E12.5))A 0111
```

```
380 CONTINUE
    IF(NX) 410,390,410
390 NDE=6
    DO 400 K=1,NBR
    IB(4,K)=IB(5,K)
    IB(5,K)=IB(6,K)
400 IB(6,K)=IB(7,K)
    IA(7)=0
    IA(8)=0
    GO TO 420
410 NDE=8
420 NH=NDE/2
    M=NH+1
    L=1
    DO 460 IK=M,NDE
    DO 440 N=L,NDE
    DO 430 J=1,NH
    IF(IA(J)-N) 430,440,430
430 CONTINUE
    GO TO 450
440 CONTINUE
450 L=N+1
460 IA(IK)=N
    DO 500 K=1,NBR
    L=1
    DO 500 IK=1,NH
    DO 480 N=L,NDE
    DO 470 J=M,NDE
    IF( IB(J,K)-N) 470,480,470
470 CONTINUE
    GO TO 490
480 CONTINUE
490 L=N+1
500 IB(IK,K)=N
    DO 510 I=1,20
    DO 510 J=1,8
510 DSC(I,J)=0.0
    DO 520 I=1,8
    Y(I)=0.0
    Z(I)=0.0
    DO 520 L=1,8
    TR1(I,L)=0.0
520 TR2(I,L)=0.0
    CALL BCOND
    DO 530 J=1,NDE
    K=IA(J)
    DO 530 I=1,NDE
530 DM(I,J)=TLI(I,K)
    DO 540 I=1,NDE
    DO 540 J=1,NDE
540 TLI(I,J)=DM(I,J)
    DO 560 K=1,NBR
    DO 550 J=1,NDE
    DO 550 I=1,NDE
    L=IB(I,K)
550 DM(I,J)=TR(L,J,K)
```

```
      DO 560 I=1,NDE                                                    A  0168
      DO 560 J=1,NDE                                                    A  0169
  560 TR(I,J,K)=DM(I,J)                                                 A  0170
      IF(NDE-7) 570,570,580                                             A  0171
  570 TR1(3,5)=1.0                                                      A  0172
      TR1(6,6)=1.0                                                      A  0173
      TR2(5,3)=1.0                                                      A  0174
      TR2(6,6)=1.0                                                      A  0175
      GO TO 590                                                         A  0176
  580 TR1(3,7)=1.0                                                      A  0177
      TR1(4,5)=1.0                                                      A  0178
      TR1(7,8)=1.0                                                      A  0179
      TR1(8,6)=1.0                                                      A  0180
      TR2(5,4)=1.0                                                      A  0181
      TR2(6,8)=1.0                                                      A  0182
      TR2(7,3)=1.0                                                      A  0183
      TR2(8,7)=1.0                                                      A  0184
  590 NPL=NDE+1                                                         A  0185
  600 IBR=1                                                             A  0186
      ISTR=ISTK                                                         A  0187
      NSTR=0                                                            A  0188
      NFF=0                                                             A  0189
      IRS=0                                                             A  0190
      NFP=2                                                             A  0191
      NF=1                                                              A  0192
  610 I=IBR                                                             A  0193
      WRITE (6,1830)                                                    A  0194    282
      WRITE (6,620) IBR,INXC                                            A  0195    283
  620 FORMAT (1H0,20X,17HLOADS FOR PART NO,I3,12H  SUBCASE NO,I3)       A  0196
      ICYL=0                                                            A  0197
      ISH=ISS(I)                                                        A  0198
      READ (5,140) (DSC(I,J),J=2,8,2)                                   A  0199    286
      WRITE (6,630) (DSC(I,J),J=2,8,2)                                  A  0200    293
  630 FORMAT (1H0,39HRING LOADS AT END OF THIS PART ARE   Q=,E12.5,     A  0201
     17H  NPHI=, E12.5, 7H  MPHI=,E12.5, 4H  N=,E12.5)                  A  0202
      S=SI(I)                                                           A  0203
      INDEX=2                                                           A  0204
      CALL INPUT                                                        A  0205    302
      INDEX=3                                                           A  0206
      CALL ORTHO                                                        A  0207    305
      CALL INPUT                                                        A  0208    307
      INDEX=4                                                           A  0209
      IF(NPRT) 730,730,640                                              A  0210
  640 CONTINUE                                                          A  0211
      JJ=MLY(IBR)+1                                                     A  0212
      WRITE (6,650) IBR                                                 A  0213    313
  650 FORMAT (1H0, 20X, 28HAT BEGINNING AND END OF PART,I3,             A  0214
     115H PARAMETERS ARE  )                                             A  0215
      WRITE (6,1830)                                                    A  0216    314
      DO 720 K=1,2                                                      A  0217
      WRITE (6,660) R1,R2,R3,SXN,CXS                                    A  0218    317
      WRITE (6,670)  PN,PL, PC,T0,T1                                    A  0219    318
      WRITE (6,680)   C11,C12,C22,C66,E11,E12                           A  0220    319
      WRITE (6,690)  E22,E66,D11,D12,D22,D66                            A  0221    320
      WRITE (6,700)  H11,H12,H22,H21,G12,G21                            A  0222    321
      WRITE (6,710)   (ZLY(IBR,J), J=1,JJ)                              A  0223    322
```

```
      WRITE (6,1830)                                                    327
  660 FORMAT (1H , 4H R1=,E12.4, 4H R2=, E12.4,4H R3=, E12.4,
     15H SXN=, E12.4, 5H CXS=, E12.4)
  670 FORMAT (1H , 4H PN=, E12.4, 4H PL=, E12.4,4H PC=, E12.4,
     14H TO=, E12.4, 4H T1=, E12.4)
  680 FORMAT (1H , 5H C11=,E12.4, 5H C12=,E12.4, 5H C22=, E12.4,
     15H C66=, E12.4, 5H E11=, E12.4, 5H E12=, E12.4)
  690 FORMAT (1H , 5H E22=, E12.4, 5H E66=, E12.4, 5H D11=, E12.4,
     15H D12=, E12.4, 5H D22=, E12.4, 5H D66=, E12.4)
  700 FORMAT (1H , 5H H11=, E12.4, 5H H12=, E12.4, 5H H22=, E12.4,
     15H H21=, E12.4, 5H G12=, E12.4, 5H G21=, E12.4)
  710 FORMAT (1H , 7HZS ARE=,10E12.4)
      S=SX(I)
      CALL ORTHO                                                        329
      CALL INPUT                                                        331
  720 CONTINUE
  730 CONTINUE
      K=IPAR(I)+NF-1
      DO 740 J=NF,K
  740 ITP(J)=0
      IF(NTP(IBR))  780,780,750
  750 DO 760 J=NF,K
  760 ITP(J)=1
      IF(NTP(IBR)-1) 780,780,770
  770 ITP(K)=2
  780 CONTINUE
      PARTS=IPAR(IBR)
      SMXX=(SX(IBR) - SI(IBR))/PARTS
      SZERO=SI(IBR)
      XINT=ING(IBR)
      XPR(IBR)=SMXX/XINT
      NFF=NFF+IPAR(IBR)
      SMAX=SZERO+SMXX
      XOUT=SMAX
      IF(NPRT-1) 810,810,790
  790 WRITE (6,60)
      WRITE (6,800) IBR,IPAR(IBR)                                       369
  800 FORMAT(1H0,62HINITIAL VALUE INTEGRATIONS (COLUMNS OF Y(X) MATRIX) 370
     1OF PART NO,I3,5H OVER,I3,16H SEGMENTS FOLLOW)
  810 CONTINUE
  820 J=0
      XLD=0.0
  830 J=J+1
      DO 840 I=1,8
  840 Y(I)=0.0
      IF(J-NDE) 850,850,860
  850 Y(J)=1.0
  860 IF(NPRT) 880,880,870
  870 WRITE  (6,950)
     1          SZERO, (Y(I),I=1,NDE)                                   389
  880 CONTINUE
      S=SZERO
      HH=0.01*SMXX
      IBCL=0
      IF(ICYL) 910,910,890
  890 DO 900 I=1,NDE
```

```
A            -  EFN    SOURCE STATEMENT  -  IFN(S)  -

      K=I+8                                        A  0280
  900 D(NF,I,J)=DM(K,J)                            A  0281
      GO TO 930                                    A  0282
  910 CALL RUNGE                                   A  0283    410
      DO 920 I=1,NDE                               A  0284
      K=I+8                                        A  0285
      DM(K,J)=Y(I)                                 A  0286
  920 D(NF,I,J)=Y(I)                               A  0287
  930 IF(NPRT-1) 970,970,940                       A  0288
  940 WRITE  (6,950)                               A  0289
     1SMAX,  (D(NF,I,J),I=1,NDE)                   A  0290    425
  950 FORMAT (1HO, F9.3,8E15.7)                    A  0291
      WRITE (6,960) IBCL                           A  0292    432
  960 FORMAT (1H ,7HPOINTS=,I5)                    A  0293
  970 CONTINUE                                     A  0294
      IF(J-NDE) 830,980,990                        A  0295
  980 XLD=1.0                                      A  0296
      ICYL=0                                       A  0297
      GO TO 830                                    A  0298
  990 IF(NF-NFF)  1010,1000,1000                   A  0299
 1000 IF(IPAR(IBR)-1) 1010,1010,1110               A  0300
 1010 IF(IRS) 1080,1080,1020                       A  0301
 1020 IRS=0                                        A  0302
      S=SI(IBR)                                    A  0303
      CALL INPUT                                   A  0304    447
      TR2(1,1)=CXS                                 A  0305
      TR2(1,2)=SXN                                 A  0306
      TR2(3,1)=-SXN                                A  0307
      TR2(3,2)=CXS                                 A  0308
      IF(NDE-7) 1030,1030,1040                     A  0309
 1030 TR2(2,4)=CXS                                 A  0310
      TR2(2,5)=SXN                                 A  0311
      TR2(4,4)=-SXN                                A  0312
      TR2(4,5)=CXS                                 A  0313
      GO TO 1050                                   A  0314
 1040 TR2(2,5)=CXS                                 A  0315
      TR2(2,6)=SXN                                 A  0316
      TR2(4,5)=-SXN                                A  0317
      TR2(4,6)=CXS                                 A  0318
 1050 DO 1060 I=1,NDE                              A  0319
      DO 1060 K=1,NDE                              A  0320
      DM(I,K)=0.0                                  A  0321
      DO 1060 L=1,NDE                              A  0322
 1060 DM(I,K)=DM(I,K)+D(NF,I,L)*TR2(L,K)           A  0323
      DO 1070 I=1,NDE                              A  0324
      DO 1070 K=1,NDE                              A  0325
 1070 D(NF,I,K)=DM(I,K)                            A  0326
 1080 IF(IPAR(IBR)-1) 1110,1110,1090               A  0327
 1090 NF=NF+1                                      A  0328
      NFP=NF+1                                     A  0329
      SZERO=SMAX                                   A  0330
      SMAX=SMXX+SZERO                              A  0331
      XOUT=SMAX                                    A  0332
      IF(ISH-2)820,1100,820                        A  0333
 1100 ICYL=1                                       A  0334
      GO TO 820                                    A  0335
```

```
 1110 DO 1120 I=1,NDE
 1120 D(NF,I,NPL)=D(NF,I,NPL)-DSC(IBR,I)
 1130 IF(IBR-IBRM) 1140,1220,1220
 1140 IF(ITP(NF)-2) 1150,1210,1210
 1150 S=SX(IBR)
      CALL INPUT
      TR1(1,1)=CXS
      TR1(1,3)=-SXN
      TR1(2,1)=SXN
      TR1(2,3)=CXS
      IF( NDE-7) 1160,1170,1170
 1160 TR1(4,2)=CXS
      TR1(4,4)=-SXN
      TR1(5,2)=SXN
      TR1(5,4)=CXS
      GO TO 1180
 1170 TR1(5,2)=CXS
      TR1(5,4)=-SXN
      TR1(6,2)=SXN
      TR1(6,4)=CXS
 1180 DO 1190 I=1,NDE
      DO 1190 K=1,NPL
      DM(I,K)=0.0
      DO 1190 L=1,NDE
 1190 DM(I,K)=DM(I,K)+TR1(I,L)*D(NF,L,K)
      DO 1200 I=1,NDE
      DO 1200 K=1,NPL
 1200 D(NF,I,K)=DM(I,K)
 1210 IBK=IBK+1
      NF=NF+1
      NFP=NF+1
      IRS=1
      GO TO 610
 1220 DO 1230 I=1,NDE
      DO 1230 J=1,NDE
      DM(I,J)=0.0
      DO 1230 L=1,NDE
 1230 DM(I,J)=DM(I,J) +D(1,I,L)*TL1(L,J)
      DO1240 I=1,NDE
      DO1240 J=1,NDE
 1240 D(1,I,J)=DM(I,J)
      KF=1
      IX=NF
      DO 1290 K=1,NBR
      DO 1250 I=1,NDE
      DO 1250 J=1,NPL
      DM(I,J)=0.0
      DO 1250 L=1,NDE
 1250 DM(I,J)=DM(I,J)+TR(I,L,K)*D(IX,L,J)
      DO 1260 I=1,NDE
      DO 1260 J=1,NPL
 1260 D(IX,I,J)=DM(I,J)
      DO 1270 IX=KF,NF
      JJ=IX
      IF(ITP(IX)-2) 1270,1280,1280
 1270 CONTINUE
```

```
1280 IX=JJ                                                          A  0392
     KF=IX+1                                                        A  0393
1290 CONTINUE                                                       A  0394
     IF(NPRI) 1320,1320,1300                                        A  0395
1300 CONTINUE                                                       A  0396
     WRITE (6,1310) (ITP(I),I=1,NF)                                 A  0397  619
1310 FORMAT(1H0,50I2)                                               A  0398
     WRITE (6,1330)                                                 A  0399  626
1320 CONTINUE                                                       A  0400
     XLD=1.0                                                        A  0401
     CALL TRIANG                                                    A  0402  628
     IF(NPRI) 1370,1370,1330                                        A  0403
1330 CONTINUE                                                       A  0404
     WRITE  (6,1340)                                                A  0405
    1              NFP,NFP,XN,  ERP                                 A  0406  631
1340 FORMAT(1H0, 5X,4HNEXT,    I3, 41H ROWS ARE VARIABLES AT ENDPOINTS A  0407
    1X1,...,X, I2, 12H OF SEGMENTS,                                 A  0408
    22X, 3HXN=, F5.1,2X,                      10H ACCURACY=,E8.1)   A  0409
     WRITE (6,1630)                                                 A  0410  632
     DO 1350 I=1,NFP                                                A  0411
1350 WRITE  (6,1360)                                                A  0412
    1             I,(DM(I,J), J=1,NDE)                              A  0413  636
1360 FORMAT (1H ,3HX=X,I2,2X,8E15.7)                                A  0414
1370 CONTINUE                                                       A  0415
     IST=0                                                          A  0416
     DO 1440 K=1,NF                                                 A  0417
     IF( ITP(K)-1) 1440,1380,1400                                   A  0418
1380 IF(IST) 1390,1390,1440                                         A  0419
1390 IST=2                                                          A  0420
     I=K                                                            A  0421
     GO TO 1440                                                     A  0422
1400 IF(IST) 1410,1410,1420                                         A  0423
1410 I=K                                                            A  0424
1420 CONTINUE                                                       A  0425
     DO 1430 J=1,NH                                                 A  0426
1430 DM(K+1,J)=DM(I,J)                                              A  0427
     IST=0                                                          A  0428
1440 CONTINUE                                                       A  0429
     DO 1450 I=1,NDE                                                A  0430
     Y(I)=0.0                                                       A  0431
     DO 1450 J=1,NDE                                                A  0432
1450 Y(I)=Y(I)+TLI(I,J)*DM(1,J)                                     A  0433
     DO 1460 I=1,NDE                                                A  0434
1460 DM(1,I)=Y(I)                                                   A  0435
1470 CONTINUE                                                       A  0436
     WRITE  (6,60)                                                  A  0437  690
     WRITE  (6,1480)       NX                                       A  0438  691
1480 FORMAT (1H0,20X,60HSTATIC SOLUTION AT POINTS ALONG MERIDIAN FOR WAA  0439
    1VE NUMBER NX=,I3)                                              A  0440
     IF(ISTR-1) 1510,1490,1490                                      A  0441
1490 WRITE  (6,1500)                                                A  0442  693
1500 FORMAT (1H0,9H    S    ,2X,12H     W     , 12H     Q     ,     A  0443
    112H    UPHI    , 12H    NPHI    , 12H    BPHI    , 12H    MPHI A  0444
    2, 12H   UTHETA   , 12H     N     , 12H   NTHETA  ,10H   MTHETA )A  0445
     GO TO 1530                                                     A  0446
1510 WRITE  (6,1520)                                                A  0447  695
```

403

```
 1520 FORMAT (1H0,9H     S    ,2X,12H     M    , 12H    UPHI    ,
     112H    UTHETA  , 12H    BPHI    , 12H   SPHI IN  ,12H  SPHI OUT
     2,12H STHETA IN  ,12H STHETA OUT , 12H  SP[illegible] IN  ,12H  SPITH OUT )
 1530 IST=1
      KF=0
      IK=1
      JPA=IPAR(1)+1
      DO 1850 N=1,IBRN
      ISN=ISS(N)
      IBR=N
      L=ING(N)+1
      S=SI(N)
      KF=IPAR(N)+KF
      IF(ITP(KF)-1) 1540,1550,1550
 1540 WRITE   (6,1560) N
      GO TO 1580
 1550 WRITE   (6,1570) N
      IST=IST+1
 1560 FORMAT (1H0, 20H  MAIN SHELL PART NO,I3)
 1570 FORMAT (1H0, 20HBRANCH SHELL PART NO,I3)
 1580 INDEX=3
      CALL ORTHC
      CALL INPUT
      IF(N-1) 1660,1660,1590
 1590 IF(NSTR) 1600,1600,1660
 1600 TR2(1,1)=CXS
      TR2(1,2)=SXN
      TR2(3,1)=-SXN
      TR2(3,2)=CXS
      IF(NDE-7) 1610,1610,1620
 1610 TR2(2,4)=CXS
      TR2(2,5)=SXN
      TR2(4,4)=-SXN
      TR2(4,5)=CXS
      GO TO 1630
 1620 TR2(2,5)=CXS
      TR2(2,6)=SXN
      TR2(4,5)=-SXN
      TR2(4,6)=CXS
 1630 DO 1640 I=1,NDE
      Y(I)=0.0
      DO 1640 J=1,NDE
 1640 Y(I)=Y(I)+TR2(I,J)*DM(JPA,J)
      DO 1650 I=1,NDE
 1650 DM(JPA,I)=Y(I)
      JPA=JPA+IPAR(N)
 1660 XX=0.0
      INDEX=4
      S=IPAR(N)
      DELS=0.01*(SK(N)-SI(1))/S
      SZERO=SI(N)
      IF(ISTR-1)  1700,1670,1670
 1670 IF(NPCH) 1700,1700,1680
 1680 NTK= IPAR(N)*ING(N)+1
      I=1
C     FOLLOWING STATEMENT IS INTENDED TO PUNCH CARDS FOR PRESTRESS STATE
```

```
        WRITE (7,1690) N,NTK,I                                          A  0504   764
 1690   FORMAT (3I5)                                                    A  0505
 1700   CONTINUE                                                        A  0506
        DO 1840 K=IX,KF                                                 A  0507
        Z(7)=0.0                                                        A  0508
        Z(8)=0.0                                                        A  0509
        DO 1710 I=1,NDE                                                 A  0510
        Z(I)=DM(K,I)                                                    A  0511
 1710   Y(I)=DM(K,I)                                                    A  0512
        MM=DELS                                                         A  0513
        DO 1820 M=1,L                                                   A  0514
        SZERO=SZERO+XPR(M)*XX                                           A  0515
        S=SZERO                                                         A  0516
        XX=1.0                                                          A  0517
        CALL ORTHO                                                      A  0518   779
        CALL INPUT                                                      A  0519   781
        XNR=XN*R2                                                       A  0520
        SXR=SXN*R2                                                      A  0521
        CXR=CXS*R2                                                      A  0522
        ELM=R1-SXR                                                      A  0523
        ELJ=R1+SXR                                                      A  0524
        ETH=XNR*Z(7)+ CXR*Z(3) +SXR*Z(1)                                A  0525
        BTH=XNR*Z(1) +SXR*Z(7)                                          A  0526
        MTH=XNR*BTH+CXR*Z(5)                                            A  0527
        EPS=-XNR*Z(3)-CXR*Z(7)                                          A  0528
        MKA= -2.0*XNR*Z(5)-2.0*XNR*CXR*Z(1)+CXR*(ELM-SXR)*Z(7)+XNR*R1*Z(3)A  0529
        DEL=C11*D11-E11*E11                                             A  0530
        EN= Z(4)-C12*ETH-E12*MTH-(H11*TO+H12*T1)                        A  0531
        EM= Z(6) -E12*ETH-D12*MTH-(H12*TO+G12*T1)                       A  0532
        EFI= (EN*D11-EM*E11)/DEL                                        A  0533
        MFI= (EM*C11-EN*E11)/DEL                                        A  0534
        IF(ISTR-1) 1720,1750,1750                                       A  0535
 1720   ZD=        (Z (8)-(C66+SXR*E66)*EPS-(E66+SXR*D66)*MKA)          A  0536
       1/(C66+2.0*E66*SXR+D66*SXR*SXR)                                  A  0537
        ETF = EPS+ZD                                                    A  0538
        MTF=MKA+SXR*ZD                                                  A  0539
        Z(2)=Z(3)                                                       A  0540
        Z(3)=Z(7)                                                       A  0541
        Z(4)=Z(5)                                                       A  0542
        JJ=MLY(IBR)                                                     A  0543
        DO 1740 I=1,JJ                                                  A  0544
        J=I+1                                                           A  0545
        Z(5)      =B11(N,I)*(EFI+ZLY(N,I)*MFI) +B12(N,I)*(ETH+ZLY(N,I)*MTH)A  0546
       1-(B11(N,I)*AL1(N,I)+B12(N,I)*AL2(N,I))*(TO+ZLY(N,I)*T1)         A  0547
        Z(6)      =B11(N,I)*(EFI+ZLY(N,J)*MFI) +B12(N,I)*(ETH+ZLY(N,J)*MTH)A  0548
       1-(B11(N,I)*AL1(N,I)+B12(N,I)*AL2(N,I))*(TO+ZLY(N,J)*T1)         A  0549
        Z(7)      =B12(N,I)*(EFI+ZLY(N,I)*MFI) +B22(N,I)*(ETH+ZLY(N,I)*MTH)A  0550
       1-(B12(N,I)*AL1(N,I)+B22(N,I)*AL2(N,I))*(TO+ZLY(N,I)*T1)         A  0551
        Z(8)      =B12(N,I)*(EFI+ZLY(N,J)*MFI) +B22(N,I)*(ETH+ZLY(N,J)*MTH)A  0552
       1-(B12(N,I)*AL1(N,I)+B22(N,I)*AL2(N,I))*(TO+ZLY(N,J)*T1)         A  0553
        Z(9)      =B66(N,I)*(ETF+ZLY(N,I)*MTF)                          A  0554
        Z(10)     =B66(N,I)*(ETF+ZLY(N,J)*MTF)                          A  0555
        WRITE (6,1730) S, (Z(LLL),LLL=1,10)                             A  0556   830
 1730   FORMAT (1X,1H ,F8.5,2X,10E12.5)                                 A  0557
 1740   CONTINUE                                                        A  0558
C    THE FOLLOWING CARD CAUSES STRESSES TO BE DOUBLE SPACED
```

```
C     WRITE (6,1830)                                              A  0559
      GO TO 1790                                                  A  0560
 1750 Z(9)=C12*EFI+C22*ETH+E12*HFI+E22*HTH+(H22*TO+H21*T1)        A  0561
      Z(10)=                                                      A  0562
     1      D12*HFI+D22*HTH+E12*EFI+E22*ETH+(H21*TO+G21*T1)       A  0563
      WRITE (6,1730) S, (Z(I),I=1,10)                             A  0564   830
      IF(NPCH) 1790,1790,1760                                     A  0565
 1760 IF(M-L) 1770,1790,1790                                      A  0566
 1770 WRITE (7,1780) S, Z(4),Z(9),Z(8)                            A  0567   848
 1780 FORMAT (4F20.8)                                             A  0568
 1790 IF(M-L) 1800,1820,1820                                      A  0569
 1800 XOUT=XPR(IBK)+SZERO                                         A  0570
      IBCL=0                                                      A  0571
      CALL RUNGE                                                  A  0572   854
      Z(7)=0.0                                                    A  0573
      Z(8)=0.0                                                    A  0574
      DO 1810 I=1,NDE                                             A  0575
 1810 Z(I)=Y(I)                                                   A  0576
 1820 CONTINUE                                                    A  0577
      XX=0.0                                                      A  0578
      WRITE  (6,1830)                                             A  0579   864
 1830 FORMAT (1H )                                                A  0580
 1840 CONTINUE                                                    A  0581
      IX=IX+IPAR(N)                                               A  0582
 1850 CONTINUE                                                    A  0583
      IF(ISTR-1) 1870,1870,1860                                   A  0584
 1860 ISTR=0                                                      A  0585
      NNN=2                                                       A  0586
      NSTR=2                                                      A  0587
      GO TO 1470                                                  A  0588
 1870 IF(INXC-NXT) 1880,10,10                                     A  0589
 1880 INXC=INXC+1                                                 A  0590
      GO TO 260                                                   A  0591
      END                                                         A  0592
```

```
      SUBROUTINE ORTHO
      DIMENSION Y(8),YD(8),DUMM(27)
      DIMENSION MLY(20),ZLY(20,5)
      DIMENSIOND(100,8,9),DM(101,9),GA(4),GB(4,4),ITP(100),IA(8),IB(8,4)
      DIMENSION TR(8,8,4),TLI(8,8),ALFR(4)
      DIMENSION B11(20,4),B12(20,4),B22(20,4),B66(20,4),AL1(20,4)
      DIMENSION AL2(20,4),RHO(20,4)
      COMMON NDE,S,Y,YD,MM,JN,JMAX,MV,XOUT,IFREQ,DUMM,IBR,ISM
      COMMON XN,XLD,OMSQ,HTT,R1,R2,R3,SXN,CXS,INDEX
      COMMON PN,PL,PC,TO,T1,RH1,RH2,RH3,MLY,ZLY,EMFI,CNTH
      COMMON TR,TLI,IBCL,ALFL,ALFR,NBR
      COMMON NPRT,D,DM,GA,GB,ITP,NF,NFP,NPL,NM,IA,IB,OMEGA
      COMMON B11,B12,B22,B66,AL1,AL2,RHO,H11,H12,H21,H22,G12,G21
      COMMON C11,C12,C22,E11,E12,E22,D11,D12,D22,C66,E66,D66
      DIMENSION IL1(20,4),IL2(20),PSR(20,4)
      GO TO (10,10,220,190),INDEX
   10 MK=MLY(IBR)
      DO 60 I=1,MK
      READ (5,20) B11(IBR,I),B12(IBR,I),B22(IBR,I),B66(IBR,I),
     1AL1(IBR,I),AL2(IBR,I),RHO(IBR,I),IL1(IBR,I)
   20 FORMAT (7F10.6, I5)
      PSR(IBR,I)=B12(IBR,I)
      IF(IL1(IBR,I)) 30,60,40
   30 L3=-IL1(IBR,I)
      DY=FGEN (L3,1)
      GO TO 60
   40 L1=IL1(IBR,I)
      L2=IL1(IBR,I)+5
      DO 50 J=L1,L2
   50 DY=FGEN (J,1)
   60 CONTINUE
      READ (5,70)  (ZLY(IBR,I),I=1, 5),IL2(IBR)
   70 FORMAT (5F10.6,I5)
      IF(IL2(IBR)) 100,100,80
   80 L2=IL2(IBR)+MK
      L1=IL2(IBR)
      DO 90 I=L1,L2
   90 DY=FGEN (I,1)
  100 CONTINUE
      MK=MLY(IBR)
      DO 180 I=1,MK
      J=I+1
      WRITE (6,110) I, ZLY(IBR,I),ZLY(IBR,J)
  110 FORMAT (1HO, 8HLAYER NO, I3, 9H  FROM Z=,E12.5,7H  TO Z=,E12.5)
      IF(B22(IBR,I)-1.0E-8) 120,120,140
  120 E=B11(IBR,I)
      P=B12(IBR,I)
      P1=1.0-P*P
      B11(IBR,I)=E/P1
      B12(IBR,I)=P*E/P1
      B22(IBR,I)=B11(IBR,I)
      B66(IBR,I)=E*0.5/(1.0+P)
      WRITE (6,130) E,P
  130 FORMAT(1H ,5X,49HCONSISTS OF ISOTROPIC MATERIAL, YOUNGS MODULUS E=
     1, E12.5, 20H  POISSONS RATIO NU=,E12.5)
```

```
      GO TO 160                                                    B   0056
140   WRITE (6,150) B11(IBR,I),B12(IBR,I),B22(IBR,I),B66(IBR,I)    B   0057   61
150   FORMAT (1H ,5X,37HCONSISTS OF ORTHOTROPIC MATERIAL WITH,     B   0058
     16H  B11=,E12.5,6H  B12=,E12.5,6H  B22=,E12.5,6H  B66=,E12.5) B   0059
160   WRITE (6,170) AL1(IBR,I),AL2(IBR,I),RHO(IBR,I)               B   0060   86
170   FORMAT (1H ,5X,33HCOEFFICIENTS OF THERMAL EXPANSION, 6H  AFI=, B 0061
     1E12.5, 9H  ATHETA=,E12.5, 19H  MASS DENSITY RHO=,E12.5)      B   0062
180   CONTINUE                                                     B   0063
      RETURN                                                       B   0064
190   IF(IL2(IBR)) 200,200,230                                     B   0065
200   DO 210 I=1,MK                                                B   0066
      IF(IL1(IBR,I)) 250,210,250                                   B   0067
210   CONTINUE                                                     B   0068
      RETURN                                                       B   0069
220   L1=IL2(IBR)                                                  B   0070
      MK=MLY(IBR)                                                  B   0071
      L2=IL2(IBR)+MK                                               B   0072
      IF(IL2(IBR)) 290,250,230                                     B   0073
230   J=0                                                          B   0074
      DO 240 I=L1,L2                                               B   0075
      J=J+1                                                        B   0076
240   ZLY(IBR,J)=FGEN(I,2)                                         B   0077   120
250   DO 280 I=1,MK                                                B   0078
      IF(IL1(IBR,I)) 260,280,270                                   B   0079
260   L3=-IL1(IBR,I)                                               B   0080
      E=FGEN (L3,2)                                                B   0081   133
      P=PSR(IBR,I)                                                 B   0082
      P1=1.0-P*P                                                   B   0083
      B11(IBR,I)=E/P1                                              B   0084
      B22(IBR,I)=B11(IBR,I)                                        B   0085
      B12(IBR,I)=P*B11(IBR,I)                                      B   0086
      B66(IBR,I)=E*0.5/(1.0+P)                                     B   0087
      GO TO 280                                                    B   0088
270   J=IL1(IBR,I)                                                 B   0089
      B11(IBR,I)= FGEN (J  ,2)                                     B   0090   145
      B12(IBR,I)= FGEN (J+1,2)                                     B   0091   147
      B22(IBR,I)= FGEN (J+2,2)                                     B   0092   149
      B66(IBR,I)= FGEN (J+3,2)                                     B   0093   151
      AL1(IBR,I)= FGEN (J+4,2)                                     B   0094   153
      AL2(IBR,I)= FGEN (J+5,2)                                     B   0095   155
280   CONTINUE                                                     B   0096
290   C11=0.0                                                      B   0097
      RHH=0.0                                                      B   0098
      C12=0.0                                                      B   0099
      C22=0.0                                                      B   0100
      C66=0.0                                                      B   0101
      E11=0.0                                                      B   0102
      E12=0.0                                                      B   0103
      E22=0.0                                                      B   0104
      E66=0.0                                                      B   0105
      D11=0.0                                                      B   0106
      D12=0.0                                                      B   0107
      D22=0.0                                                      B   0108
      D66=0.0                                                      B   0109
      H11=0.0                                                      B   0110
      H12=0.0                                                      B   0111
```

```
      H22=0.0                                                          B 0112
      H21=0.0                                                          B 0113
      G12=0.0                                                          B 0114
      G21=0.0                                                          B 0115
      RH1=0.0                                                          B 0116
      RH2=0.0                                                          B 0117
      RH3=C.0                                                          B 0118
      HTT=  ZLY(IBR,MK+1)-ZLY(IBR,1)                                   B 0119
      DO 300 I=1,MK                                                    B 0120
      J=I+1                                                            B 0121
      Z1=ZLY(IBR,J) -ZLY(IBR,I)                                        B 0122
      Z2=ZLY(IBR,J)*ZLY(IBR,J)-ZLY(IBR,I)*ZLY(IBR,I)                   B 0123
      Z3=ZLY(IBR,J)*ZLY(IBR,J)*ZLY(IBR,J)-ZLY(IBR,I)*ZLY(IBR,I)*ZLY(IBR,B 0124
     1I)                                                               B 0125
      Z2=0.5*Z2                                                        B 0126
      Z3=0.333333*Z3                                                   B 0127
      C11=C11+B11(IBR,I)*Z1                                            B 0128
      C12=C12+B12(IBR,I)*Z1                                            B 0129
      C22=C22+B22(IBR,I)*Z1                                            B 0130
      C66=C66+B66(IBR,I)*Z1                                            B 0131
      E11=E11+B11(IBR,I)*Z2                                            B 0132
      E12=E12+B12(IBR,I)*Z2                                            B 0133
      E22=E22+B22(IBR,I)*Z2                                            B 0134
      E66=E66+B66(IBR,I)*Z2                                            B 0135
      D11=D11+B11(IBR,I)*Z3                                            B 0136
      D12=D12+B12(IBR,I)*Z3                                            B 0137
      D22=D22+B22(IBR,I)*Z3                                            B 0138
      D66=D66+B66(IBR,I)*Z3                                            B 0139
      A1=B11(IBR,I)*AL1(IBR,I)+B12(IBR,I)*AL2(IBR,I)                   B 0140
      A2=B12(IBR,I)*AL1(IBR,I)+B22(IBR,I)*AL2(IBR,I)                   B 0141
      H11=H11-A1*Z1                                                    B 0142
      H12=H12-A1*Z2                                                    B 0143
      H22=H22-A2*Z1                                                    B 0144
      H21=H21-A2*Z2                                                    B 0145
      G12=G12-A1*Z3                                                    B 0146
      G21=G21-A2*Z3                                                    B 0147
      RH1=RH1+RHO(IBR,I)*Z1                                            B 0148
      RH2=RH2+RHO(IBR,I)*Z2                                            B 0149
      RH3=RH3+RHO(IBR,I)*Z3                                            B 0150
  300 CONTINUE                                                         B 0151
      RETURN                                                           B 0152
      END                                                              B 0153
```

```
      SUBROUTINE DIFFEQ
      DIMENSION Y(8),YD(8),DUMM(27)
      DIMENSION MLY(20),ZLY(20,5)
      DIMENSIONDD(100,8,9),DP(101,9),GA(4),GB(4,4),ITP(100),IA(8),IB(8,4)
      DIMENSION TR(8,8,4),TLI(8,8),ALFR(4)
      DIMENSION B11(20,4),B12(20,4),B22(20,4),B66(20,4),AL1(20,4)
      DIMENSION AL2(20,4),RHO(20,4)
      COMMON NDE,S,Y,YD,MM,J9,JMAX,M9,XOUT,IFREQ,DUMM,IBK,ISH
      COMMON XN,XLD,OMSQ,MTT,R1,R2,R3,SXN,CXS,INDEX
      COMMON PN,PL,PC,TO,T1,RH1,RH2,RH3,MLY,ZLY,EMFI,EMTH
      COMMON TR,TLI,IBCL,ALFL,ALFR,NBK
      COMMON NPRT,D,DM,GA,GB,ITP,NF,NFP,NPL,NH,IA,IB,OMEGA
      COMMON B11,B12,B22,B66,AL1,AL2,RHO,H11,H12,H21,H22,G12,G21
      COMMON C11,C12,C22,E11,E12,E22,D11,D12,D22,C66,E66,D66
      IBCL=IBCL+1
      IF(IBCL-500) 130,130,10
   10 WRITE (6,20)
   20 FORMAT (1H0, 96HTHERE IS SOMETHING WRONG IN THIS SEGMENT. MORE THAC
     1N 500 POINTS HAVE BEE  USED IN INTEGRATION      )
      WRITE (6,30)
   30 FORMAT (1H0, 28HAT THIS POINT PARAMETERS ARE)
      WRITE (6,40) S,MM,XOUT
   40 FORMAT (1H0,2HX=,E12.4, 5H  MM=, E12.4, 7H  XOUT=, E12.4)
      WRITE (6,50)  (Y(I),I=1,NDE)
   50 FORMAT (1H0, 2HY=, 8E14.7)
      WRITE (6,60)  (YD(I),I=1,NDE)
   60 FORMAT (1H0, 3HDY=,  8E14.7)
      WRITE (6,70) R1,R2,R3,SXN,CXS
   70 FORMAT (1H0, 4H R1=,E12.4, 4H R2=, E12.4,4H R3=, E12.4,
     15H SXN=, E12.4, 5H CXS=, E12.4)
      WRITE (6,80)  PN,PL, PC,TO,T1
   80 FORMAT (1H0, 4H PN=, E12.4, 4H PL=, E12.4,4H PC=, E12.4,
     14H TO=, E12.4, 4H T1=, E12.4)
      WRITE (6,90)    C11,C12,C22,C66,E11,E12
   90 FORMAT (1H0, 5H C11=,E12.4, 5H C12=,E12.4, 5H C22=, E12.4,
     15H C66=, E12.4, 5H E11=, E12.4, 5H E12=, E12.4)
      WRITE (6,100)  E22,E66,D11,D12,D22,D66
  100 FORMAT (1H0, 5H E22=, E12.4, 5H E66=, E12.4, 5H D11=, E12.4,
     15H D12=, E12.4, 5H D22=, E12.4, 5H D66=, E12.4)
      WRITE (6,110)  H11,H12,H22,H21,G12,G21
  110 FORMAT (1H0, 5H H11=, E12.4, 5H H12=, E12.4, 5H H22=, E12.4,
     15H H21=, E12.4, 5H G12=, E12.4, 5H G21=, E12.4)
      JJ=MLY(IBK)+1
      WRITE (6,120)    (ZLY(IBK,J), J=1,JJ)
  120 FORMAT (1H0, 7HZS ARE=,10E12.4)
      CALL EXIT
  130 CONTINUE
      CALL ORTHO
      CALL INPUT
      CXR=CXS*R2
      SXR=SXN*R2
      IF(NDE-7) 140,140,150
  140 ETH=          CXR*Y(3) +SXR*Y(1)
      MTH=        CXR*Y(5)
      DEL=C11*D11-E11*E11
```

```
      EN= V(4)-C12*(TM-E.2*H(M-(H11*(CM12*T1))*XLD
      EM= V(6) -C12*(TM-D12*HM-(H12*(CG12*T1))*XLD
      EF1= (EN*D11-EM*F11)/DEL
      MF1= (EM*C11-EN*E11)/DEL
      VD(1)=-1*V(3)-V(5)
      VD(3)= EF1-Z1*V(1)
      VD(5)=MF1
      THN= C12*F1*C22*E(TM*E12*MF1*E22*MTM*(H22*TGM21*T1)*XLD
      THM= D12*M1*D22*MTM*E12*EF1*E22*E(TM*(H21*TGG21*T1)*XLD
      VD(2)=-CA*V(2)*SA*(TM-A)*V(4)-P*XLD
      VD(4)=CA*(TM-V(6))-A*V(2)-PL*XLD
      VD(6)= CA*(TM-V(6))*V(2)-F*XLD
      GO TO 160
150   ANG=ANG+?
      CLM=A1-SA
      CL2=A1*SA
      ETM=AMK*V(7)*C(4)*V(3) *SA*V(1)
      DTM=AN*V(1) *SA*V(7)
      HTM=ANM*FTM*CA*V(5)
      EPS=AN*V(1)-CA*V(7)
      MAS -2.00 *V(5)-2.00*C*V(1)*CA*(CLM-SA*V(7)*AN*A1*V(1))
      DEL=C11*D11-E11*F11
      EN= V(4)-C12*(TM-E.2*HM-(H11*(CM12*T1))*XLD
      EM= V(6) -C12*(TM-D12*HM-(H12*(CG12*T1))*XLD
      EF1= (EN*D11-EM*F11)/DEL
      MF1= (EN*C11-EM*E11)/DEL
      VD(1)=-1*V(3)-V(5)
      VD(3)= EF1-A1*V(1)
      VD(5)=MF1
      VD(7)=   ( V(4)-(C*SA*(  )*(PS-(1.0*SA*D)*MA)
     1/(C*2.0*SA*C*SA*SA)
      THN= C12*F1*C22*E(TM*E12*MF1*E22*MTM*(H22*TGM21*T1)*XLD
      THM= D12*M1*D22*MTM*C12*EF1*E22*E(TM*(H21*TGG21*T1)*XLD
      TTM= (  *VD(7))* *SA*VD(7))
      VD(2)= -2.  *C*  (TM- C*V(2)*SA*  *A1*V(4)*  *  -
     .P*XLD
      VD(4)=  -   *C*  (TM-V(4))   -A1*V(2)-PL*XLD
      VD(6)=  -2.  *C*  (TM-V(6)) *V(2)
      VD(6)= CLM*C*  -2.  *C*V(6) *  *  *SA*  -PC*XLD
160   IF(A)-1.0) 170,170,170
170   DO 180 I=1,
180   VD(I)=VD(I)/
190   RETURN
      END
```

```
      SUBROUTINE TRIANG
      DIMENSION V(8),VD(8),DUMM(27)
      DIMENSION PLV(20),ZLV(20,5)
      DIMENSION T4(8,8,4),TLI(8,9),ALFR(4)
      DIMENSIOND(100,8,9),DM(101,9),GA(4),GB(4,4),ITP(100),IA(8),IB(8,4)
      COMMON NDE,S,V,VD,NN,J9,JMAX,N9,XOUT,IFREQ,DUMM,IBR,ISN
      COMMON XM,ALD,CMSQ,MTT,K1,K2,K3,SXN,CXS,INDEX
      COMMON PN,PL,PC,TO,T1,NN1,NN2,NN3,NLV,ZLV,ENFI,ENTH
      COMMON TR,TLI,IBCL,ALFL,ALFR,NBX
      COMMON VPHI,D,DM,GA,GB,ITP,NF,NFP,NPL,NH,IA,IB,OMEGA
      DIMENSION D1(4,5),D2(4,4),U1(4,4),U2(4,4)
      IST=0
      IRA=1
      DO 10 I=1,NH
      DO 10 J=1,NH
      JJ=J+NH
   10 U1(I,J) =  D(1,I,JJ)
      CALL INVERT (U1,NH,DET)
      DO 20 I=1,NH
      II=I+NH
      DO 20 J=1,NH
      U2(I,J) = 0.0
      DO 20 L=1,NH
      LL=L+NH
   20 U2(I,J) = U2(I,J) + D(1,II,LL) * U1(L,J)
      DM(1,1)=DET
      DO 30 I=1,NH
      II=I+NH
      D1(1,I) = - D(1,I,NPL)
      D1(2,I) = - D(1,II,NPL)
      DO 30 J=1,NH
      D1(1,I) = D1(1,I) - D(1,I,J) * GA(J)
   30 D1(2,I) = D1(2,I) - D(1,II,J) * GA(J)
      DO 40 I=1,NH
      DO 40 J=1,NH
   40 D1(2,I) = D1(2,I) - U2(I,J) * D1(1,J)
      DO 50 I=1,NH
      II = I+NH
      D(1,I,NPL) = D1(1,I)
      D(1,II,NPL) = D1(2,I)
      DO 50 J=1,NH
   50 D(1,II,J) = U1(I,J)
      IF(NF-1) 520,520,60
   60 CALL INVERT (U2,NH,DET)
      DM(1,2) = DET
      DO 70 I=1,NH
      II=I+NH
      DO 70 J=1,NH
      JJ=J+NH
   70 D(1,II,JJ) = U2(I,J)
      DO 510 K=2,NF
      DO 80 I=1,NH
      DO 80 J=1,NH
      JJ=J+NH
      D2(I,J) = 0.0
```

```
      DO 80 L=1,NH                                          D   0056
      LL= L+NH                                              D   0057
   80 D2(I,J) = D2(I,J) + D(K,I,L) * D(K-1,LL,JJ)           D   0058
      IF(ITP(K-1)-2) 110,90,90                              D   0059
   90 DO 100 I=1,NH                                         D   0060
      DO 100 J=1,NH                                         D   0061
      JJ=J+NH                                               D   0062
      U1(I,J) = D(K,I,JJ)                                   D   0063
      DO 100 L=1,NH                                         D   0064
  100 U1(I,J) = U1(I,J) - D2(I,L) *D(K-1,L,JJ)              D   0065
      GO TO 130                                             D   0066
  110 DO 120 I=1,NH                                         D   0067
      DO 120 J=1,NH                                         D   0068
      JJ=J+NH                                               D   0069
  120 U1(I,J) = D(K,I,JJ) + D2(I,J)                         D   0070
  130 IF( ITP(K)-1) 200,140,140                             D   0071
  140 IF(IST  -1) 150,150,180                               D   0072
  150 IRR=IRR+1                                             D   0073
      IST=K                                                 D   0074
      DO 170 I=1,NH                                         D   0075
      II=I+NH                                               D   0076
      DO 160 J=1,NH                                         D   0077
      JJ=J+NH                                               D   0078
      D(K-1,I,JJ)=0.0                                       D   0079
  160 D(K-1,I,J )=0.0                                       D   0080
  170 D(K-1,I,II)=-1.0                                      D   0081
  180 DO 190 I=1,NH                                         D   0082
      DO 190 J=1,NH                                         D   0083
      JJ=J+NH                                               D   0084
      D(K,I,J)=0.0                                          D   0085
      DO 190 L=1,NH                                         D   0086
  190 D(K,I,J) = D(K,I,J) - D2(I,L) * D(K-1,L,JJ)           D   0087
  200 CALL INVERT (U1,NH,DET)                               D   0088   193
      DM(K,1)=DET                                           D   0089
      DO 210 I=1,NH                                         D   0090
      D1(1,I) = - D(K,I,NPL)                                D   0091
      DO 210 J=1,NH                                         D   0092
      JJ=J+NH                                               D   0093
  210 D1(1,I) = D1(1,I) - D2(I,J) * D(K-1,JJ,NPL)           D   0094
      DO 220 I=1,NH                                         D   0095
      II=I+NH                                               D   0096
      DO 220 J=1,NH                                         D   0097
      JJ = J + NH                                           D   0098
      D2(I,J) = 0.0                                         D   0099
      DO 220 L=1,NH                                         D   0100
      LL = L + NH                                           D   0101
  220 D2(I,J) = D2(I,J) + D(K,II,L) * D(K-1,LL,JJ)          D   0102
      IF( ITP(K)-1) 250,230,230                             D   0103
  230 DO 240 I=1,NH                                         D   0104
      DO 240 J=1,NH                                         D   0105
      JJ=J+NH                                               D   0106
      D(K,I,JJ)=0.0                                         D   0107
      DO 240 L=1,NH                                         D   0108
  240 D(K,I,JJ) = D(K,I,JJ) - D2(I,L) * D(K-1,L,JJ)         D   0109
  250 DO 260 I=1,NH                                         D   0110
      II=I+NH                                               D   0111
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D          - EFN    SOURCE STATEMENT  -  IFN(S)  -

      D1(2,I) = -D(K,II,NPL)                              D   0112
      DO 260 J=1,NH                                       D   0113
      JJ=J+NH                                             D   0114
  260 D1(2,I) = D1(2,I) - D2(I,J) * D(K-1,JJ,NPL)         D   0115
      IF(ITP(K-1)-2) 300,270,270                          D   0116
  270 DO 280 I=1,NH                                       D   0117
      DO 280 J=1,NH                                       D   0118
      JJ=J+NH                                             D   0119
      U2(I,J) = 0.0                                       D   0120
      DO 280 L=1,NH                                       D   0121
  280 U2(I,J) = U2(I,J) - D2(I,L) * D(K-1,L,JJ)           D   0122
      DO 290 I=1,NH                                       D   0123
      DO 290 J=1,NH                                       D   0124
  290 D2(I,J) = U2(I,J)                                   D   0125
  300 DO 310 I=1,NH                                       D   0126
      II=I+NH                                             D   0127
      DO 310 J=1,NH                                       D   0128
      JJ=J+NH                                             D   0129
  310 D2(I,J) = D(K,II,JJ) + D2(I,J)                      D   0130
      DO 320 I=1,NH                                       D   0131
      DO 320 J=1,NH                                       D   0132
      U2(I,J) = 0.0                                       D   0133
      DO 320 L=1,NH                                       D   0134
  320 U2(I,J) = U2(I,J) + D2(I,L) * U1(L,J)               D   0135
      IF( ITP(K)-1) 350,330,330                           D   0136
  330 DO 340 I=1,NH                                       D   0137
      DO 340 J=1,NH                                       D   0138
      JJ=J+NH                                             D   0139
      DO 340 L=1,NH                                       D   0140
  340 D(K,I,JJ) = D(K,I,JJ) - U2(I,L) * D(K,L,J)          D   0141
  350 DO 360I=1,NH                                        D   0142
      DO 360J=1,NH                                        D   0143
  360 D1(2,I) = D1(2,I)- U2(I,J) * D1(1,J)                D   0144
      DO 370 I=1,NH                                       D   0145
      II=I+NH                                             D   0146
      D(K,I,NPL) = D1(1,I)                                D   0147
      D(K,II,NPL) = D1(2,I)                               D   0148
      DO 370 J=1,NH                                       D   0149
  370 D(K,II,J) = U1(I,J)                                 D   0150
      IF(K-NF) 380,520,520                                D   0151
  380 CALL INVERT (U2,NH,DET)                             D   0152     362
      DM(K,2) = DET                                       D   0153
      DO 390 I=1,NH                                       D   0154
      II=I+NH                                             D   0155
      DO 390 J=1,NH                                       D   0156
      JJ=J+NH                                             D   0157
  390 D(K,II,JJ) = U2(I,J)                                D   0158
      IF( ITP(K)-1) 510,510,400                           D   0159
  400 DO 410 I=1,NH                                       D   0160
      II=I+NH                                             D   0161
  410 D(K,II,NPL) = D(K,II,NPL) + GB(I,IKK)               D   0162
      DO 420 I=1,NH                                       D   0163
      II=I+NH                                             D   0164
      DO 420 J=1,NH                                       D   0165
      U1(I,J)=D(K+1,I,J)                                  D   0166
  420 D1(I,J)=D(K+1,II,J)                                 D   0167
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      DO 490 N=IST,K                                              D 0168
      DO 430 I=1,NH                                               D 0169
      DO 430 J=1,NH                                               D 0170
      JJ=J+NH                                                     D 0171
      U2(I,J)=0.0                                                 D 0172
      D2(I,J)=0.0                                                 D 0173
      DO 430 L=1,NH                                               D 0174
      LL=L+NH                                                     D 0175
      U2(I,J) = U2(I,J) + U1(I,L) * D(N-1,LL,JJ)                  D 0176
  430 D2(I,J) = D2(I,J) + D1(I,L) * D(N-1,LL,JJ)                  D 0177
      DO 440 I=1,NH                                               D 0178
      II=I+NH                                                     D 0179
      DO 440 J=1,NH                                               D 0180
      JJ=J+NH                                                     D 0181
      DO 440 L=1,NH                                               D 0182
      D(K+1,I,JJ )= D(K+1,I ,JJ)- U2(I,L)*D(N-1,L,JJ)             D 0183
  440 D(K+1,II,JJ)= D(K+1,II,JJ)- D2(I,L)*D(N-1,L,JJ)             D 0184
      DO 450 I=1,NH                                               D 0185
      II=I+NH                                                     D 0186
      DO 450 J=1,NH                                               D 0187
      JJ=J+NH                                                     D 0188
      D(K+1,I,NPL) = D(K+1, I,NPL)+ U2(I,J) * D(N-1,JJ,NPL)       D 0189
  450 D(K+1,II,NPL) = D(K+1,II,NPL) + D2(I,J) * D(N-1,JJ,NPL)     D 0190
      DO 460 I=1,NH                                               D 0191
      DO 460 J=1,NH                                               D 0192
      U1(I,J)=0.0                                                 D 0193
      D1(I,J)=0.0                                                 D 0194
      DO 460 L=1,NH                                               D 0195
      LL=L+NH                                                     D 0196
      U1(I,J) = U1(I,J) + U2(I,L) * D(N,LL,J)                     D 0197
  460 D1(I,J) = D1(I,J) + D2(I,L) * D(N,LL,J)                     D 0198
      DO 470 I=1,NH                                               D 0199
      II=I+NH                                                     D 0200
      DO 470 J=1,NH                                               D 0201
      JJ=J+NH                                                     D 0202
      DO 470 L=1,NH                                               D 0203
      D(K+1,I ,JJ)= D(K+1,I ,JJ) -U1(I,L)*D(N,L,J)                D 0204
  470 D(K+1,II,JJ)= D(K+1,II,JJ) -D1(I,L)*D(N,L,J)                D 0205
      DO 480 I=1,NH                                               D 0206
      II=I+NH                                                     D 0207
      DO 480 J=1,NH                                               D 0208
      D(K+1,I, NPL) = D(K+1,I, NPL) + U1(I,J) * D(N,J,NPL)        D 0209
  480 D(K+1,II,NPL) = D(K+1,II,NPL) + D1(I,J) * D(N,J,NPL)        D 0210
  490 CONTINUE                                                    D 0211
      DO 500 I=1,NH                                               D 0212
      II=I+NH                                                     D 0213
      DO 500 J=1,NH                                               D 0214
      D(K+1,I ,J )= U1(I,J)                                       D 0215
  500 D(K+1,II,J )= D1(I,J)                                       D 0216
      IST=0                                                       D 0217
  510 CONTINUE                                                    D 0218
  520 DO 530 I=1,NH                                               D 0219
      II=I+NH                                                     D 0220
  530 D(NF,II,NPL) = D(NF,II,NPL) + GB(I,1)                       D 0221
      CALL INVERT (U2,NH,DET)                                     D 0222    542
      IF(NPRT) 610,610,540                                        D 0223
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540 CONTINUE                                                    D  0224
    WRITE (6,550)                                               D  0225   545
550 FORMAT (1H0,40X,45HSOLUTION IS BASED ON FOLLOWING C-SUB-M MATRIX) D  0226
    WRITE (6,560)                                               D  0227
   1            ((U2(I,J),     J=1,NH),I=1,NH)                  D  0228   546
560 FORMAT (1H , 22X,4E18.8)                                    D  0229
    WRITE (6,570)                                               D  0230
   1            (DM(I,1),I=1,NF )                               D  0231   556
570 FORMAT(1H ,8HDET(UE)=,8E15.7)                               D  0232
    J=NF-1                                                      D  0233
    WRITE (6,580)                                               D  0234
   1           (DM(I,2),I=1,J)                                  D  0235   563
580 FORMAT(1H ,8HDET(UC)=,8E15.7)                               D  0236
    DIAG=1.0                                                    D  0237
    DO 590 I=1,NH                                               D  0238
590 DIAG=DIAG*U2(I,I)                                           D  0239
    WRITE (6,600) DET,DIAG                                      D  0240   576
600 FORMAT (1H0, 12HDETERMINANT=,E16.8,7H  DIAG=,E14.7)         D  0241
610 CONTINUE                                                    D  0242
    DO 620 I=1,NH                                               D  0243
    II=I+NH                                                     D  0244
    DM(NFP,II)=0.0                                              D  0245
    DO 620 J=1,NH                                               D  0246
    JJ=J+NH                                                     D  0247
620 D(NF,II,JJ)=U2(I,J)                                         D  0248
    IST=0                                                       D  0249
    DO 760 N=1,NF                                               D  0250
    K=NFP-N                                                     D  0251
    IF( ITP(K)-1) 630,660,650                                   D  0252
630 IF(IST-1) 660,680,640                                       D  0253
640 IST=0                                                       D  0254
    GO TO 660                                                   D  0255
650 IK=K+1                                                      D  0256
    IST=2                                                       D  0257
660 DO 670 I=1,NH                                               D  0258
    II=I+NH                                                     D  0259
    DO 670 J=1,NH                                               D  0260
    JJ=J+NH                                                     D  0261
670 D(K,II,NPL) = D(K,II,NPL) - D(K,I,JJ) * DM(IK,JJ)           D  0262
    IF(ITP(K)-1) 680,680,700                                    D  0263
680 DO 690 I=1,NH                                               D  0264
    II=I+NH                                                     D  0265
690 D(K,II,NPL) = D(K,II,NPL) + DM(K+1,II)                      D  0266
700 DO 710 I=1,NH                                               D  0267
    II=I+NH                                                     D  0268
    DM(K+1,I) =0.0                                              D  0269
    DO 710 L=1,NH                                               D  0270
    LL=L+NH                                                     D  0271
710 DM(K+1,I) = DM(K+1,I) + D(K,II,LL) * D(K,LL,NPL)            D  0272
    IF(ITP(K)-1) 740,720,720                                    D  0273
720 DO 730 I=1,NH                                               D  0274
    DO 730 J=1,NH                                               D  0275
    JJ=J+NH                                                     D  0276
730 D(K,I,NPL) = D(K,I,NPL) - D(K,I,J) * DM(IK,JJ)              D  0277
740 DO 750 I=1,NH                                               D  0278
750 D(K,I,NPL) = D(K,I,NPL) + DM(K+1,I)                         D  0279
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      DO 760 I=1,NH                                          D    0280
      II=I+NH                                                D    0281
      DM(K,II)=0.0                                           D    0282
      DO 760 L=1,NH                                          D    0283
  760 DM(K,II) = DM(K,II) + D(K,II,L) * D(K,L,NPL)           D    0284
      DO 770 I=1,NH                                          D    0285
  770 DM(1,I)=GA(I)                                          D    0286
      RETURN                                                 D    0287
      END                                                    D    0288
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      SUBROUTINE BOUND                                              E  0001
      DIMENSION Y(8),YD(8),DUMM(27)                                 E  0002
      DIMENSION NLY(20),ZLY(20,5)                                   E  0003
      DIMENSION TR(8,8,4),TLI(8,6),ALFR(4)                          E  0004
      COMMON NDE,S,Y,YD,HH,J9,JMAX,M9,XOUT,IFREQ,DUMM,IBR,ISH       E  0005
      COMMON XN,XLD,OMSQ,HTT,R1,R2,R3,SXN,CXS,INDEX                 E  0006
      COMMON PN,PL,PC,TO,T1,RH1,RH2,RH3,MLY,ZLY,EMFI,EMTH           E  0007
      COMMON TR,TLI,IACL,ALFL,ALFR,NBR                              E  0008
C     NOTE THAT MATRIX TLI HERE IS Y(A)=TLI*U(A)                    E  0009
C     TR MATRIX IS U(B)=TR*Y(B)                                     E  0010
      DO 10 I=1,NDE                                                 E  0011
      DO 10 J=1,NDE                                                 E  0012
      TLI(I,J)=0.0                                                  E  0013
      DO 10 K=1,NBR                                                 E  0014
   10 TR (I,J,K)=0.0                                                E  0015
      DO 20 I=1,NDE                                                 E  0016
      TLI(I,I)=1.0                                                  E  0017
      DO 20 K=1,NBR                                                 E  0018
   20 TR (I,I,K)=1.0                                                E  0019
      ALFA=ALFL*1.74533E-02                                         E  0020
      SIL=SIN (ALFA)                                                E  0021      27
      COL=COS (ALFA)                                                E  0022      28
      TLI(1,1)=  COL                                                E  0023
      TLI(1,3)=  SIL                                                E  0024
      TLI(2,2)=  COL                                                E  0025
      TLI(2,4)=  SIL                                                E  0026
      TLI(3,1)= -SIL                                                E  0027
      TLI(3,3)=  COL                                                E  0028
      TLI(4,2)= -SIL                                                E  0029
      TLI(4,4)=  COL                                                E  0030
      DO 30 K=1,NBR                                                 E  0031
      ALFA=ALFR(K)*1.74533E-02                                      E  0032
      SIR=SIN (ALFA)                                                E  0033      33
      COR=COS (ALFA)                                                E  0034      34
      TR(1,1,K)=COR                                                 E  0035
      TR(1,3,K)=-SIR                                                E  0036
      TR(2,2,K)=COR                                                 E  0037
      TR(2,4,K)=-SIR                                                E  0038
      TR(3,1,K)=SIR                                                 E  0039
      TR(3,3,K)=COR                                                 E  0040
      TR(4,2,K)=SIR                                                 E  0041
   30 TR(4,4,K)=COR                                                 E  0042
   40 RETURN                                                        E  0043
      END                                                           E  0044
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      SUBROUTINE INPUT                                                   F
      DIMENSION Y(6),YC(3),DUMR(27)                                      F
      DIMENSION RLY(20),ZLY(20,5)                                        F
      COMMON NLE,S,Y,YD,IF,JV,JMAX,N9,XCUT,IFREQ,DUMF,IBR,ISH            F
      COMMON XN,XLD,CMSQ,HTT,R1,R2,R3,SX4,CXS,INDEX                      F
      COMMON PN,PL,PC,TC,T1,RH1,RH2,RH3,RLY,ZLY,EMFI,EXTH                F
      DIMENSION VN(10,4),IL1(10),IL2(10),IFG(10,4)                       F
      DIMENSION VK(10,5),IK2(10),IK1(10),IKG(10,5),IWORK(10)             F
      GO TO (10,250,390,480),INDEX                                       F
   10 READ  (5,20) (VN(IBR,I), I=1,4), IL2(IBR)                          F    3
   20 FORMAT (4F10.5,I5)                                                 F
      IF(IL2(IBR)) 70,70,30                                              F
   30 READ  (5,40) (IFG(IBR,I),I=1,3), IL1(IBR)                          F   13
   40 FORMAT (10I5)                                                      F
      WRITE (6,50)  (IFG(IBR,I),I=1,3)                                   F   21
   50 FORMAT (1H0, 29HVARIABLE SHELL PARAMETERS ARE,  4I5)               F
      L1=IL1(IBR)                                                        F
      L2=IL2(IBR)+IL1(IBR)-1                                             F
      DO 60 I=L1,L2                                                      F
   60 DY=FGEN (I,1)                                                      F   37
   70 GO TO (80,100,120,140,160,200,220,240,180), ISH                    F
   80 WRITE (6,90)                                                       F   41
   90 FORMAT (1H0,  30HNO SHELL NO. 1 IN THIS PROGRAM)                   F
      CALL EXIT                                                          F   42
  100 WRITE  (6,110) ISH,VN(IBR,1),VN(IBR,2), VN(IBR,3)                  F   43
  110 FORMAT (1H0,      20HCYLINDRICAL SHELL NO,I3,2X,                   F
     1 4H  H=,E12.5,2X,4H  R=,E12.5,  2X, 4HPHI=, F10.3, 9H  DEGREES)    F
      RETURN                                                             F
  120 WRITE (6,130)  ISH,VN(IBR,1),VN(IBR,2), VN(IBR,4)                  F   47
  130 FORMAT (1H0,       18HSPHERICAL SHELL NO,I3,4X,                    F
     1 4H  H=,E12.5,2X,4H  R=,E12.5, 12H  DIRECTION=, F3.0 )             F
      RETURN                                                             F
  140 WRITE (6,150)  ISH,VN(IBR,1),VN(IBR,2), VN(IBR,4)                  F   51
  150 FORMAT (1H0,      21HPARABOLOIDAL SHELL NO,I3,3X,                  F
     1 4H  H=,E12.5,2X,5H  2P=,E12.5,  12H  DIRECTION=, F3.0)            F
      RETURN                                                             F
  160 WRITE (6,170)  ISH,VN(IBR,1),VN(IBR,2),VN(IBR,3),VN(IBR,4)         F   55
  170 FORMAT (1H0,       20HELLIPSOIDAL SHELL NO,I3,3X,                  F
     1 4H  H=,E12.5,2X,4H  A=,E12.5, 2X,4H  B=,E12.5,10H  DIRECTN=,F3.0)F
      RETURN                                                             F
  180 WRITE(6,190)  ISH,VN(IBR,1),VN(IBR,2),VN(IBR,3),VN(IBR,4)          F   60
  190 FORMAT (1H0,       20HHYPERBOLIC  SHELL NO,I3,3X,                  F
     1 4H  H=,E12.5,2X,4H  A=,E12.5, 2X,4H  B=,E12.5,10H  DIRECTN=,F3.0)F
      RETURN                                                             F
  200 WRITE  (6,210) ISH,VN(IBR,1),VN(IBR,2),VN(IBR,3)                   F   65
  210 FORMAT (1H0,       16HCONICAL SHELL NO,I3,2X,    4H  H=,F10.6, 2X, F
     14HPHI=,F10.3, 8H DEGREES, 2X,4H  A=,E12.5)                         F
      RETURN                                                             F
  220 WRITE (6,230)  ISH,VN(IBR,1),VN(IBR,2),VN(IBR,3),VN(IBR,4)         F   69
  230 FORMAT (1H0,      17HTOROIDAL SHELL NO,I3,3X,                      F
     1 4H  H=,E12.5,2X,4H  A=,E12.5, 2X,4H  B=,E12.5,10H  DIRECTN=,F3.0)F
      RETURN                                                             F
  240 WRITE  (6,250) ISH, (VN(IBR,I),I=1,4)                              F   74
  250 FORMAT (1H0,  16HGENERAL SHELL NO, I3, 4H  H=,E12.5,               F
     17H 1/RFI=,E12.5, 5H   R=, E12.5, 4H FI=, F10.3,  4H DEG)           F
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    RETURN                                                        F  0056
260 READ (5,270)  (VK(IBR,I),I=1,5), IK2(IBR),INORM(IBR),RPM      F  0057   61
270 FORMAT (5F10.5,2I5,F10.5)                                     F  0058
    RPS=6.28318*RPM/60.0                                          F  0059
    RPQ=RPS*RPS                                                   F  0060
    EMTH=0.0                                                      F  0061
    EMFI=0.0                                                      F  0062
    IF(INORM(IBR)) 300,290,300                                    F  0063
280 WRITE (6,290) (VK(IBR,I),I=1,5)                               F  0064   92
290 FORMAT (1H0,30HSURFACE AND TEMP LOADS ARE  P=,E12.5,6H  PFI=,E12.5F  0065
   1,9H  PTHETA=,E12.5,5H  TL=,E12.5,5H  TU=,E12.5)              F  0066
    GO TO 340                                                     F  0067
300 WRITE(6,310)(VK(IBR,I),I=1,5)                                 F  0068   100
310 FORMAT (1H0,30HSURFACE AND TEMP LOADS ARE P1=,E12.5,6H   P2=,E12.5F  0069
   1,9H  PTHETA=,E12.5,5H  TL=,E12.5,5H  TU=,E12.5)              F  0070
    IF(RPM) 340,340,320                                           F  0071
320 WRITE (6,330) RPM                                             F  0072   108
330 FORMAT (1H0, 45HSHELL IS SPINNING ABOUT AXIS OF SYMMETRY WITH,  F  0073
   1E12.5, 5H  RPM)                                               F  0074
340 CONTINUE                                                      F  0075
    IF(IK2(IBR)) 380,380,350                                      F  0076
350 READ  (5,40) (IKG(IBR,I),I=1,5), IK1(IBR)                     F  0077   112
    WRITE (6,360)    (IKG(IBR,I),I=1,5)                           F  0078   120
360 FORMAT (1H0,  28HVARIABLE LOAD PARAMETERS ARE,  5I5)          F  0079
    K1=IK1(IBR)                                                   F  0080
    K2=IK2(IBR)+IK1(IBR)-1                                        F  0081
    DO 370 I=K1,K2                                                F  0082
370 DY= FGEN(I,1)                                                 F  0083   136
380 RETURN                                                        F  0084
390 CONTINUE                                                      F  0085
    L1=IL1(IBR)                                                   F  0086
    L2=IL2(IBR)+IL1(IBR)-1                                        F  0087
    K1=IK1(IBR)                                                   F  0088
    K2=IK2(IBR)+IK1(IBR)-1                                        F  0089
    MK=MLY(IBR)                                                   F  0090
    GO TO(80,400,410,420,430,450,460,470,440), ISH                F  0091
400 R2=ABS (1.0/VN(IBR,2))                                        F  0092
    R1=0.0                                                        F  0093
    R3=1.0                                                        F  0094
    CXS=0.0                                                       F  0095
    ALFA=VN(IBR,3)*1.745329E-02                                   F  0096
    SXN=SIN (ALFA)                                                F  0097   156
    GO TO 480                                                     F  0098
410 R1=1.0/VN(IBR,2)                                              F  0099
    R3=ABS (R1)                                                   F  0100
420 GO TO 480                                                     F  0101
430 BSC= (VN(IBR,3)/VN(IBR,2))*(VN(IBR,3)/VN(IBR,2))              F  0102
    GO TO 480                                                     F  0103
440 BSC= (VN(IBR,3)/VN(IBR,2))*(VN(IBR,3)/VN(IBR,2))              F  0104
    GO TO 480                                                     F  0105
450 R1=0.0                                                        F  0106
    R3=1.0                                                        F  0107
    ALFA = VN(IBR,2)*0.1745329E-01                                F  0108
    SXN=SIN (ALFA)                                                F  0109   171
    CXS=COS (ALFA)                                                F  0110   172
    GO TO 480                                                     F  0111
```

```
F            -  EFN   SOURCE STATEMENT  -  IFN(S)  -

460 R1=1.0/VN(IBR,3)                                                F   0112
    R3=ABS (R1)                                                     F   0113
    GC TO 460                                                       F   0114
470 R3=1.0                                                          F   0115
480 IF(IL2(IBR)) 510,510,490                                        F   0116
490 IK=0                                                            F   0117
    DO 500 I=L1,L2                                                  F   0118
    IK=IK+1                                                         F   0119
    J=IFG(IBR,IK)                                                   F   0120
500 VN(IBR,J)=FGEN (1,2)                                            F   0121   189
510 IF(IK2(IBR)) 540,540,520                                        F   0122
520 IK=0                                                            F   0123
    DO 530 I=K1,K2                                                  F   0124
    IK=IK+1                                                         F   0125
    J=IKG(IBR,IK)                                                   F   0126
530 VK(IBR,J)=FGEN (1,2)                                            F   0127   204
540 PN=VK(IBR,1)                                                    F   0128
    PL=VK(IBR,2)                                                    F   0129
    PC=VK(IBR,3)                                                    F   0130
    TO= (VK(IBR,4)*ZLY(IBR,MK+1) -VK(IBR,5)*ZLY(IBR,1))/HTT         F   0131
    T1= (VK(IBR,5)-VK(IBR,4))/HTT                                   F   0132
    GO TO(80,620,550,560,570,590,600,610,580), ISH                  F   0133
550 ARG=S*VN(IBR,4)                                                 F   0134
    SXN=SIN (ARG)                                                   F   0135   221
    CXS=COS (ARG)                                                   F   0136   222
    R2=ABS (R1/SXN)                                                 F   0137
    GO TO 620                                                       F   0138
560 ARG=S*VN(IBR,4)                                                 F   0139
    SXN=SIN (ARG)                                                   F   0140   226
    CXS=COS (ARG)                                                   F   0141   227
    R2=ABS (CXS/(SXN*VN(IBR,2)))                                    F   0142
    R1= ABS(CXS*CXS*CXS)/VN(IBR,2)                                  F   0143
    R3=ABS (R1)                                                     F   0144
    GO TO 620                                                       F   0145
570 ARG=S*VN(IBR,4)                                                 F   0146
    SXN=SIN (ARG)                                                   F   0147   233
    CXS=COS (ARG)                                                   F   0148   234
    R=SQRT (BSQ+(1.-BSQ)*SXN*SXN)                                   F   0149   235
    R1=R*R*R/(BSQ*VN(IBR,2))                                        F   0150
    R2=ABS (R   /(SXN*VN(IBR,2)))                                   F   0151
    R3=ABS (R1)                                                     F   0152
    GO TO 620                                                       F   0153
580 ARG=S*VN(IBR,4)                                                 F   0154
    SXN=SIN (ARG)                                                   F   0155   241
    CXS=COS (ARG)                                                   F   0156   242
    R=SQRT (SXN*SXN -BSQ*CXS*CXS)                                   F   0157   243
    R1= -VN(IBR,2)*R*R*R/(VN(IBR,3)*VN(IBR,3))                      F   0158
    R2=ABS (R   /(SXN*VN(IBR,2)))                                   F   0159
    R3=ABS (R1)                                                     F   0160
    GO TO 620                                                       F   0161
590 R2= ABS (1.0/((VN(IBR,3)+S)*CXS))                               F   0162
    GO TO 620                                                       F   0163
600 ARG=S*VN(IBR,4)                                                 F   0164
    SXN=SIN (ARG)                                                   F   0165   254
    CXS=COS (ARG)                                                   F   0166   255
    R2=ABS (1.0/(VN(IBR,2)+VN(IBR,3)*SXN))                          F   0167
```

```
    GO TO 620                                          F    0168
610 R1=VN(IBR,2)                                       F    0169
    R2=ABS (1.0/VN(IBR,3))                             F    0170
    ARG=  VN(IBR,4)*0.1745329E-01                      F    0171
    CXS=COS(ARG)                                       F    0172      263
    SXN=SIN(ARG)                                       F    0173      264
620 IF(XLD-0.5) 650,650,630                            F    0174
630 IF(INORM(IBR)) 650,650,640                         F    0175
640 PHR= VK(IBR,2)+RPQ*(RH1/R2+     SXN*RH2)           F    0176
    EMFI=RPQ*CXS*(RH2/R2+SXN*RH3)                      F    0177
    PN=VK(IBR,1)*CXS+PHR*SXN                           F    0178
    PL=-VK(IBR,1)*SXN+PHR*CXS                          F    0179
650 RETURN                                             F    0180
    END                                                F    0181
```

```
      FUNCTION FGEN (NM,S1)
      DIMENSION XP(30,20),YP(30,20),SL(30,20),M(30)
      COMMON NAMES,S
      IF(NM-30) 30,30,10
   10 WRITE (6,20)
   20 FORMAT(1H0,50HMAXIMUM NUMBER OF 30 FGEN SETS HAVE BEEN EXCEEDED )
      CALL EXIT
   30 CONTINUE
      GO TO (40,120),S1
   40 READ  (5,50) NAMEG,  M(NM)
   50 FORMAT (A5,I5)
      WRITE  (6,60) NAMEG, NM,M(NM)
   60 FORMAT (1H0,  10X, A5,  32H  LINEAR FUNCTION GENERATOR NO. ,
     1I3, 5H FROM, I4, 7H POINTS)
      MK=M(NM)
      READ  (5,70) (XP(NM,I),  YP(NM,I), I=1,MK)
   70 FORMAT (8F10.5)
      MU=0
      MZ=10
      MX= (M(NM)-1)/10+1
      DO 100 J=1,MX
      L=AMINO (MK,   MZ)
      L1=MU+1
      WRITE  (6,80) (YP(NM,I), I=L1,L)
      WRITE  (6,90)  (XP(NM,I), I=L1,L)
   80 FORMAT (1H0, 13HY COORDINATES,3X,10F10.5)
   90 FORMAT (1H0, 13HX COORDINATES,3X,10F10.5)
      MZ=MZ+10
  100 MU=MU+10
      MM=M(NM)-1
      DO 110 I=1,MM
      SL(NM,I)= (YP(NM,I+1)-YP(NM,I))/ (XP(NM,I+1)-XP(NM,I))
  110 YP(NM,I)=YP(NM,I) -SL(NM,I)*XP(NM,I)
      RETURN
  120 MK=M(NM)
      DO 130 I=1,MK
      J=I
      IF(XP(NM,I)-S) 130,130,140
  130 CONTINUE
  140 IF(J-1) 150,150,160
  150 J=2
  160 FGEN= SL(NM,J-1)*S +YP(NM,J-1)
      RETURN
      END
```

```
      SUBROUTINE INVERT (DP,MAX,DETERM)
      DIMENSION DP(4,4),M(4),C(4)
      DETERM = 1.
      DO 10 I = 1, MAX
      M(I) = - I
   10 CONTINUE
      DO 140 II = 1, MAX
      D = 0.0
      DO 60 K = 1, MAX
      IF (M(K)) 20,20,60
   20 DO 50 L = 1, MAX
      IF (M(L)) 30,30,50
   30 IF ( ABS(D)- ABS(DP(K,L))) 40,40,50
   40 LD = L
      KD = K
      D = DP(K,L)
   50 CONTINUE
   60 CONTINUE
      IF(KD-LD) 70,80,70
   70 DETERM =-DETERM
   80 DETERM = D*DETERM
      NEMP = -M(LD)
      M(LD)=M(KD)
      M(KD)= NEMP
      DO 90 I = 1, MAX
      C(I) = DP(I,LD)
      DP(I,LD) = DP(I,KD)
      DP(I,KD) = 0.0
   90 CONTINUE
      DP(KD,KD) = 1.
      DO 100 J = 1, MAX
      DP(KD,J) = DP(KD,J)/D
  100 CONTINUE
      DO 130 I = 1, MAX
      IF (I-KD) 110,130,110
  110 DO 120 J = 1, MAX
      DP(I,J) = DP(I,J) - C(I)*DP(KD,J)
  120 CONTINUE
  130 CONTINUE
  140 CONTINUE
      DO 170 I = 1, MAX
      L=0
  150 L=L+1
      IF (M(L)-I) 150,160,150
  160 M(L)=M(I)
      M(I)=I
      DO 170 J = 1, MAX
      TEMP = DP(L,J)
      DP(L,J) = DP(I,J)
      DP(I,J) = TEMP
  170 CONTINUE
      RETURN
      END
```

```
SUBROUTINE RUNGE
COMMON N,  X,Y,DY,HH,J, JMAX,M, XOUT,IFREQ,X1,X2,X3,Y1,Y2,Y3
DIMENSION Y(8),DY(8),Y1(8),Y2(8),Y3(8)
J=1
JMAX=1
IFREQ=3
M=1
CALL RUNKUT                                                    6
RETURN
END
```

```
      SUBROUTINE RUNKUT                                          K  0001
      COMMON N,  X,Y,DY,HH,J, JMAX,M, XOUT,IFREQ,X1,X2,X3,Y1,Y2,Y3  K  0002
      DIMENSION Y(8),DY(8),Y1(8),Y2(8),Y3(8)                    K  0003
      INDE9     = 0                                              K  0004
      CALL        ADJSTP                                         K  0005    3
      IF(J-JMAX) 10,10,50                                        K  0006
   10 INDE9 = INDE9 + 1                                          K  0007
      CALL INTPOL                                                K  0008    8
      IF(J-JMAX) 20,20,50                                        K  0009
   20 CALL STEP                                                  K  0010   12
      X1        = X2                                             K  0011
      X2        = X3                                             K  0012
      X3        = X                                              K  0013
      DO 30 I = 1, N                                             K  0014
      Y1(I)     = Y2(I)                                          K  0015
      Y2(I)     = Y3(I)                                          K  0016
   30 Y3(I)     = Y(I)                                           K  0017
      IF (INDE9 - IFREQ) 10,40,40                                K  0018
   40 INDE9     = 0                                              K  0019
      CALL        ADJSTP                                         K  0020   29
      IF(J-JMAX) 10,10,50                                        K  0021
   50 RETURN                                                     K  0022
      END                                                        K  0023
```

```
      SUBROUTINE ADJSTP
      COMMON N,  X,Y,DY,HH,J, JMAX,M, XOUT,IFREQ,X1,X2,X3,Y1,Y2,Y3
      DIMENSION Y(8),DY(8),Y1(8),Y2(8),Y3(8)
      KSL=0
      HFACT     = 1.0 E+31
      HFACT1    = 1.0E+30
      GO TO (30,10), K
   10 H1        = HH
      HH        = 2.0 * HH
      X         = X1
      DO 20 I = 1, N
   20 Y(I)      = Y1(I)
      GO TO 100
   30 KSL=1
   40 H1        = HH
      XXX       = X
      DO 50 I = 1, N
   50 Y1(I)     = Y(I)
      X1        = X
      CALL INTPOL                                                   24
      IF(J-JMAX) 60,60,250
   60 CALL STEP                                                     28
      DO 70 I = 1, N
   70 Y2(I)     = Y(I)
      X2        = X
      CALL INTPOL                                                   38
      IF(J-JMAX) 80,80,250
   80 CALL STEP                                                     42
      DO 90 I = 1, N
      Y3(I)     = Y(I)
   90 Y(I)      = Y1(I)
      X3        = X
      X         = XXX
      HH        = 2.0 * HH
  100 CALL STEP                                                     55
      DO 150 I = 1, N
      DELY    = ABS ( Y(I)-Y3(I))/30.0
      IF(DELY  -ABS (Y2(I))*1.E-05 )120,110,110
  110 IF( ABS (Y2(I))-1.0E-05) 120,130,130
  120 HFIRST    = 1.0E+30
      GO TO 140
  130 HFIRST=  (ABS (Y2(I))* 1.0E-05/DELY    ) **0.2                71
  140 CONTINUE
  150 HFACT=AMIN1 (HFACT, HFIRST   )
      IF (HFACT1 - HFACT) 160,160,170
  160 HH        = 2.0 * H1
      GO TO (40,230), M
  170 HH        = H1 * HFACT
      GO TO (180,230), M
  180 IF(KSL) 220,220,190
  190 KSL=0
      IF(ABS (HH)-ABS (H1)) 200,220,220
  200 DO 210 I = 1, N
  210 Y(I)      = Y1(I)
      X         = XXX
```

```
L          - EFN   SOURCE STATEMENT  -  IFN(S)  -

    GO TO 40                                            L   0056
220 KSL=0                                               L   0057
    M         = 2                                       L   0058
230 DO 240 I = 1, N                                     L   0059
240 Y(I)        = Y3(I)                                 L   0060
250 RETURN                                              L   0061
    END                                                 L   0062
```

```
      SUBROUTINE STEP                                             M  0001
      COMMON N,  A,Y,DY,HH,J, JMAX,M, XOUT,IFREQ,X1,X2,X3,Y1,Y2,Y3  M  0002
      DIMENSION Y(8),DY(8),Y1(8),Y2(8),Y3(8)                      M  0003
      DIMENSION Y0(8),P1(8)                                       M  0004
      DO 10 I = 1, N                                              M  0005
   10 Y0(I)     = Y(I)                                            M  0006
      X0        =X                                                M  0007
      CALL DIFFEQ                                                 M  0008    10
      DO 20 I = 1, N                                              M  0009
      P1(I)     = DY(I) * HH                                      M  0010
   20 Y(I)      = Y0(I) + P1(I)*0.5                               M  0011
      X         = X0 + HH*0.5                                     M  0012
      CALL DIFFEQ                                                 M  0013    23
      DO 30 I = 1, N                                              M  0014
      P1(I)= P1(I)+2.0*HH*DY(I)                                   M  0015
   30 Y(I)      = Y0(I) + 0.5*HH*DY(I)                            M  0016
      CALL DIFFEQ                                                 M  0017    37
      DO 40 I = 1, N                                              M  0018
      P1(I)= P1(I)+2.0*HH*DY(I)                                   M  0019
   40 Y(I)      = Y0(I) + HH*DY(I)                                M  0020
      X         = X0 + HH                                         M  0021
      CALL DIFFEQ                                                 M  0022    51
      DO 50 I = 1, N                                              M  0023
   50 Y(I)=Y0(I) + (P1(I)+HH*DY(I))*0.1666667                     M  0024
      RETURN                                                      M  0025
      END                                                         M  0026
```

```
      SUBROUTINE INTPCL                                              N 0001
      COMMON N, X,Y,DY,HH,J, JMAX,M, XOUT,IFREQ,X1,X2,X3,Y1,Y2,Y3    N 0002
      DIMENSION Y(8),DY(8),Y1(8),Y2(8),Y3(8)                         N 0003
      IF(ABS (XOUT    - X)-ABS (HH)) 10,10,20                        N 0004
   10 HH=XOUT-X                                                      N 0005
      CALL STEP                                                      N 0006
      J        = J + 1                                               N 0007
   20 RETURN                                                         N 0008
      END                                                            N 0009
```

```
C     EIGENVALUE PROGRAM FOR AXISYMMETRIC PRESTRESS FOR FREE VIBR + STABD 0001
      DIMENSION Y(8),YD(8),DUMM(27),PAR(2)                            0 0002
      DIMENSION NLY(20),ZLY(20,5)                                     0 0003
      DIMENSION TR(8,8,4),TLI(8,8),ALFR(4)                            0 0004
      DIMENSIOND(100,8,9),DP(101,9),GA(4),GB(4,4),ITP(100),IA(8),IB(8,4)0 0005
      DIMENSION E11(20,4),B12(20,4),B22(20,4),B66(20,4),AL1(20,4)     0 0006
      DIMENSION AL2(20,4),RHO(20,4)                                   0 0007
      COMMON NOE,S,Y,YD,HH,J9,JMAX,M9,XOUT,IFREQ,DUMM,IBR,ISH         0 0008
      COMMON PAR,LC,NFG,XMM,IVB                                       0 0009
      COMMON XN,XLD,OMSQ,HTT,R1,R2,R3,SXN,CXS,INDEX                   0 0010
      COMMON PN,PL,PC,TO,T1,RH1,RH2,RH3,NLY,ZLY                       0 0011
      COMMON TR,TLI,IBCL,ALFL,ALFR,NBR                                0 0012
      COMMON LD,ACC,IM,XA,NAC,IFR,NB,XXC,XXE,DETA,DETB,DETC           0 0013
      COMMON NPRT,D,DN,GA,GB,ITP,NF,NFP,NPL,NH,IA,IB,OMEGA            0 0014
      COMMON B11,B12,B22,B66,AL1,AL2,RHO,H11,H12,H21,H22,G12,G21      0 0015
      COMMON C11,C12,C22,E11,E12,E22,D11,D12,D22,C66,E66,D66          0 0016
      DIMENSION          SI(20),SX(20),XPR(20), ING(20), IPAR(20),Z(20)0 0017
      DIMENSION ISS(20),TR1(8,8),TR2(8,8),NTP(20),LCST(20),IFTP(50)   0 0018
      DIMENSION DTASV(50),DTBSV(50),XCSAV(50),XESAV(50),OMSAV(50),CY(50)0 0019
C     IVB=1 IS STABILITY, IVB=0 IS FREE-VIBRATION                     0 0020
   10 READ (5,20)    IBRM,ISTK,NBR,NXT,IVB,NPRT                       0 0021     1
   20 FORMAT(16I5)                                                    0 0022
      IF(IBRM)          30,30,50                                      0 0023
   30 WRITE (6,40)IBRM                                                0 0024     9
   40 FORMAT (1H0,50HIBRM=0 WHICH INDICATES END OF JOB. EXIT IS CALLED )0 0025
      CALL EXIT                                                       0 0026    10
   50 CONTINUE                                                        0 0027
      ACC=0.01                                                        0 0028
      XMM=1.0                                                         0 0029
      ERP=1.0E-05                                                     0 0030
      IBCL=0                                                          0 0031
      INXC=1                                                          0 0032
      WRITE (6,60)                                                    0 0033    13
   60 FORMAT (1H1, 10X, 109HSTABILITY AND FREE VIBRATION OF SHELLS WITH 0 0034
     1AXISYMMETRIC PRESTRESS, BY A. KALNINS, LEHIGH UNIV, BETHLEHEM, PA/0 0035
     220X,80HWRIGHT PATTERSON AIR FORCE BASE FLIGHT DYNAMICS LABORATORY 0 0036
     3VERSION, 22 JULY 1968)                                          0 0037
      IF(IVB) 90,90,70                                                0 0038
   70 WRITE (6,80) IBRM,NBR,NXT                                       0 0039    15
   80 FORMAT (1H0, 20X, 18HSTABILITY ANALYSIS, 8H  PARTS=,I3,         0 0040
     111H  BRANCHES=,I3,21H  NUMBER OF SUBCASES=,I3)                  0 0041
      GO TO 110                                                       0 0042
   90 WRITE (6,100) IBRM,NBR,NXT                                      0 0043    17
  100 FORMAT (1H0, 20X, 18HFREE VIBRATION    , 8H  PARTS=,I3,         0 0044
     111H  BRANCHES=,I3,21H  NUMBER OF SUBCASES=,I3)                  0 0045
  110 CONTINUE                                                        0 0046
      IF(NBR-3) 140,140,120                                           0 0047
  120 WRITE (6,130) NBR                                               0 0048    20
  130 FORMAT (1H0,I3,38H  BRANCHES EXCEED ALLOWED MAXIMUM OF 3)       0 0049
      CALL EXIT                                                       0 0050    21
  140 CONTINUE                                                        0 0051
      IF(IBRM-20) 170,170,150                                         0 0052
  150 WRITE (6,160) IBRM                                              0 0053    24
  160 FORMAT (1H0,I3,36H  PARTS EXCEED ALLOWED MAXIMUM OF 20)         0 0054
      CALL EXIT                                                       0 0055    25
```

```
170 CONTINUE                                                          0  0056
      NBR=NBR+1                                                       0  0057
      READ (5,180) ALFL, (ALFR(I),I=1,NBR)                            C  0058   27
180 FORMAT (7F10.5)                                                   0  0059
      WRITE (6,190) ALFL, (ALFR(I),I=1,NBR)                           C  0060   34
190 FORMAT (1H0, 51HANGLES OF ROTATION OF BOUNDARY CONDITIONS ARE ALFLO  0061
     1=,  E12.5, 8H  ALFRS=, 4E12.5)                                  0  0062
      IGCT=0                                                          0  0063
      DO 370 I=1,IBRN                                                 0  0064
      IBR=I                                                           C  0065
      WRITE (6,200) I                                                 U  0066   45
200 FORMAT (1H0, 10X, 7HPART NO,I3)                                   0  0067
      READ (5,210)                                                    C  0068
     1SI(I),SX(I),IPAR(I),ING(I),ISS(I),NTP(I),MLY(I)                 0  0069   46
210 FORMAT (2F10.5,5I5)                                               0  0070
      WRITE (6,220)                                                   0  0071
     1SI(I),SX(I),IPAR(I),ING(I),ISS(I),NTP(I),MLY(I)                 C  0072   54
220 FORMAT (1H0,3HSI=,E12.5,6H   SX=,E12.5, 8H   IPAR=,I3,7H   ING=,I30  0073
     1,14H    SHELL TYPE,I2, 7H   NTP=,I2, 14H   LAYERS MLY=,I2)      0  0074
      IGCT=IGCT+IPAR(I)                                               0  0075
      ISH=ISS(I)                                                      C  0076
      INDEX=1                                                         0  0077
      CALL INPUT                                                      U  0078   67
      CALL ORTHO                                                      0  0079   69
      IF(MLY(I)-4) 250,250,230                                        0  0080
230 WRITE (6,240) MLY(I),I                                            0  0081   73
240 FORMAT(1H0,I3,19H  LAYERS IN PART NO,I3,22H  EXCEED ALLOWED MAX 4)0  0082
      CALL EXIT                                                       C  0083   75
250 CONTINUE                                                          0  0084
      READ (5,260) IBRT,NTR,LCST(IBR)                                 0  0085   76
260 FORMAT (10I5)                                                     0  0086
C     LCST=-1 MEANS CONSTANT, 0 MEANS ZERO, AND 1 MEANS VARIABLE PRESTR 0  0087
      IF(LCST(IBR)) 290,270,290                                       0  0088
270 WRITE (6,280) LCST(IBR)                                           0  0089   82
280 FORMAT (1H0, 5HLCST=,I4, 42H  SO THAT PRESTRESS IS ZERO OVER THIS 0  0090
     1PART)                                                           C  0091
      GO TO 370                                                       C  0092
290 CONTINUE                                                          0  0093
      IF(NTR-50) 320,320,300                                          0  0094
300 WRITE (6,310) NTR,IBR                                             0  0095   87
310 FORMAT (1H0, 31HNUMBER OF POINTS FOR PRESTRESS=,I5,24H EXCEEDS 50 0  0096
     1OVER PART NO,I3)                                                U  0097
      CALL EXIT                                                       0  0098   88
320 CONTINUE                                                          0  0099
      WRITE (6,330) IPRT,NTR,LCST(IBR)                                C  0100   89
330 FORMAT (1H0,  14HPRESTRESS PART,I2,9H  POINTS=,I3,5H  LC=,I2,     0  0101
     1 16H        S          ,16H       NPHI         ,16H      NTHETA       )  C  0102
      WRITE (6,1910)                                                  U  0103   91
      NFG=1                                                           C  0104
      CALL PGEN                                                       0  0105   93
      DO 350 M=1,NTR                                                  C  0106
      READ  (5,340) S, (PAR(J),J=1,2)                                 C  0107   96
340 FORMAT (3F20.8)                                                   C  0108
C 340 FORMAT (E16.8,16X,2E16.8)                                       C  0109
      NFG=2                                                           C  0110
      CALL PGEN                                                       0  0111   102
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 0          -  EFN   SOURCE STATEMENT  -  IFN(S)  -

350 WRITE (6,360)  S, (PAR(J),J=1,2)                                0   0112   103
360 FORMAT (1H , 35X,  3E16.8)                                      0   0113
370 WRITE (6,1910)                                                  0   0114   109
380 LD=0                                                            0   0115
    IF(IGCT-100) 410,410,390                                        0   0116
390 WRITE (6,400) IGCT                                              0   0117   115
400 FORMAT (1H0, I3, 40H  SEGMENTS EXCEED ALLOWED MAXIMUM OF 100)   0   0118
    CALL EXIT                                                       0   0119   116
410 CONTINUE                                                        0   0120
    IFRCT=0                                                         0   0121
    WRITE (6,60)                                                    0   0122   118
    READ (5,420) OMZER,DELOM,OMFIN,NFIN,NX                          0   0123   119
420 FORMAT (3F10.3,2I5)                                             0   0124
    NAC=5                                                           0   0125
    XN=NX                                                           0   0126
    WRITE (6,430)  INXC,NX                                          0   0127   123
430 FORMAT (1H0,40X, 10HSUBCASE NO,I3,18H  WITH WAVE NUMBER,I3)     0   0128
    WRITE (6,440) OMZER,DELOM,OMFIN,NFIN                            0   0129   124
440 FORMAT (1H0,  15HSTARTING OMEGA=, E12.5, 12H  INCREMENT=,       0   0130
   1E12.5,14H  FINAL OMEGA=, E12.5, I5,  20H EIGENVALUES        )   0   0131
    IM=0                                                            0   0132
    IFR=1                                                           0   0133
    NB=0                                                            0   0134
    READ (5,450) (IA(I),GA(I),I=1,4)                                0   0135   127
450 FORMAT (4(I5,F10.5))                                            0   0136
    WRITE  (6,460)                                                  0   0137
   1(IA(I),GA(I),I=1,4)                                             0   0138   135
460 FORMAT(1H0,39HBOUNDARY CONDITIONS AT STARTING EDGE   ,4(I5,E12.5))0 0139
    DO 510 K=1,NBR                                                  0   0140
    READ (5,450)                                                    0   0141
   1 (IB(I+4,K),GB(I,K),I=1,4)                                      0   0142   145
    IF(K-1) 470,470,490                                             0   0143
470 WRITE (6,480)  (IB(I+4,K), GB(I,K),I=1,4)                       0   0144   152
480 FORMAT(1H0,39HBOUNDARY CONDITIONS AT FINAL    EDGE   ,4(I5,E12.5))0 0145
    GO TO 510                                                       0   0146
490 J=K-1                                                           0   0147
    WRITE (6,500) J,                                                0   0148
   1(IB(I+4,K),GB(I,K),I=1,4)                                       0   0149   160
500 FORMAT(1H0,36HBOUNDARY CONDITION AT BRANCH EDGE NO,I3,4(I5,E12.5))0 0150
510 CONTINUE                                                        0   0151
    IF(NX)  520,520,540                                             0   0152
520 NDE=6                                                           0   0153
    DO 530 K=1,NBR                                                  0   0154
    IB(4,K)=IB(5,K)                                                 0   0155
    IB(5,K)=IB(6,K)                                                 0   0156
530 IB(6,K)=IB(7,K)                                                 0   0157
    IA(7)=0                                                         0   0158
    IA(8)=0                                                         0   0159
    GO TO 550                                                       0   0160
540 NDE=8                                                           0   0161
550 NH=NDE/2                                                        0   0162
    M=NH+1                                                          0   0163
    L=1                                                             0   0164
    DO 590 IK=M,NDE                                                 0   0165
    DO 570 N=L,NDE                                                  0   0166
    DO 560 J=1,NH                                                   0   0167
```

```
      IF(IA(J)-N) 560,570,560                    O   0168
  560 CONTINUE                                   C   0169
      GO TO 580                                  C   0170
  570 CONTINUE                                   O   0171
  580 L=N+1                                      O   0172
  590 IA(K)=N                                    C   0173
      DO 630 K=1,NB4                             O   0174
      L=1                                        C   0175
      DO 630 IK=1,NM                             O   0176
      DO 610 N=L,NCE                             C   0177
      DO 600 J=N,NDE                             C   0178
      IF( IB(J,K)-N) 600,610,600                 O   0179
  600 CONTINUE                                   O   0180
      GO TO 620                                  O   0181
  610 CONTINUE                                   O   0182
  620 L=N+1                                      O   0183
  630 IB(IK,K)=N                                 O   0184
      DO 640 I=1,8                               O   0185
      Y(I)=0.0                                   C   0186
      Z(I)=0.0                                   O   0187
      DO 640 L=1,4                               O   0188
      TA1(I,L)=0.0                               C   0189
  640 TA2(I,L)=0.0                               O   0190
      CALL SECOND                                C   0191      245
      DO 650 J=1,NCE                             O   0192
      K=IA(J)                                    C   0193
      DO 650 I=1,NDE                             C   0194
  650 DM(I,J)=TL(I,I,K)                          O   0195
      DO 660 I=1,NDE                             C   0196
      DO 660 J=1,NDE                             C   0197
  660 TL(I,J)=DM(I,J)                            O   0198
      DO 680 K=1,NB4                             C   0199
      DO 670 J=1,NDE                             C   0200
      DO 670 I=1,NDE                             C   0201
      L=IP(I,K)                                  O   0202
  670 DM(I,J)=TA(L,J,K)                          C   0203
      DO 680 I=1,NDE                             O   0204
      DO 680 J=1,NDE                             O   0205
  680 TA(I,J,K)=DM(I,J)                          O   0206
      IF(NDE-7) 690,690,700                      O   0207
  690 TA1(5,5)=1.0                               O   0208
      TA1(6,6)=1.0                               C   0209
      TA2(5,5)=1.0                               C   0210
      TA2(6,6)=1.0                               O   0211
      GO TO 710                                  C   0212
  700 TA1(5,7)=1.0                               C   0213
      TA1(6,5)=1.0                               O   0214
      TA1(7,8)=1.0                               O   0215
      TA1(8,6)=1.0                               O   0216
      TA2(5,6)=1.0                               C   0217
      TA2(6,8)=1.0                               C   0218
      TA2(7,5)=1.0                               O   0219
      TA2(8,7)=1.0                               O   0220
  710 NPL=NBE+1                                  C   0221
      OMEGA=CP2EN                                C   0222
  720 ID=1                                       O   0223
```

```
      WRITE (6,60)                                                      0  0224   302
      IFRCT=IFRCT+1                                                     0  0225
      IF(IFRCT-50) 740,730,730                                          0  0226
730   IFRCT=50                                                          C  0227
740   OMSAV(IFRCT)=OMEGA                                                0  0228
      ISTR=ISTK                                                         0  0229
      NSTK=0                                                            0  0230
      NFF=0                                                             0  0231
      IRS=0                                                             0  0232
      NFP=2                                                             0  0233
      NF=1                                                              C  0234
      XMR=1.0                                                           C  0235
      OME=OMEGA                                                         0  0236
      OMSQ=OMEGA*OMEGA*39.4784                                          0  0237
      IF(IVB) 760,760,750                                               0  0238
750   XMR=OMEGA                                                         0  0239
      OMSQ=0.0                                                          0  0240
760   CONTINUE                                                          0  0241
770   I=IBR                                                             0  0242
      ISH=ISS(I)                                                        0  0243
      S=SI(I)                                                           0  0244
      INDEX=3                                                           0  0245
      CALL ORTHO                                                        0  0246   323
      CALL INPUT                                                        C  0247   325
      INDEX=4                                                           0  0248
      LC =LCST(IFR)                                                     C  0249
      NFG=3                                                             C  0250
      PAR(1)=0.0                                                        C  0251
      PAR(2)=0.0                                                        0  0252
      IF(LC) 780,790,780                                                0  0253
780   CALL PGEN                                                         0  0254   333
790   CONTINUE                                                          C  0255
      IF(NPRT) 890,890,800                                              0  0256
800   CONTINUE                                                          0  0257
      JJ=MLY(IBR)+1                                                     0  0258
      WRITE (6,810) IBR                                                 0  0259   338
810   FORMAT (1H0, 20X, 28HAT BEGINNING AND END OF PART,I3,             0  0260
     115H PARAMETERS ARE  )                                             0  0261
      WRITE (6,1910)                                                    0  0262   339
      DO 880 K=1,2                                                      C  0263
      WRITE (6,830) R1,R2,R3,SXN,CXS                                    0  0264   342
      WRITE (6,840)    C11,C12,C22,C66,E11,E12                          0  0265   343
      WRITE (6,850)   E22,E66,D11,D12,D22,D66                           0  0266   344
      WRITE (6,860)    (ZLY(IBR,J), J=1,JJ)                             0  0267   345
      WRITE (6,820) OMEGA,OMSQ,XMR,LC, (PAR(J),J=1,2)                   0  0268   350
820   FORMAT (1H ,  6HOMEGA=,E12.5, 7H  OMSQ=,E12.5, 6H  XMR=,E12.5,    0  0269
     15H  LC=,I3, 12H  PRESTRESS=,2E16.7)                               0  0270
      WRITE (6,1910)                                                    0  0271   355
830   FORMAT (1H , 4H R1=,E12.4, 4H R2=, E12.4,4H R3=, E12.4,           0  0272
     15H SXN=, E12.4, 5H CXS=, E12.4)                                   0  0273
840   FORMAT (1H , 5H C11=,E12.4, 5H C12=,E12.4, 5H C22=, E12.4,        0  0274
     15H C66=, E12.4, 5H E11=, E12.4, 5H E12=, E12.4)                   C  0275
850   FORMAT (1H , 5H E22=, E12.4, 5H E66=, E12.4, 5H D11=, E12.4,      0  0276
     15H D12=, E12.4, 5H D22=, E12.4, 5H D66=, E12.4)                   0  0277
860   FORMAT (1H , 7HZS ARE=,10E12.4)                                   C  0278
      S=SX(I)                                                           C  0279
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0          - EFN    SOURCE STATEMENT  -  IFN(S)  -

      CALL ORTHO                                                  0  0280   357
      CALL INPUT                                                  0  0281   359
      IF(LC) 870,880,670                                          0  0282
  870 CALL PGEN                                                   0  0283   363
  880 CONTINUE                                                    0  0284
  890 CONTINUE                                                    0  0285
      K=IPAR(I)+NF-1                                              0  0286
      DO 900 J=NF,K                                               0  0287
  900 ITP(J)=0                                                    0  0288
      IF(NTP(IBR))  940,940,910                                   0  0289
  910 DO 920 J=NF,K                                               0  0290
  920 ITP(J)=1                                                    0  0291
      IF(NTP(IBR)-1) 940,940,930                                  0  0292
  930 ITP(K)=2                                                    0  0293
  940 CONTINUE                                                    0  0294
      PARTS=IPAR(IBR)                                             0  0295
      SMXX=(SX(IBR) - SI(IBR))/PARTS                              0  0296
      SZERO=SI(IBR)                                               0  0297
      XINT=ING(IBR)                                               0  0298
      XPR(IBR)=SMXX/XINT                                          0  0299
      NFF=NFF+IPAR(IBR)                                           0  0300
      SMAX=SZERO+SMXX                                             0  0301
      XOUT=SMAX                                                   0  0302
      IF(NPRT-1) 970,970,950                                      0  0303
  950 WRITE (6,60)                                                0  0304   401
      WRITE (6,960) IBR,IPAR(IBR)                                 0  0305   402
  960 FORMAT(1H0,62HINITIAL VALUE INTEGRATIONS (COLUMNS OF Y(X) MATRIX) 0  0306
     1OF PART NO,I3,5H OVER,I3,16H SEGMENTS FOLLOW)               0  0307
  970 CONTINUE                                                    0  0308
  980 J=0                                                         0  0309
      XLD=0.0                                                     0  0310
  990 J=J+1                                                       0  0311
      DO 1000 I=1,8                                               0  0312
 1000 Y(I)=0.0                                                    0  0313
      IF(J-NDE) 1010,1010,1020                                    0  0314
 1010 Y(J)=1.0                                                    0  0315
 1020 IF(NPRT-2) 1040,1030,1030                                   0  0316
 1030 WRITE  (6,1090)                                             0  0317
     1          SZERO, (Y(I),I=1,NDE)                             0  0318   421
 1040 S=SZERO                                                     0  0319
      HH=0.01*SMXX                                                0  0320
      IBCL=0                                                      0  0321
 1050 CALL RUNGE                                                  0  0322   430
      DO 1060 I=1,NDE                                             0  0323
 1060 D(NF,I,J)=Y(I)                                              0  0324
 1070 IF(NPRT-2) 1110,1080,1060                                   0  0325
 1080 WRITE  (6,1090)                                             0  0326
     1 S,     (D(NF,I,J),I=1,NDE)                                 0  0327   442
 1090 FORMAT (1H0, F9.3,6E15.7)                                   0  0328
      WRITE (6,1100) IBCL                                         0  0329   449
 1100 FORMAT (1H ,7HPOINTS=,I5)                                   0  0330
 1110 IF(J-NDE) 990,1120,1120                                     0  0331
 1120 IF(NF-NFF)  1140,1130,1130                                  0  0332
 1130 IF(IPAR(IBR)-1) 1140,1140,1230                              0  0333
 1140 IF(IRS) 1210,1210,1150                                      0  0334
 1150 IRS=0                                                       0  0335
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0           - EFN   SOURCE STATEMENT  -  IFN(S)  -

      S=SI(IBR)                                          0   0336
      CALL INPUT                                         0   0337   462
      TR2(1,1)=CXS                                       0   0338
      TR2(1,2)=SXN                                       0   0339
      TR2(3,1)=-SXN                                      0   0340
      TR2(3,2)=CXS                                       0   0341
      IF(NDE-7) 1160,1160,1170                           0   0342
 1160 TR2(2,4)=CXS                                       0   0343
      TR2(2,5)=SXN                                       0   0344
      TR2(4,4)=-SXN                                      0   0345
      TR2(4,5)=CXS                                       0   0346
      GO TO 1180                                         0   0347
 1170 TR2(2,5)=CXS                                       0   0348
      TR2(2,6)=SXN                                       0   0349
      TR2(4,5)=-SXN                                      0   0350
      TR2(4,6)=CXS                                       0   0351
 1180 DO 1190 I=1,NDE                                    0   0352
      DO 1190 K=1,NDE                                    0   0353
      DM(I,K)=0.0                                        0   0354
      DO 1190 L=1,NDE                                    0   0355
 1190 DM(I,K)=DM(I,K)+D(NF,I,L)*TR2(L,K)                 0   0356
      DO 1200 I=1,NDE                                    0   0357
      DO 1200 K=1,NDE                                    0   0358
 1200 D(NF,I,K)=DM(I,K)                                  0   0359
 1210 IF(IPAR(IPR)-1) 1230,1230,1220                     0   0360
 1220 NF=NF+1                                            0   0361
      NFP=NF+1                                           0   0362
      SZERO=SMAX                                         0   0363
      SMAX=SMAX+SZERO                                    0   0364
      XOUT=SMAX                                          0   0365
      GO TO 980                                          0   0366
 1230 CONTINUE                                           0   0367
 1240 IF(IBR-IBRP) 1250,1330,1330                        0   0368
 1250 IF(ITP(NF)-2) 1260,1320,1320                       0   0369
 1260 S=SX(IBR)                                          0   0370
      CALL INPUT                                         0   0371   512
      TR1(1,1)=CXS                                       0   0372
      TR1(1,3)=-SXN                                      0   0373
      TR1(2,1)=SXN                                       0   0374
      TR1(2,3)=CXS                                       0   0375
      IF( NDE-7) 1270,1280,1280                          0   0376
 1270 TR1(4,2)=CXS                                       0   0377
      TR1(4,4)=-SXN                                      0   0378
      TR1(5,2)=SXN                                       0   0379
      TR1(5,4)=CXS                                       0   0380
      GO TO 1290                                         0   0381
 1280 TR1(5,2)=CXS                                       0   0382
      TR1(5,4)=-SXN                                      0   0383
      TR1(6,2)=SXN                                       0   0384
      TR1(6,4)=CXS                                       0   0385
 1290 DO 1300 I=1,NDE                                    0   0386
      DO 1300 K=1,NDE                                    0   0387
      DM(I,K)=0.0                                        0   0388
      DO 1300 L=1,NDE                                    0   0389
 1300 DM(I,K)=DM(I,K)+TR1(I,L)*D(NF,L,K)                 0   0390
      DO 1310 I=1,NDE                                    0   0391
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0          - EFN    SOURCE STATEMENT  -  IFN(S)  -

      DO 1310 K=1,NDE                                                0   0392
 1310 D(NF,I,K)=DM(I,K)                                              0   0393
 1320 IBR=IBR+1                                                      0   0394
      NF=NF+1                                                        0   0395
      NFP=NF+1                                                       0   0396
      IRS=1                                                          0   0397
      GO TO 770                                                      0   0398
 1330 DO 1340 I=1,NDE                                                0   0399
      DO 1340 J=1,NDE                                                0   0400
      DM(I,J)=0.0                                                    0   0401
      DO 1340 L=1,NDE                                                0   0402
 1340 DM(I,J)=DM(I,J) +D(1,I,L)*TLI(L,J)                             0   0403
      DO1350 I=1,NDE                                                 0   0404
      DO1350 J=1,NDE                                                 0   0405
 1350 D(1,I,J)=DM(I,J)                                               0   0406
      KF=1                                                           0   0407
      IX=NF                                                          0   0408
      DO 1400 K=1,NBR                                                0   0409
      DO 1360 I=1,NDE                                                0   0410
      DO 1360 J=1,NDE                                                0   0411
      DM(I,J)=0.0                                                    0   0412
      DO 1360 L=1,NDE                                                0   0413
 1360 DM(I,J)=DM(I,J)+TR(I,L,K)*D(IX,L,J)                            0   0414
      DO 1370 I=1,NDE                                                0   0415
      DO 1370 J=1,NDE                                                0   0416
 1370 D(IX,I,J)=DM(I,J)                                              0   0417
      DO 1380 IX=KF,NF                                               0   0418
      JJ=IX                                                          0   0419
      IF(ITP(IX)-2) 1380,1390,1390                                   0   0420
 1380 CONTINUE                                                       0   0421
 1390 IX=JJ                                                          0   0422
      KF=IX+1                                                        0   0423
 1400 CONTINUE                                                       0   0424
      IF(NPRT) 1430,1430,1410                                        0   0425
 1410 CONTINUE                                                       0   0426
      WRITE (6,1420) (ITP(I),I=1,NF)                                 0   0427   623
 1420 FORMAT(1H0,50I2)                                               0   0428
      WRITE (6,1910)                                                 0   0429   630
 1430 CONTINUE                                                       0   0430
      CALL TRIANG                                                    0   0431   632
      DTASV(IFRCT)=DETA                                              0   0432
      DTBSV(IFRCT)=DETB                                              0   0433
      DV(IFRCT)=DETC                                                 0   0434
      XCSAV(IFRCT)=XXC                                               0   0435
      XESAV(IFRCT)=AXE                                               0   0436
      IFTP(IFRCT)=1                                                  0   0437
      IF(NPRT) 1460,1480,1440                                        0   0438
 1440 CONTINUE                                                       0   0439
      WRITE  (6,1450)                                                0   0440
     1                  NFP,NFP,XN,  EXP                             0   0441   641
 1450 FORMAT(1H0, 5X,4HNEXT,     I3, 41H ROWS ARE VARIABLES AT ENDPOINTS 0 0442
     1X1,....X, I2, 12H OF SEGMENTS,                                 0   0443
     22X, 3HXN=, F5.1,2X,                       13H ACCURACY=,E8.1)  0   0444
      WRITE (6,1910)                                                 0   0445   642
      DO 1460 I=1,NFP                                                0   0446
 1460 WRITE  (6,1470)                                                0   0447
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1              1,(CP(I,J), J=1,NDE)                              0  0448   646
1470 FORMAT (1H ,3HX=X,I2,2X,8E15.7)                             0  0449
1480 CONTINUE                                                    0  0450
     IST=0                                                       0  0451
     DO 1550 K=1,NF                                              0  0452
     IF( ITP(K)-1) 1550,1490,1510                                0  0453
1490 IF(IST) 1500,1500,1550                                      0  0454
1500 IST=2                                                       0  0455
     I=K                                                         0  0456
     GO TO 1550                                                  0  0457
1510 IF(IST) 1520,1520,1530                                      0  0458
1520 I=K                                                         0  0459
1530 CONTINUE                                                    0  0460
     DO 1540 J=1,NM                                              0  0461
1540 DM(K-1,J)=DM(I,J)                                           0  0462
     IST=0                                                       0  0463
1550 CONTINUE                                                    0  0464
     DO 1560 I=1,NDE                                             0  0465
     Y(I)=0.0                                                    0  0466
     DO 1560 J=1,NDE                                             0  0467
1560 Y(I)=Y(I)+TLI(I,J)*DM(I,J)                                  0  0468
     DO 1570 I=1,NDE                                             0  0469
1570 DM(1,I)=Y(I)                                                0  0470
1580 WRITE (6,63)                                                0  0471   700
     IF(IVB) 1600,1600,1590                                      0  0472
1590 OMC=XMM                                                     0  0473
1600 CONTINUE                                                    0  0474
     WRITE (6,1610) OME,KX                                       0  0475   703
1610 FORMAT (1H0,   64HSOLUTION AT POINTS ALONG MERIDIAN AT EIGENVALUE 0  0476
    1PARAMETER OMEGA=,E14.7, 21H  FOR WAVE NUMBER KX=,I3)        0  0477
     IF(ISTR-1) 1640,1620,1620                                   0  0478
1620 WRITE  (6,1630)                                             0  0479   705
1630 FORMAT (1H0,9H    S    , 12H     W     , 12H     V     ,    0  0480
    112H    UPHI    , 12H    NPHI    , 12H    BPHI    , 12H    MPHI  0  0481
    2, 12H   UTHETA   , 12H    N    , 12H   NTHETA   ,12H   MTHETA 10 0  0482
     GO TO 1660                                                  0  0483
1640 WRITE  (6,1650)                                             0  0484   707
1650 FORMAT (1H0,9H    S    , 12H     W     , 12H    UPHI    ,   0  0485
    112H    UTHETA  ,  12H    BPHI    , 12H   SPHI IN  ,12H  SPHI OUT 0  0486
    2,12H STHETA IN  ,12H STHETA OUT , 12H  SFITH IN  ,12H  SFITH OUT 10 0  0487
1660 IST=1                                                       0  0488
     KF=0                                                        0  0489
     IX=1                                                        0  0490
     JPA=IPAR(1)+1                                               0  0491
     TC2=0.0                                                     0  0492
     ITC1=IR(NM+1,1)                                             0  0493
     DO 1930 N=1,IBRM                                            0  0494
     ISM=ISS(N)                                                  0  0495
     IBR=N                                                       0  0496
     L=ING(N)+1                                                  0  0497
     S=SI(N)                                                     0  0498
     KF=IPAR(N)+KF                                               0  0499
     IF(ITP(KF)-1) 1670,1680,1680                                0  0500
1670 WRITE   (6,1690) N                                          0  0501   728
     GO TO 1710                                                  0  0502
1680 WRITE   (6,1700) N                                          0  0503   730
```

```
      IST=IST+1                                                C 0504
 1690 FORMAT (1H0, 20H  MAIN SHELL PART NO,I3)                 0 0505
 1700 FORMAT (1H0, 20HBRANCH SHELL PART NO,I3)                 0 0506
 1710 INDEX=3                                                  C 0507
      CALL ORTHO                                               0 0508   733
      CALL INPUT                                               0 0509   735
      LC =LCST(IBR)                                            0 0510
      NFG=3                                                    0 0511
      PAR(1)=0.0                                               C 0512
      PAR(2)=0.0                                               0 0513
      IF(LC) 1720,1730,1720                                    0 0514
 1720 CALL PGEN                                                0 0515   742
 1730 CONTINUE                                                 0 0516
C     REMOVE FOLLOWING CARD IF PRESTRESS WORKS OK              0 0517
      WRITE (6,820) ONE,  OMSG,XMR,LC, (PAR(J),J=1,2)          0 0518   743
      IF(N-1) 1010,1010,1740                                   0 0519
 1740 IF(NSTR) 1750,1750,1010                                  C 0520
 1750 TR2(1,1)=CXS                                             0 0521
      TR2(1,2)=SXN                                             0 0522
      TR2(3,1)=-SXN                                            0 0523
      TR2(3,2)=CXS                                             C 0524
      IF(NDE-7) 1760,1760,1770                                 0 0525
 1760 TR2(2,4)=CXS                                             C 0526
      TR2(2,5)=SXN                                             0 0527
      TR2(4,4)=-SXN                                            C 0528
      TR2(4,5)=CXS                                             0 0529
      GO TO 1780                                               0 0530
 1770 TR2(2,5)=CXS                                             C 0531
      TR2(2,6)=SXN                                             0 0532
      TR2(4,5)=-SXN                                            C 0533
      TR2(4,6)=CXS                                             0 0534
 1780 DO 1790 I=1,NDE                                          0 0535
      Y(I)=0.0                                                 C 0536
      DO 1790 J=1,NDE                                          C 0537
 1790 Y(I)=Y(I)+TR2(I,J)*DM(JPA,J)                             0 0538
      DO 1800 I=1,NDE                                          0 0539
 1800 DM(JPA,I)=Y(I)                                           C 0540
      JPA=JPA+IPAR(N)                                          0 0541
 1010 XX=0.0                                                   0 0542
      INDEX=4                                                  C 0543
      S=IPAR(N)                                                C 0544
      DELS=0.01*(SX(N)-SI(N))/S                                0 0545
      SZERO=SI(N)                                              0 0546
      WRITE (6,1910)                                           C 0547   782
      DO 1920 K=1K,KF                                          0 0548
      DO 1820 I=1,NDE                                          0 0549
      Z(I)=DM(K,I)                                             0 0550
 1820 Y(I)=DM(K,I)                                             C 0551
      H=DELS                                                   C 0552
      DO 1900 M=1,L                                            0 0553
      TC1=ABS (Y(ITC1))                                        0 0554
      TC2=AMAX1 (TC1,TC2)                                      C 0555
      SZERO=SZERO+XPR(N)*XX                                    0 0556
      S=SZERO                                                  C 0557
      XX=1.0                                                   0 0558
      CALL ORTHO                                               C 0559   797
```

```
      CALL INPUT                                                      0  0560   799
      XNR=XN*R2                                                       0  0561
      SXR=SXN*R2                                                      0  0562
      CXR=CXS*R2                                                      0  0563
      ELH=R1-SXR                                                      0  0564
      ELJ=R1+SXR                                                      0  0565
      ETH=XNR*Z(7)+ CXR*Z(3) +SXR*Z(1)                                0  0566
      BTH=XNR*Z(1) +SXR*Z(7)                                          0  0567
      HTH=XNR*BTH+CXR*Z(5)                                            0  0568
      EPS=-XNR*Z(3)-CXR*Z(7)                                          0  0569
      HKA= -2.0*XNR*Z(5)-2.0*XNR*CXR*Z(1)+CXR*(ELH-SXR)*Z(7)+XNR*R1*Z(3)0  0570
      DEL=C11*D11-E11*E11                                             0  0571
      EN= Z(4)-C12*ETH-E12*HTH                                        0  0572
      EM= Z(6) -E12*ETH-D12*HTH                                       0  0573
      EFI= (EN*D11-EM*E11)/DEL                                        0  0574
      HFI= (EM*C11-EN*E11)/DEL                                        0  0575
      IF(ISTR-1) 1830,1860,1860                                       0  0576
 1830 ZD=       (Z (8)-(C66+SXR*E66)*EPS-(E66+SXR*D66)*HKA)           0  0577
     1/(C66+2.0*E66*SXR+D66*SXR*SXR)                                  0  0578
      ETF = EPS+ZD                                                    0  0579
      HTF=HKA+SXR*ZD                                                  0  0580
      Z(2)=Z(3)                                                       0  0581
      Z(3)=Z(7)                                                       0  0582
      Z(4)=Z(5)                                                       0  0583
      JJ=MLY(IBR)                                                     0  0584
      DO 1850 I=1,JJ                                                  0  0585
      J=I+1                                                           0  0586
      Z(5)=     B11(N,I)*(EFI+ZLY(N,I)*HFI) +B12(N,I)*(ETH+ZLY(N,I)*HTH)0  0587
      Z(6)=     B11(N,I)*(EFI+ZLY(N,J)*HFI) +B12(N,I)*(ETH+ZLY(N,J)*HTH)0  0588
      Z(7)=     B12(N,I)*(EFI+ZLY(N,I)*HFI) +B22(N,I)*(ETH+ZLY(N,I)*HTH)0  0589
      Z(8)=     B12(N,I)*(EFI+ZLY(N,J)*HFI) +B22(N,I)*(ETH+ZLY(N,J)*HTH)0  0590
      Z(9)=     B66(N,I)*(ETF+ZLY(N,I)*HTF)                           0  0591
      Z(10)=    B66(N,I)*(ETF+ZLY(N,J)*HTF)                           0  0592
      WRITE (6,1840) S, (Z(LLL),LLL=1,10)                             0  0593   828
 1840 FORMAT (1H ,1X,F8.5,10E12.5)                                    0  0594
 1850 CONTINUE                                                        0  0595
      WRITE (6,1910)                                                  0  0596   834
      GO TO 1870                                                      0  0597
 1860 Z(9)=C12*EFI+C22*ETH+E12*HFI+E22*HTH                            0  0598
      Z(10)=                                                          0  0599
     1     D12*HFI+D22*HTH+E12*EFI+E22*ETH                            0  0600
      WRITE (6,1840) S, (Z(I),I=1,10)                                 0  0601   837
 1870 IF(M-L) 1880,1900,1900                                          0  0602
 1880 XOUT=XPR(IBR)+SZERO                                             0  0603
      IBCL=0                                                          0  0604
      CALL RUNGE                                                      0  0605   847
      DO 1890 I=1,NDE                                                 0  0606
 1890 Z(I)=Y(I)                                                       0  0607
 1900 CONTINUE                                                        0  0608
      XX=0.0                                                          0  0609
      WRITE  (6,1910)                                                 0  0610   857
 1910 FORMAT (1H )                                                    0  0611
 1920 CONTINUE                                                        0  0612
      IX=IX+IPAR(N)                                                   0  0613
 1930 CONTINUE                                                        0  0614
      GO TO (1940,2010,1940), IFR                                     0  0615
```

```
1940 IF(ISTR-1) 2010,1950,1950
1950 TC4= ABS (DETA)
     WRITE (6,1960) TC2,TC4,ACC
1960 FORMAT (1HO, 47HMAX OF 1ST PRESCRIBED VARIABLE AT FINAL EDGE IS,
    1E12.5,5H DET=, E12.5, 6H  ACC=,E12.5)
     IF(TC4-ACC*TC2) 1970,1970,1990
1970 NB=NB+1
     WRITE (6,1980) ONE
1980 FORMAT (1HO, 6HOMEGA=,E12.5, 17H IS AN EIGENVALUE)
     IFR=1
     IFTP(IFRCT)=2
     GO TO 2010
1990 WRITE (6,2000) LD
2000 FORMAT(1HO,21HTHIS WAS ITERATION NO,I3,23H  ACCURACY NOT ATTAINED)
2010 CONTINUE
     IF( ISTR-1) 2030,2030,2020
2020 ISTR=0
     NSTR=2
     GO TO 1500
2030 IF(NB-NFIN) 2040,2060,2060
2040 GO TO (2050,2050,720),IFR
2050 OMEGA=XA+DELOM
     LD=L
     IF(OMEGA-OMFIN) 720,720,2060
2060 CONTINUE
     WRITE (6,60)
     WRITE (6,2070) NR
2070 FORMAT (1HO,       25HSUMMARY OF RESULTS FOR N=,I3)
     WRITE (6,2080)
2080 FORMAT (1HO, 2CX,16H      OMEGA      ,16H   ACTUAL DET    ,
    116H  ADJUSTED DET   ,16H    NORMAL DET   ,8H    XNC    ,8H   XNE  )
     DO 2130 I=1,IFRCT
     J=IFTP(I)
     GO TO (2090,2110),J
2090 WRITE (6,2100) OPSAV(I),DIASV(I),DIRSV(I),CV(I),XCSAV(I),XESAV(I)
2100 FORMAT (1HO, 20X,4E16.7,2F8.1)
     GO TO 2130
2110 WRITE (6,1910)
     WRITE (6,2120) OPSAV(I),DIASV(I),DIGSV(I),CV(I),XCSAV(I),XESAV(I)
2120 FORMAT (1HO,  20H   EIGENVALUE AT    ,4E16.7,2F8.1)
     WRITE (6,1910)
2130 CONTINUE
     IF(INRC-NRT) 2140,10,10
2140 INRC=INRC+1
     GO TO 320
     END
```

```
      SUBROUTINE ORTHO
      DIMENSION Y(8),YD(8),DUMM(27),PAR(2)
      DIMENSION MLY(20),ZLY(20,5)
      DIMENSION TR(8,8,4),TLI(8,8),ALFR(4)
      DIMENSIOND(100,8,9),DM(101,9),GA(4),GB(4,4),ITP(100),IA(8),IB(8,4)
      DIMENSION B11(20,4),B12(20,4),B22(20,4),B66(20,4),AL1(20,4)
      DIMENSION AL2(20,4),RHO(20,4)
      COMMON NDE,S,Y,YD,HH,J9,JMAX,M9,XOUT,IFREQ,DUMM,IBR,ISH
      COMMON PAR,LC,NFG,XMR,IVB
      COMMON XN,XLD,OMSQ,HTT,R1,R2,R3,SXN,CXS,INDEX
      COMMON PN,PL,PC,T0,T1,RH1,RH2,RH3,MLY,ZLY
      COMMON TR,TLI,IBCL,ALFL,ALFR,NBR
      COMMON LD,ACC,IM,XA,NAC,IFR,NB,XXC,AXE,DETA,DETB,DETC
      COMMON NPRT,D,DM,GA,GB,ITP,NF,NFP,NPL,NH,IA,IB,OMEGA
      COMMON B11,B12,B22,B66,AL1,AL2,RHO,H11,H12,H21,H22,G12,G21
      COMMON C11,C12,C22,E11,E12,E22,D11,D12,D22,C66,E66,D66
      DIMENSION IL1(20,4),IL2(20),PSR(20,4)
      GO TO (10,10,220,190),INDEX
   10 MK=MLY(IBR)
      DO 60 I=1,MK
      READ (5,20) B11(IBR,I),B12(IBR,I),B22(IBR,I),B66(IBR,I),
     1AL1(IBR,I),AL2(IBR,I),RHO(IBR,I),IL1(IBR,I)                      8
   20 FORMAT (7F10.6, I5)
      PSR(IBR,I)=B12(IBR,I)
      IF(IL1(IBR,I)) 30,60,40
   30 L3=-IL1(IBR,I)
      DY=FGEN (L3,1)
      GO TO 60                                                         24
   40 L1=IL1(IBR,I)
      L2=IL1(IBR,I)+5
      DO 50 J=L1,L2
   50 DY=FGEN (J,1)
   60 CONTINUE                                                         34
      READ (5,70)  (ZLY(IBR,I),I=1, 5),IL2(IBR)
   70 FORMAT (5F10.6,I5)                                               38
      IF(IL2(IBR)) 100,100,80
   80 L2=IL2(IBR)+MK
      L1=IL2(IBR)
      DO 90 I=L1,L2
   90 DY=FGEN (I,1)
  100 CONTINUE                                                         57
      MK=MLY(IBR)
      DO 180 I=1,MK
      J=I+1
      WRITE (6,110) I, ZLY(IBR,I),ZLY(IBR,J)
  110 FORMAT (1H0, 8HLAYER NO, I3, 9H  FROM Z=,E12.5,7H  TO Z=,E12.5)  66
      IF(B22(IBR,I)-1.0E-8) 120,120,140
  120 E=B11(IBR,I)
      P=B12(IBR,I)
      P1=1.0-P*P
      B11(IBR,I)=E/P1
      B12(IBR,I)=P*E/P1
      B22(IBR,I)=B11(IBR,I)
      B66(IBR,I)=E*0.5/(1.0+P)
      WRITE (6,130) E,P                                                79
```

```
      RM1=0.0
      RM2=0.0
      RM3=0.0
      DO 300 I=1,MR
      J=I+1
      Z1=ZLV(IBR,J) -ZLV(IBR,I)
      Z2=ZLV(IBR,J)*ZLV(IBR,J)-ZLV(IBR,I)*ZLV(IBR,I)
      Z3=ZLV(IBR,J)*ZLV(IBR,J)*ZLV(IBR,J)-ZLV(IBR,I)*ZLV(IBR,I)*ZLV(IBR,
     1I)
      Z2=0.5*Z2
      Z3=0.333333*Z3
      C11=C11+B11(IBR,I)*Z1
      C12=C12+B12(IBR,I)*Z1
      C22=C22+B22(IBR,I)*Z1
      C66=C66+B66(IBR,I)*Z1
      E11=E11+B11(IBR,I)*Z2
      E12=E12+B12(IBR,I)*Z2
      E22=E22+B22(IBR,I)*Z2
      E66=E66+B66(IBR,I)*Z2
      D11=D11+B11(IBR,I)*Z3
      D12=D12+B12(IBR,I)*Z3
      D22=D22+B22(IBR,I)*Z3
      D66=D66+B66(IBR,I)*Z3
      RM1=RM1+RHO(IBR,I)*Z1
      RM2=RM2+RHO(IBR,I)*Z2
      RM3=RM3+RHO(IBR,I)*Z3
  300 CONTINUE
      RETURN
      END
```

```
      SUBROUTINE DIFFEC
      DIMENSION Y(8),YD(8),DUMM(27),PAR(2)
      DIMENSION MLY(20),ZLY(20,5)
      DIMENSION TK(6,8,4),TLI(6,8),ALFK(4)
      DIMENSION D(100,8,9),DM(101,9),GA(4),GB(4,4),ITP(100),IA(8),IB(8,4)
      DIMENSION B11(20,4),B12(20,4),B22(20,4),B66(20,4),AL1(20,4)
      DIMENSION AL2(20,4),RHO(20,4)
      COMMON NDE,S,Y,YD,HH,J9,JMAX,M9,XOUT,IFREQ,DUMM,IBK,ISM
      COMMON PAR,LC,NFG,XMR,IVB
      COMMON XN,XLD,OMSQ,HTT,R1,R2,R3,SXN,CXS,INDEX
      COMMON PN,PL,PC,TO,T1,RH1,RH2,RH3,MLY,ZLY
      COMMON TK,TLI,IBCL,ALFL,ALFR,N5R
      COMMON LD,ACC,IP,AA,NAC,IFR,N6,XXC,XXE,DETA,DETB,DETC
      COMMON NPRT,D,DM,GA,GB,ITP,NF,NFP,NPL,NH,IA,IB,OMEGA
      COMMON B11,B12,B22,B66,AL1,AL2,RHO,H11,H12,H21,H22,G12,G21
      COMMON C11,C12,C22,E11,E12,E22,D11,D12,D22,C66,E66,D66
      IBCL=IBCL+1
      IF(IBCL-500) 120,120,10
   10 WRITE (6,20)                                                       4
   20 FORMAT (1H0, 96HTHERE IS SOMETHING WRONG IN THIS SEGMENT. MORE THA
     1N 500 POINTS HAVE BEEN USED IN INTEGRATION     )
      WRITE (6,30)                                                       5
   30 FORMAT (1H0, 28HAT THIS POINT PARAMETERS ARE)
      WRITE (6,40) S,HH,XOUT                                             6
   40 FORMAT (1H0,2HX=,E12.4, 5H  HH=, E12.4, 7H  XOUT=, E12.4)
      WRITE (6,50)  (Y(I),I=1,NDE)                                       7
   50 FORMAT (1H0, 2HY=, 8E14.7)
      WRITE (6,60)  (YD(I),I=1,NDE)                                      14
   60 FORMAT (1H0, 3HDY=,  8E14.7)
      WRITE (6,70) R1,R2,R3,SXN,CXS                                      21
   70 FORMAT (1H0, 4H R1=,E12.4, 4H R2=, E12.4,4H R3=, E12.4,
     15H SXN=, E12.4, 5H CXS=, E12.4)
      WRITE (6,80) OMEGA,OMSQ,XMR,LC, (PAR(J),J=1,2)                     22
   80 FORMAT (1H , 6HOMEGA=,E12.5, 7H  OMSQ=,E12.5, 6H  XMR=,F12.5,
     15H  LC=,I3, 12H  PRESTRESS=,2E16.7)
      WRITE (6,90)   C11,C12,C22,C66,E11,E12                             29
   90 FORMAT (1H0, 5H C11=,E12.4, 5H C12=,E12.4, 5H C22=, E12.4,
     15H C66=, E12.4, 5H E11=, E12.4, 5H E12=, E12.4)
      WRITE (6,100)  E22,E66,D11,D12,D22,D66                             30
  100 FORMAT (1H0, 5H E22=, E12.4, 5H E66=, E12.4, 5H D11=, E12.4,
     15H D12=, E12.4, 5H D22=, E12.4, 5H D66=, E12.4)
      JJ=MLY(IBK)+1
      WRITE (6,110)   (ZLY(IBK,J), J=1,JJ)                               32
  110 FORMAT (1H0, 7HZS ARE=,10E12.4)
      CALL EXIT                                                          39
  120 CONTINUE
      CALL ORTHO                                                         41
      CALL INPUT                                                         43
      CXR=CXS*R2
      SXR=SXN*R2
      IF(NDE-7) 130,130,140
  130 ETH=          CXR*Y(3) +SXR*Y(1)
      HTH=        CXR*Y(5)
      DEL=C11*D11-E11*E11
      EN= Y(4)-C12*ETH-E12*HTH
```

```
      EM= Y(6) -E12*ETH-D12*HTH
      EFI= (EN*D11-EM*E11)/DEL
      HFI= (EM*C11-EN*E11)/DEL
      YD(1)=R1*Y(3)-Y(5)
      YD(3)= EFI-R1*Y(1)
      YD(5)=HFI
      THN= C12*EFI+C22*ETH+E12*HFI+E22*HTH
      THM= D12*HFI+D22*HTH+E12*EFI+E22*ETH
      YD(2)=-CXR*Y(2)+SXR*THN+R1*Y(4)
      YD(4)=CXR*(THN-Y(4))-R1*Y(2)
      YD(6)=                CXR*(THM-Y(6)) +Y(2)
      GO TO 150
  140 XNR=XN*R2
      ELJ=R1+SXR
      ELH=R1-SXR
      ETH=XNR*Y(7)+ CXR*Y(3) +SXR*Y(1)
      BTH=XNR*Y(1) +SXR*Y(7)
      HTH=XNR*BTH+CXR*Y(5)
      EPS=-XNR*Y(3)-CXR*Y(7)
      HKA= -2.0*XNR*Y(5)-2.0*XNR*CXR*Y(1)+CXR*(ELH-SXR)*Y(7)+XNR*R1*Y(3)
      DEL=C11*D11-E11*E11
      EN= Y(4)-C12*ETH-E12*HTH
      EM= Y(6) -E12*ETH-D12*HTH
      EFI= (EN*D11-EM*E11)/DEL
      HFI= (EM*C11-EN*E11)/DEL
      YD(1)=R1*Y(3)-Y(5)
      YD(3)= EFI-R1*Y(1)
      YD(5)=HFI
      YD(7)=    ( Y(8)-(C66+SXR*E66)*EPS-(E66+SXR*D66)*HKA)
     1/(C66+2.0*E66*SXR+D66*SXR*SXR)
      THN= C12*EFI+C22*ETH+E12*HFI+E22*HTH
      THM= D12*HFI+D22*HTH+E12*EFI+E22*ETH
      TFM= E66*(EPS+YD(7))+ D66*(HKA+SXR*YD(7))
      YD(2)= -2.0*XNR*CXR*TFM- CXR*Y(2)+SXR*THN+ R1*Y(4)+ XNR*XNR*THM
      YD(4)=  -XNR*Y(8)+XNR*ELJ*TFM+CXR*(THN-Y(4))   -R1*Y(2)
      YD(6)=  -2.0*XNR*TFM+CXR*(THM-Y(6)) +Y(2)
      YD(8)= ELH*CXR*TFM -2.0*CXR*Y(8) +XNR*THN+ XNR*SXR*THM
  150 CONTINUE
      IF(IVB) 160,160,180
  160 YD(2)=YD(2)-OMSQ*RH1*Y(1)
      YD(4)=YD(4)-OMSQ*(RH1*Y(3)+RH2*Y(5))
      YD(6)=YD(6)-OMSQ*(RH2*Y(3)+RH3*Y(5))
      IF(NDE-7) 180,180,170
  170 YD(8)=YD(8)-OMSQ*(RH1*Y(7)+RH2*BTH)
  180 CONTINUE
      IF(LC) 200,210,190
  190 CALL PGEN
  200 YD(2)=YD(2)+XMR*(PAR(1)*HFI+PAR(2)*HTH)
  210 IF(R3-1.0) 220,240,220
  220 DO 230 I=1,NDE
  230 YD(I)=YD(I)/R3
  240 RETURN
      END
```

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```
      SUBROUTINE TRIANG                                                    R  0001
      DIMENSION Y(8),YD(8),DUMM(27),PAR(2)                                 R  0002
      DIMENSION MLY(20),ZLY(20,5)                                          R  0003
      DIMENSION TR(8,8,4),TLI(8,8),ALFR(4)                                 R  0004
      DIMENSIOND(100,8,9),DM(101,9),GA(4),GB(4,4),ITP(100),IA(8),IB(8,4)R  0005
      COMMON NDE,S,Y,YD,MM,J9,JKAX,M9,XOUT,IFREQ,DUMM,IBK,ISH              R  0006
      COMMON PAR,LC,NFG,XMR,IYR                                            R  0007
      COMMON XN,XLD,OMSQ,HTT,R1,R2,R3,SXN,CXS,INDEX                        R  0008
      COMMON PN,PL,PC,TO,T1,RH1,RH2,RH3,MLY,ZLY                            R  0009
      COMMON TR,TLI,IBCL,ALFL,ALFR,NBR                                     R  0010
      COMMON LD,ACC,IM,XA,NAC,IFR,NB,XXC,XXE,DETA,DETB,DETC                R  0011
      COMMON NPRT,D,DM,GA,GB,ITP,NF,NFP,NPL,NH,IA,IB,OMEGA                 R  0012
      DIMENSION D1(4,4),D2(4,4),U1(4,4),U2(4,4),DZ(4)                      R  0013
      XLC=0.0                                                              R  0014
      IST=0                                                                R  0015
      IRK=1                                                                R  0016
      DO 10 K=1,NF                                                         R  0017
      DO 10 I=1,NCE                                                        R  0018
   10 D(K,I,NPL)=0.0                                                       R  0019
      DO 20 I=1,NH                                                         R  0020
      DO 20 J=1,NH                                                         R  0021
      JJ=J+NH                                                              R  0022
   20 U1(I,J) =  D(1,I,JJ)                                                 R  0023
      CALL INVERT (U1,NH,DET)                                              R  0024   25
      DO 30 I=1,NH                                                         R  0025
      II=I+NH                                                              R  0026
      DO 30 J=1,NH                                                         R  0027
      U2(I,J) = 0.0                                                        R  0028
      DO 30 L=1,NH                                                         R  0029
      LL=L+NH                                                              R  0030
   30 U2(I,J) = U2(I,J) + D(1,II,LL) * U1(L,J)                             R  0031
      DM(1,1)=DET                                                          R  0032
      DO 40 I=1,NH                                                         R  0033
      II = I+NH                                                            R  0034
      DO 40 J=1,NH                                                         R  0035
   40 D(1,II,J) = U1(I,J)                                                  R  0036
      IF(NF-1) 450,450,50                                                  R  0037
   50 CALL INVERT (U2,NH,DET)                                              R  0038   59
      DM(1,2) = DET                                                        R  0039
      DO 60 I=1,NH                                                         R  0040
      II=I+NH                                                              R  0041
      DO 60 J=1,NH                                                         R  0042
      JJ=J+NH                                                              R  0043
   60 D(1,II,JJ) = U2(I,J)                                                 R  0044
      DO 440 K=2,NF                                                        R  0045
      DO 70 I=1,NH                                                         R  0046
      DO 70 J=1,NH                                                         R  0047
      JJ=J+NH                                                              R  0048
      D2(I,J) = 0.0                                                        R  0049
      DO 70 L=1,NH                                                         R  0050
      LL= L+NH                                                             R  0051
   70 D2(I,J) = D2(I,J) + D(K,I,L) * D(K-1,LL,JJ)                          R  0052
      IF(ITP(K-1)-2) 100,80,80                                             R  0053
   80 DO 90 I=1,NH                                                         R  0054
      DO 90 J=1,NH                                                         R  0055
```

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R           - EFN   SOURCE STATEMENT  -  IFN(S)  -
    JJ=J+NH                                                     R   0056
    U1(I,J) = D(K,I,JJ)                                         R   0057
    DO 90 L=1,NH                                                R   0058
 90 U1(I,J) = U1(I,J) - D2(I,L) *D(K-1,L,JJ)                    R   0059
    GO TO 120                                                   R   0060
100 DO 110 I=1,NH                                               R   0061
    DO 110 J=1,NH                                               R   0062
    JJ=J+NH                                                     R   0063
110 U1(I,J) = D(K,I,JJ) + D2(I,J)                               R   0064
120 IF( ITP(K)-1) 190,130,130                                   R   0065
130 IF(IST  -1) 140,140,170                                     R   0066
140 IRR=IRR+1                                                   R   0067
    IST=K                                                       R   0068
    DO 160 I=1,NH                                               R   0069
    II=I+NH                                                     R   0070
    DO 150 J=1,NH                                               R   0071
    JJ=J+NH                                                     R   0072
    D(K-1,I,JJ)=0.0                                             R   0073
150 D(K-1,I,J )=0.C                                             R   0074
160 D(K-1,I,II)=-1.0                                            R   0075
170 DO 180 I=1,NH                                               R   0076
    DO 180 J=1,NH                                               R   0077
    JJ=J+NH                                                     R   0078
    D(K,I,J)=0.0                                                R   0079
    DO 180 L=1,NH                                               R   0080
180 D(K,I,J) = D(K,I,J) - D2(I,L) * D(K-1,L,JJ)                 R   0081
190 CALL INVERT (U1,NH,DET)                                     R   0082     163
    DM(K,1)=DET                                                 R   0083
    DO 200 I=1,NH                                               R   0084
    II=I+NH                                                     R   0085
    DO 200 J=1,NH                                               R   0086
    JJ = J + NH                                                 R   0087
    D2(I,J) = 0.0                                               R   0088
    DO 200 L=1,NH                                               R   0089
    LL = L + NH                                                 R   0090
200 D2(I,J) = D2(I,J) + D(K,II,L) * D(K-1,LL,JJ)                R   0091
    IF( ITP(K)-1) 230,210,210                                   R   0092
210 DO 220 I=1,NH                                               R   0093
    DO 220 J=1,NH                                               R   0094
    JJ=J+NH                                                     R   0095
    D(K,I,JJ)=0.0                                               R   0096
    DO 220 L=1,NH                                               R   0097
220 D(K,I,JJ) = D(K,I,JJ) - D2(I,L) * D(K-1,L,JJ)               R   0098
230 CONTINUE                                                    R   0099
    IF(ITP(K-1)-2) 270,240,240                                  R   0100
240 DO 250 I=1,NH                                               R   0101
    DO 250 J=1,NH                                               R   0102
    JJ=J+NH                                                     R   0103
    U2(I,J) = 0.0                                               R   0104
    DO 250 L=1,NH                                               R   0105
250 U2(I,J) = U2(I,J) - D2(I,L) * D(K-1,L,JJ)                   R   0106
    DO 260 I=1,NH                                               R   0107
    DO 260 J=1,NH                                               R   0108
260 D2(I,J) = U2(I,J)                                           R   0109
270 DO 280 I=1,NH                                               R   0110
    II=I+NH                                                     R   0111
```

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      R              - EFN    SOURCE STATEMENT   -  IFN(S)  -

      DO 280 J=1,NH                                          R    0112
      JJ=J+NH                                                R    0113
  280 D2(I,J) = D(K,II,JJ) + D2(I,J)                         R    0114
      DO 290 I=1,NH                                          R    0115
      DO 290 J=1,NH                                          R    0116
      U2(I,J) = 0.0                                          R    0117
      DO 290 L=1,NH                                          R    0118
  290 U2(I,J) = U2(I,J) + D2(I,L) * U1(L,J)                  R    0119
      IF( ITP(K)-1) 320,300,300                              R    0120
  300 DO 310 I=1,NH                                          R    0121
      DO 310 J=1,NH                                          R    0122
      JJ=J+NH                                                R    0123
      DO 310 L=1,NH                                          R    0124
  310 D(K,I,JJ) = D(K,I,JJ) - U2(I,L) * D(K,L,J)             R    0125
  320 CONTINUE                                               R    0126
      DO 330 I=1,NH                                          R    0127
      II=I+NH                                                R    0128
      DO 330 J=1,NH                                          R    0129
  330 D(K,II,J) = U1(I,J)                                    R    0130
      IF(K-NF) 340,450,450                                   R    0131
  340 CALL INVERT (U2,NH,DET)                                R    0132      289
      DM(K,2) = DET                                          R    0133
      DO 350 I=1,NH                                          R    0134
      II=I+NH                                                R    0135
      DO 350 J=1,NH                                          R    0136
      JJ=J+NH                                                R    0137
  350 D(K,II,JJ) = U2(I,J)                                   R    0138
      IF( ITP(K)-1) 440,440,360                              R    0139
  360 CONTINUE                                               R    0140
      DO 370 I=1,NH                                          R    0141
      II=I+NH                                                R    0142
      DO 370 J=1,NH                                          R    0143
      U1(I,J)=D(K+1,I,J)                                     R    0144
  370 D1(I,J)=D(K+1,II,J)                                    R    0145
      DO 420 N=IST,K                                         R    0146
      DO 380 I=1,NH                                          R    0147
      DO 380 J=1,NH                                          R    0148
      JJ=J+NH                                                R    0149
      U2(I,J)=0.0                                            R    0150
      D2(I,J)=0.0                                            R    0151
      DO 380 L=1,NH                                          R    0152
      LL=L+NH                                                R    0153
      U2(I,J) = U2(I,J) + U1(I,L) * D(N-1,LL,JJ)             R    0154
  380 D2(I,J) = D2(I,J) + D1(I,L) * D(N-1,LL,JJ)             R    0155
      DO 390 I=1,NH                                          R    0156
      II=I+NH                                                R    0157
      DO 390 J=1,NH                                          R    0158
      JJ=J+NH                                                R    0159
      DO 390 L=1,NH                                          R    0160
      D(K+1,I,JJ )= D(K+1,I ,JJ)- U2(I,L)*D(N-1,L,JJ)        R    0161
  390 D(K+1,II,JJ)= D(K+1,II,JJ)- D2(I,L)*D(N-1,L,JJ)        R    0162
      DO 400 I=1,NH                                          R    0163
      DO 400 J=1,NH                                          R    0164
      U1(I,J)=0.0                                            R    0165
      D1(I,J)=0.0                                            R    0166
      DO 400 L=1,NH                                          R    0167
```

```
      LL=L+NH
      U1(I,J) = U1(I,J) + U2(I,L) * D(N,LL,J)
  400 D1(I,J) = D1(I,J) + D2(I,L) * D(N,LL,J)
      DO 410 I=1,NH
      II=I+NH
      DO 410 J=1,NH
      JJ=J+NH
      DO 410 L=1,NH
      D(K+1,I ,JJ)= D(K+1,I ,JJ) -U1(I,L)*D(N,L,J)
  410 D(K+1,II,JJ)= D(K+1,II,JJ) -D1(I,L)*D(N,L,J)
  420 CONTINUE
      DO 430 I=1,NH
      II=I+NH
      DO 430 J=1,NH
      D(K+1,I ,J )= U1(I,J)
  430 D(K+1,II,J )= D1(I,J)
      IST=0
  440 CONTINUE
  450 CONTINUE
      IF(NDE-7) 470,470,460
  460 V1=U2(3,3)*U2(4,4)-U2(3,4)*U2(4,3)
      V2=U2(3,1)*U2(4,2)-U2(3,2)*U2(4,1)
      V3=U2(3,2)*U2(4,4)-U2(3,4)*U2(4,2)
      V4=U2(3,2)*U2(4,3)-U2(4,2)*U2(3,3)
      V5=U2(3,1)*U2(4,3)-U2(3,3)*U2(4,1)
      V6=U2(3,1)*U2(4,4)-U2(3,4)*U2(4,1)
      DZ(1)= U2(2,2)*V1-U2(2,3)*V3+U2(2,4)*V4
      DZ(2)=-U2(2,1)*V1+U2(2,3)*V6-U2(2,4)*V5
      DZ(3)= U2(2,1)*V3-U2(2,2)*V6+U2(2,4)*V2
      DZ(4)=-U2(2,1)*V4+U2(2,2)*V5-U2(2,3)*V2
      GO TO 480
  470 DZ(1)=U2(2,2)   *U2(3,3)     -U2(2,3)    *U2(3,2)
      DZ(2)= -U2(2,1)   *U2(3,3)       +U2(2,3)    *U2(3,1)
      DZ(3)= U2(2,1)    *U2(3,2)     -U2(3,1)    *U2(2,2)
  480 CONTINUE
      DET=0.0
      DO 490 I=1,NH
  490 DET=DET+U2(1,I)*DZ(I)
      DETA=DET
      IF(NPKT) 560,560,500
  500 CONTINUE
      WRITE (6,510)                                                   434
  510 FORMAT (1H0,38X, 42HSOLUTION IS BASED ON FOLLOWING DETERMINANT)
      WRITE (6,520)
     1            ((U2(I,J),    J=1,NH),I=1,NH)                       435
  520 FORMAT (1H , 22X,4E18.8)
      DIAG=1.0
      DO 530 I=1,NH
  530 DIAG=DIAG*U2(I,I)
      WRITE (6,540)
     1            (DM(I,1),I=1,NF)                                    451
  540 FORMAT(1H ,8HDET(UE)=,8E15.7)
      J=NF-1
      WRITE (6,550)
     1            (DM(I,2),I=1,J)                                     458
  550 FORMAT(1H ,8HDET(UC)=,8E15.7)
```

```
  560 CONTINUE                                             R  0224
      XXE=1.0                                              R  0225
      DO 570 I=1,NF                                        R  0226
  570 XXE=XXE*DM(I,1)/ABS (DM(I,1))                        R  0227
      XXC=1.0                                              R  0228
      DO 580 I=1,J                                         R  0229
  580 XXC=XXC*DM(I,2)/ABS (DM(I,2))                        R  0230
      DETB=DET*XXC*XXE                                     R  0231
      DO 590 I=1,NH                                        R  0232
      II=I+NH                                              R  0233
      DM(NFP,II)=0.0                                       R  0234
  590 DM(NFP,I)=DZ(I)                                      R  0235
      IST=0                                                R  0236
      DO 740 N=1,NF                                        R  0237
      K=NFP-N                                              R  0238
      IF(N-1) 720,720,600                                  R  0239
  600 CONTINUE                                             R  0240
      IF( ITP(K)-1) 610,640,630                            R  0241
  610 IF(IST-1) 660,660,620                                R  0242
  620 IST=0                                                R  0243
      GO TO 640                                            R  0244
  630 IK=K+1                                               R  0245
      IST=2                                                R  0246
  640 DO 650 I=1,NH                                        R  0247
      II=I+NH                                              R  0248
      DO 650 J=1,NH                                        R  0249
      JJ=J+NH                                              R  0250
  650 D(K,II,NPL) = D(K,II,NPL) - D(K,I,JJ) * DM(IK,JJ)    R  0251
      IF(ITP(K)-1) 660,660,680                             R  0252
  660 DO 670 I=1,NH                                        R  0253
      II=I+NH                                              R  0254
  670 D(K,II,NPL) = D(K,II,NPL) + DM(K+1,II)               R  0255
  680 DO 690 I=1,NH                                        R  0256
      II=I+NH                                              R  0257
      DM(K+1,I) =0.0                                       R  0258
      DO 690 L=1,NH                                        R  0259
      LL=L+NH                                              R  0260
  690 DM(K+1,I) = DM(K+1,I) + D(K,II,LL) * D(K,LL,NPL)     R  0261
      IF(ITP(K)-1) 720,700,700                             R  0262
  700 DO 710 I=1,NH                                        R  0263
      DO 710 J=1,NH                                        R  0264
      JJ=J+NH                                              R  0265
  710 D(K,I,NPL) = D(K,I,NPL) - D(K,I,J) * DM(IK,JJ)       R  0266
  720 DO 730 I=1,NH                                        R  0267
  730 D(K,I,NPL) = D(K,I,NPL) + DM(K+1,I)                  R  0268
      DO 740 I=1,NH                                        R  0269
      II=I+NH                                              R  0270
      DM(K,II)=0.0                                         R  0271
      DO 740 L=1,NH                                        R  0272
  740 DM(K,II) = DM(K,II) + D(K,II,L) * D(K,L,NPL)         R  0273
      DO 750 I=1,NH                                        R  0274
      II=I+NH                                              R  0275
      DM(NFP,II)=0.0                                       R  0276
      DM(1,I)=0.0                                          R  0277
      DO 750 J=1,NH                                        R  0278
  750 DM(NFP,II)=DM(NFP,II)+  U2(I,J)*DZ(J)                R  0279
```

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R            -  EFN   SOURCE STATEMENT  -  IFN(S)  -

      CHK=DM(1,NH+1)
      DETC=DET/CHK
      DET=DETA
      IF(NPRT) 780,780,760
  760 CONTINUE
      WRITE (6,770) DET,DETA,DETB,OMEGA,DIAG,XXE,XXC                          599
  770 FORMAT (1H0,4HDET=,E13.6,  9H ACT DET=,E13.6,  9H ADJ DET=,E13.6,
     1 7H OMEGA=,E13.6, 6H DIAG=,E13.6, 5H XXE=,F4.1, 5H XXC=,F4.1)
  780 CONTINUE
      IF(LD) 840,840,790
  790 CONTINUE
      IF(LD-NAC) 810,800,800
  800 IFR=1
      XA=X1
      YA=Y1
      RETURN
  810 IF(DET/YA) 830,950,820
  820 XA=OMEGA
      YA=DET
      LD=LD+1
      GO TO 880
  830 XD=XC
      XC=XB
      XB=OMEGA
      YE=YC
      YC=YB
      YB=DET
      LD=LD+1
      IM=IM+1
      GO TO 880
  840 IF(IM) 850,850,860
  850 IM=1
      XA=OMEGA
      YA=DET
      XD=0.0
      XC=0.0
      XB=0.0
      YE=0.0
      YC=0.0
      YB=0.0
      GO TO 960
  860 IM=IM+1
      XD=XC
      XC=XB
      XB=XA
      XA=OMEGA
      YE=YC
      YC=YB
      YB=YA
      YA=DET
      IF(YA/YB) 870,950,960
  870 LD=1
      X1=XA
      Y1=YA
  880 G1=  YA/(XA-XB)  +YB/(XB-XA)
      G2=0.0
```

```
    G3=0.0                                                          R   0336
    IF(IP-2) 900,900,890                                            R   0337
890 G2=  YA/((XA-XB)*(XA-XC))  +YB/((XB-XA)*(XB-XC))  +YC/((XC-XA)* R   0338
   1(XC-XB))                                                        R   0339
900 X=  -YA/G1+XA                                                   R   0340
    V1=X                                                            R   0341
    X=  -(YA+(X-XA)*(X-XB)*G2)/G1+XA                                R   0342
    V2=X                                                            R   0343
    V3=X                                                            R   0344
    OMEGA=X                                                         R   0345
    IF(NPRT) 940,940,910                                            R   0346
910 CONTINUE                                                        R   0347
    WRITE(6,920)  V1,V2,V3                                          R   0348    628
920 FORMAT (1H0,  29HITERATIONS FOR ZERO POINT ARE, 3E16.8)         R   0349
    WRITE (6,930)                                                   R   0350
   1           XD,YE,XC,YC,XB,YB,XA,YA                              R   0351    629
930 FORMAT (1H ,13HXS AND YS ARE,  4( E12.5,E12.5))                 R   0352
940 CONTINUE                                                        R   0353
    IFR=3                                                           R   0354
    RETURN                                                          R   0355
950 IFR=1                                                           R   0356
    NB=NB+1                                                         R   0357
    XA=X1                                                           R   0358
    YA=Y1                                                           R   0359
    RETURN                                                          R   0360
960 IFR=2                                                           R   0361
    RETURN                                                          R   0362
    END                                                             R   0363
```

```
      SUBROUTINE RCOND                                            S 0001
      DIMENSION Y(8),YD(8),DUMM(27),PAR(2)                        S 0002
      DIMENSION PLY(20),ZLY(20,5)                                 S 0003
      DIMENSION TR(8,8,4),TLI(8,8),ALFR(4)                        S 0004
      COMMON NDE,S,Y,YD,HH,J9,JMAX,N9,XOUT,IFREQ,DUMM,IPK,ISH     S 0005
      COMMON PAR,LC,KFG,KMR,IVD                                   S 0006
      COMMON XN,XLD,CKSQ,HTT,R1,R2,R3,SXN,CXS,INDEX               S 0007
      COMMON PN,PL,PC,T0,T1,RH1,RH2,RH3,PLY,ZLY                   S 0008
      COMMON TR,TLI,IBCL,ALFL,ALFR,NBR                            S 0009
C     NOTE THAT MATRIX TLI HERE IS Y(A)=TLI*U(A)                  S 0010
C     TR MATRIX IS U(B)=TR*Y(B)                                   S 0011
      DO 10 I=1,NDE                                               S 0012
      DO 10 J=1,NDE                                               S 0013
      TLI(I,J)=0.0                                                S 0014
      DO 10 K=1,NBR                                               S 0015
   10 TR (I,J,K)=0.0                                              S 0016
      DO 20 I=1,NDE                                               S 0017
      TLI(I,I)=1.0                                                S 0018
      DO 20 K=1,NBR                                               S 0019
   20 TR (I,I,K)=1.0                                              S 0020
      ALFA=ALFL*1.74533E-02                                       S 0021
      SIL=SIN (ALFA)                                              S 0022   27
      COL=COS (ALFA)                                              S 0023   28
      TLI(1,1)=  COL                                              S 0024
      TLI(1,3)=  SIL                                              S 0025
      TLI(2,2)=  COL                                              S 0026
      TLI(2,4)=  SIL                                              S 0027
      TLI(3,1)= -SIL                                              S 0028
      TLI(3,3)=  COL                                              S 0029
      TLI(4,2)= -SIL                                              S 0030
      TLI(4,4)=  COL                                              S 0031
      DO 30 K=1,NBR                                               S 0032
      ALFA=ALFR(K)*1.74533E-02                                    S 0033
      SIR=SIN (ALFA)                                              S 0034   33
      COR=COS (ALFA)                                              S 0035   34
      TR(1,1,K)=COR                                               S 0036
      TR(1,3,K)=-SIR                                              S 0037
      TR(2,2,K)=COR                                               S 0038
      TR(2,4,K)=-SIR                                              S 0039
      TR(3,1,K)=SIR                                               S 0040
      TR(3,3,K)=COR                                               S 0041
      TR(4,2,K)=SIR                                               S 0042
   30 TR(4,4,K)=COR                                               S 0043
   40 RETURN                                                      S 0044
      END                                                         S 0045
```

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T         - EFN   SOURCE STATEMENT  -  IFN(S)  -

      SUBROUTINE INPUT                                                  T
      DIMENSION Y(6),YD(8),DUMM(27),PAR(2)                              T
      DIMENSION MLY(20),ZLY(20,5)                                       T
      COMMON NDE,S,Y,YD,HH,J9,JMAX,M9,XOUT,IFREQ,DUMM,IBR,ISH           T
      COMMCN PAR,LC,NFG,XMR,IVB                                         T
      COMMON XN,XLD,OMSQ,HTT,R1,R2,R3,SXN,CXS,INDEX                     T
      COMMON PN,PL,PC,TO,T1,RH1,RH2,RH3,MLY,ZLY                         T
      DIMENSION VN(10,4),IL1(10),IL2(10),IFG(10,4)                      T
      DIMENSION VK(10,5),IK2(10),IK1(10),IKG(10,5)                      T
      GO TO (10,260,270,360),INDEX                                      T
   10 READ  (5,20) (VN(IBR,I), I=1,4), IL2(IBR)                         T      3
   20 FORMAT (4F10.5,I5)                                                T
      IF(IL2(IBR)) 70,70,30                                             T
   30 READ  (5,40) (IFG(IBR,I),I=1,3), IL1(IBR)                         T     13
   40 FORMAT (10I5)                                                     T
      WRITE (6,50)   (IFG(IBR,I),I=1,3)                                 T     21
   50 FORMAT (1H0, 29HVARIABLE SHELL PARAMETERS ARE,  4I5)              T
      L1=IL1(IBR)                                                       T
      L2=IL2(IBR)+IL1(IBR)-1                                            T
      DO 60 I=L1,L2                                                     T
   60 DY=FGEN (I,1)                                                     T     37
   70 GO TO (80,100,120,140,160,200,220,240,180), ISH                   T
   80 WRITE  (6,90)                                                     T     41
   90 FORMAT (1H0,   30HNO SHELL NO. 1 IN THIS PROGRAM)                 T     42
      CALL EXIT                                                         T     43
  100 WRITE  (6,110) ISH,VN(IBR,1),VN(IBR,2), VN(IBR,3)                 T
  110 FORMAT (1H0,      20HCYLINDRICAL SHELL NO,I3,2X,                  T
     1 4H  H=,E12.5,2X,4H  R=,E12.5,  2X, 4HPHI=, F10.3, 9H  DEGREES)   T
      RETURN                                                            T
  120 WRITE (6,130)  ISH,VN(IBR,1),VN(IBR,2), VN(IBR,4)                 T     47
  130 FORMAT (1H0,       18HSPHERICAL SHELL NO,I3,4X,                   T
     1 4H  H=,E12.5,2X,4H  R=,E12.5, 12H  DIRECTION=, F3.0 )            T
      RETURN                                                            T
  140 WRITE (6,150)  ISH,VN(IBR,1),VN(IBR,2), VN(IBR,4)                 T     51
  150 FORMAT (1H0,      21HPARABOLOIDAL SHELL NO,I3,3X,                 T
     1 4H  H=,E12.5,2X,5H  2P=,E12.5,  12H  DIRECTION=, F3.0)           T
      RETURN                                                            T
  160 WRITE (6,170)  ISH,VN(IBR,1),VN(IBR,2),VN(IBR,3),VN(IBR,4)        T     55
  170 FORMAT (1H0,       20HELLIPSOIDAL SHELL NO,I3,3X,                 T
     1 4H  H=,E12.5,2X,4H  A=,E12.5, 2X,4H  B=,E12.5,10H  DIRECTN=,F3.0)T
      RETURN                                                            T
  180 WRITE(6,190)  ISH,VN(IBR,1),VN(IBR,2),VN(IBR,3),VN(IBR,4)         T     60
  190 FORMAT (1H0,       20HHYPERBOLIC  SHELL NO,I3,3X,                 T
     1 4H  H=,E12.5,2X,4H  A=,E12.5, 2X,4H  B=,E12.5,10H  DIRECTN=,F3.0)T
      RETURN                                                            T
  200 WRITE  (6,210) ISH,VN(IBR,1),VN(IBR,2),VN(IBR,3)                  T     65
  210 FORMAT (1H0,       16HCONICAL SHELL NO,I3,2X,   4H  H=,F10.6, 2X, T
     14HPHI=,F10.3, 8H DEGREES, 2X,4H  A=,E12.5)                        T
      RETURN                                                            T
  220 WRITE (6,230)  ISH,VN(IBR,1),VN(IBR,2),VN(IBR,3),VN(IBR,4)        T     69
  230 FORMAT (1H0,      17HTOROIDAL SHELL NO,I3,3X,                     T
     1 4H  H=,E12.5,2X,4H  A=,E12.5, 2X,4H  B=,E12.5,10H  DIRECTN=,F3.0)T
      RETURN                                                            T
  240 WRITE  (6,250) ISH, (VN(IBR,I),I=1,4)                             T     74
  250 FORMAT (1H0,  16HGENERAL SHELL NO, I3, 4H  H=,E12.5,              T
```

```
T           - EFN   SOURCE STATEMENT  -  IFN(S)  -

      17H 1/RFI=,E12.5, 5H   R=, E12.5, 4H FI=, F10.3,  4H DEG)      T  0056
      RETURN                                                         T  0057
  260 CONTINUE                                                       T  0058
      RETURN                                                         T  0059
  270 CONTINUE                                                       T  0060
      L1=IL1(IBR)                                                    T  0061
      L2=IL2(IBR)+IL1(IBR)-1                                         T  0062
      GO TO(60,280,290,300,310,330,340,350,320), ISH                 T  0063
  280 R2=ABS (1.0/VN(IBR,2))                                         T  0064
      R1=0.0                                                         T  0065
      R3=1.0                                                         T  0066
      CXS=0.0                                                        T  0067
      ALFA=VN(IBR,3)*1.745329E-02                                    T  0068
      SXN=SIN (ALFA)                                                 T  0069    91
      GO TO 360                                                      T  0070
  290 R1=1.0/VN(IBR,2)                                               T  0071
      R3=ABS (R1)                                                    T  0072
  300 GO TO 360                                                      T  0073
  310 BSQ= (VN(IBR,3)/VN(IBR,2))*(VN(IBR,3)/VN(IBR,2))               T  0074
      GO TO 360                                                      T  0075
  320 BSQ= (VN(IBR,3)/VN(IBR,2))*(VN(IBR,3)/VN(IBR,2))               T  0076
      GO TO 360                                                      T  0077
  330 R1=0.0                                                         T  0078
      R3=1.0                                                         T  0079
      ALFA = VN(IBR,2)*0.1745329E-01                                 T  0080
      SXN=SIN (ALFA)                                                 T  0081   106
      CXS=COS (ALFA)                                                 T  0082   107
      GO TO 360                                                      T  0083
  340 R1=1.0/VN(IBR,3)                                               T  0084
      R3=ABS (R1)                                                    T  0085
      GO TO 360                                                      T  0086
  350 R3=1.0                                                         T  0087
  360 IF(IL2(IBR)) 390,390,370                                       T  0088
  370 IK=0                                                           T  0089
      DO 380 I=L1,L2                                                 T  0090
      IK=IK+1                                                        T  0091
      J=IFG(IBR,IK)                                                  T  0092
  380 VN(IBR,J)=FGEN (I,2)                                           T  0093   124
  390 H=VN(IBR,1)                                                    T  0094
      GO TO(80,470,400,410,420,440,450,460,430), ISH                 T  0095
  400 ARG=S*VN(IBR,4)                                                T  0096
      SXN=SIN (ARG)                                                  T  0097   133
      CXS=COS (ARG)                                                  T  0098   134
      R2=ABS (R1/SXN)                                                T  0099
      GO TO 470                                                      T  0100
  410 ARG=S*VN(IBR,4)                                                T  0101
      SXN=SIN (ARG)                                                  T  0102   138
      CXS=COS (ARG)                                                  T  0103   139
      R2=ABS (CXS/(SXN*VN(IBR,2)))                                   T  0104
      R1= ABS(CXS*CXS*CXS)/VN(IBR,2)                                 T  0105
      R3=ABS (R1)                                                    T  0106
      GO TO 470                                                      T  0107
  420 ARG=S*VN(IBR,4)                                                T  0108
      SXN=SIN (ARG)                                                  T  0109   145
      CXS=COS (ARG)                                                  T  0110   146
      R=SQRT (BSQ+(1.-BSQ)*SXN*SXN)                                  T  0111   147
```

```
T         - EFN   SOURCE STATEMENT  -  IFN(S)  -

      R1=R*R*R/(BSG*VN(IBR,2))                              T   0112
      R2=ABS (R   /(SXN*VN(IBR,2)))                         T   0113
      R3=ABS (R1)                                           T   0114
      GO TO 470                                             T   0115
  430 ARG=S*VN(IBR,4)                                       T   0116
      SXN=SIN (ARG)                                         T   0117    153
      CXS=COS (ARG)                                         T   0118    154
      R=SQRT (SXN*SXN -BSG*CXS*CXS)                         T   0119    155
      R1= -VN(IBR,2)*R*R*R/(VN(IBR,3)*VN(IBR,3))            T   0120
      R2=ABS (R   /(SXN*VN(IBR,2)))                         T   0121
      R3=ABS (R1)                                           T   0122
      GO TO 470                                             T   0123
  440 R2= ABS (1.0/((VN(IBR,3)+S)*CXS))                     T   0124
      GO TO 470                                             T   0125
  450 ARG=S*VN(IBR,4)                                       T   0126
      SXN=SIN (ARG)                                         T   0127    166
      CXS=COS (ARG)                                         T   0128    167
      R2=ABS (1.0/(VN(IBR,2)+VN(IBR,3)*SXN))                T   0129
      GO TO 470                                             T   0130
  460 R1=VN(IBR,2)                                          T   0131
      R2=ABS (1.0/VN(IBR,3))                                T   0132
      ARG=  VN(IBR,4)*0.1745329E-01                         T   0133
      CXS=COS(ARG)                                          T   0134    175
      SXN=SIN(ARG)                                          T   0135    176
  470 RETURN                                                T   0136
      END                                                   T   0137
```

```
      SUBROUTINE PGEN
      DIMENSION Y(8),YD(8),DUMM(27),PAR(2)
      COMMON NDE,S,Y,YD,HH,J9,JMAX,M9,XOUT,IFREQ,DUMM,IBR,ISH
      COMMON PAR,LC,NFG,XMR,IVB
      DIMENSION        XP(31,20),YP(2,30,20),SL(2,30,20),NFT(20)
      GO TO (10,20,70),NFG
   10 NCT=0
      RETURN
   20 NCT=NCT+1
      NFT(IBR)=NCT
      XP(NCT,IBR)=S
      DO 30 I=1,2
   30 YP(I,NCT,IBR)=PAR(I)
      IF(NCT-1) 60,60,40
   40 L=NCT-1
      DO 50 I=1,2
      SL(I,L,IBR)=  (YP(I,NCT,IBR)-YP(I,L,IBR))/(S-XP(L,IBR))
   50 YP(I,L,IBR)=  YP(I,L,IBR)-SL(I,L,IBR)*XP(L,IBR)
   60 RETURN
   70 J=NFT(IBR)
      DO 80 I=1,J
      L=I
      IF(XP(I,IBR)-S) 80,80,90
   80 CONTINUE
   90 IF(L-1) 100,100,110
  100 L=2
  110 DO 120 I=1,2
  120 PAR(I)=  SL(I,L-1 ,IBR)*S+YP(I,L-1,IBR)
      RETURN
      END
```

```
      FUNCTION FGEN (NM,N1)                                          V
      DIMENSION XP(30,20),YP(30,20),SL(30,20),M(30)                  V
      COMMON NDE,S                                                   V
      IF(NM-30) 30,30,10                                             V
   10 WRITE (6,20)                                                   V      3
   20 FORMAT(1H0,50HMAXIMUM NUMBER OF 30 FGEN SETS HAVE BEEN EXCEEDED ) V
      CALL EXIT                                                      V      4
   30 CONTINUE                                                       V
      GO TO (40,120),N1                                              V
   40 READ  (5,50) NAMEG,  M(NM)                                     V      7
   50 FORMAT (A5,I5)                                                 V
      WRITE  (6,60) NAMEG, NM,M(NM)                                  V      10
   60 FORMAT (1H0,  10X, A5,  32H  LINEAR FUNCTION GENERATOR NO. ,   V
     1I3, 5H FROM, I4, 7H POINTS)                                    V
      MK=M(NM)                                                       V
      READ  (5,70) (XP(NM,I),  YP(NM,I), I=1,MK)                     V      13
   70 FORMAT (8F10.5)                                                V
      MU=0                                                           V
      MZ=10                                                          V
      MX= (M(NM)-1)/10+1                                             V
      DO 100 J=1,MX                                                  V
      L=AMINO (MK,    MZ)                                            V
      L1=MU+1                                                        V
      WRITE  (6,80) (YP(NM,I), I=L1,L)                               V      29
      WRITE  (6,90)  (XP(NM,I), I=L1,L)                              V      34
   80 FORMAT (1H0, 13HY COORDINATES,3X,10F10.5)                      V
   90 FORMAT (1H0, 13HX COORDINATES,3X,10F10.5)                      V
      MZ=MZ+10                                                       V
  100 MU=MU+10                                                       V
      MM=M(NM)-1                                                     V
      DO 110 I=1,MM                                                  V
      SL(NM,I)= (YP(NM,I+1)-YP(NM,I))/ (XP(NM,I+1)-XP(NM,I))         V
  110 YP(NM,I)=YP(NM,I) -SL(NM,I)*XP(NM,I)                           V
      RETURN                                                         V
  120 MK=M(NM)                                                       V
      DO 130 I=1,MK                                                  V
      J=I                                                            V
      IF(XP(NM,I)-S) 130,130,140                                     V
  130 CONTINUE                                                       V
  140 IF(J-1) 150,150,160                                            V
  150 J=2                                                            V
  160 FGEN= SL(NM,J-1)*S +YP(NM,J-1)                                 V
      RETURN                                                         V
      END                                                            V
```

```
      SUBROUTINE INVERT  (DP,MAX,DETERM)
      DIMENSION DP(4,4),M(4) ,C(4)
      DETERM = 1.
      DO 10 I = 1, MAX
      M(I) = - I
   10 CONTINUE
      DO 140 II = 1, MAX
      D = 0.0
      DO 60 K = 1, MAX
      IF (M(K)) 20,20,60
   20 DO 50 L = 1, MAX
      IF (M(L)) 30,30,50
   30 IF(ABS(D) - ABS(DP(K,L))) 40,40,50
   40 LD = L
      KD = K
      D = DP(K,L)
   50 CONTINUE
   60 CONTINUE
      IF(KD-LD) 70,80,70
   70 DETERM =-DETERM
   80 DETERM = D*DETERM
      NEMP = -M(LD)
      M(LD)=M(KD)
      M(KD)= NEMP
      DO 90 I = 1, MAX
      C(I) = DP(I,LD)
      DP(I,LD) = DP(I,KD)
      DP(I,KD) = 0.0
   90 CONTINUE
      DP(KD,KD) = 1.
      DO 100 J = 1, MAX
      DP(KD,J) = DP(KD,J)/D
  100 CONTINUE
      DO 130 I = 1, MAX
      IF (I-KD) 110,130,110
  110 DO 120 J = 1, MAX
      DP(I,J) = DP(I,J) - C(I)*DP(KD,J)
  120 CONTINUE
  130 CONTINUE
  140 CONTINUE
      DO 170 I = 1, MAX
      L=0
  150 L=L+1
      IF (M(L)-I) 150,160,150
  160 M(L)=M(I)
      M(I)=I
      DO 170 J = 1, MAX
      TEMP = DP(L,J)
      DP(L,J) = DP(I,J)
      DP(I,J) = TEMP
  170 CONTINUE
      RETURN
      END
```

```
SUBROUTINE RUNGE                                                  X  0001
COMMON N,  X,Y,DY,HH,J, JMAX,M, XOUT,IFREQ,X1,X2,X3,Y1,Y2,Y3      X  0002
DIMENSION Y(8),DY(8),Y1(8),Y2(8),Y3(8)                            X  0003
J=1                                                               X  0004
JMAX=1                                                            X  0005
IFREQ=3                                                           X  0006
M=1                                                               X  0007
CALL RUNKUT                                                       X  0008    6
RETURN                                                            X  0009
END                                                               X  0010
```

```
      SUBROUTINE RUNKUT
      COMMON N,  X,Y,DY,HH,J, JMAX,M, XOUT,IFREQ,X1,X2,X3,Y1,Y2,Y3
      DIMENSION Y(8),DY(8),Y1(8),Y2(8),Y3(8)
      INDE9      = 0
      CALL        ADJSTP                                          3
      IF(J-JMAX) 10,10,50
   10 INDE9      = INDE9 + 1
      CALL INTPOL                                                 8
      IF(J-JMAX) 20,20,50
   20 CALL STEP                                                  12
      X1         = X2
      X2         = X3
      X3         = X
      DO 30 I = 1, N
      Y1(I)      = Y2(I)
      Y2(I)      = Y3(I)
   30 Y3(I)      = Y(I)
      IF (INDE9 - IFREQ) 10,40,40
   40 INDE9      = 0
      CALL        ADJSTP                                         29
      IF(J-JMAX) 10,10,50
   50 RETURN
      END
```

```
      SUBROUTINE ADJSTP
      COMMON N,  X,Y,DY,HH,J, JMAX,M, XOUT,IFREQ,X1,X2,X3,Y1,Y2,Y3
      DIMENSION Y(8),DY(8),Y1(8),Y2(8),Y3(8)
      KSL=0
      HFACT     = 1.0 E+31
      HFACT1    = 1.0E+30
      GO TO (30,10), M
   10 H1        = HH
      HH        = 2.0 * HH
      X         = X1
      DO 20 I = 1, N
   20 Y(I)      = Y1(I)
      GO TO 100
   30 KSL=1
   40 H1        = HH
      XXX       = X
      DO 50 I = 1, N
   50 Y1(I)     = Y(I)
      X1        = X
      CALL INTPOL                                                          24
      IF(J-JMAX) 60,60,250
   60 CALL STEP                                                            28
      DO 70 I = 1, N
   70 Y2(I)     = Y(I)
      X2        = X
      CALL INTPOL                                                          38
      IF(J-JMAX) 80,80,250
   80 CALL STEP                                                            42
      DO 90 I = 1, N
      Y3(I)     = Y(I)
   90 Y(I)      = Y1(I)
      X3        = X
      X         = XXX
      HH        = 2.0 * HH
  100 CALL STEP                                                            55
      DO 150 I = 1, N
      DELY   = ABS ( Y(I)-Y3(I))/30.0
      IF(DELY  -ABS (Y2(I))*1.E-05 )120,110,110
  110 IF( ABS (Y2(I))-1.0E-05) 120,130,130
  120 HFIRST    = 1.0E+30
      GO TO 140
  130 HFIRST=  (ABS (Y2(I))* 1.0E-05/DELY    ) **0.2                       71
  140 CONTINUE
  150 HFACT=AMIN1 (HFACT, HFIRST   )
      IF (HFACT1 - HFACT) 160,160,170
  160 HH        = 2.0 * H1
      GO TO (40,230), M
  170 HH        = H1 * HFACT
      GO TO (180,230), M
  180 IF(KSL) 220,220,190
  190 KSL=0
      IF(ABS (HH)-ABS (H1)) 200,220,220
  200 DO 210 I = 1, N
  210 Y(I)      = Y1(I)
      X         = XXX
```

```
      Z          - EFN   SOURCE STATEMENT  -  IFN(S)  -

      GO TO 40                                             Z   0056
  220 KSL=0                                                Z   0057
      M         = 2                                        Z   0058
  230 DO 240 I = 1, N                                      Z   0059
  240 Y(I)        = Y3(I)                                  Z   0060
  250 RETURN                                               Z   0061
      END                                                  Z   0062
```

```
      SUBROUTINE STEP
      COMMCN N,  X,Y,DY,HH,J, JMAX,M, XOUT,IFREQ,X1,X2,X3,Y1,Y2,Y3
      DIMENSION Y(8),DY(8),Y1(8),Y2(8),Y3(8)
      DIMENSION YO(8),P1(8)
      DO 10 I = 1, N
   10 YO(I)       = Y(I)
      XO          =X
      CALL DIFFEQ                                                  10
      DO 20 I = 1, N
      P1(I)       = DY(I) * HH
   20 Y(I)        = YO(I) + P1(I)*0.5
      X           = XO + HH*0.5
      CALL DIFFEQ                                                  23
      DO 30 I = 1, N
      P1(I)= P1(I)+2.0*HH*DY(I)
   30 Y(I)        = YO(I) + 0.5*HH*DY(I)
      CALL DIFFEQ                                                  37
      DO 40 I = 1, N
      P1(I)= P1(I)+2.0*HH*DY(I)
   40 Y(I)        = YO(I) + HH*DY(I)
      X           = XO + HH
      CALL DIFFEQ                                                  51
      DO 50 I = 1, N
   50 Y(I)=YO(I) + (P1(I)+HH*DY(I))*0.1666667
      RETURN
      END
```

```
      SUBROUTINE INTPOL                                             ZB   0001
      COMMON N,  X,Y,DY,HH,J, JMAX,M, XOUT,IFREQ,X1,X2,X3,Y1,Y2,Y3 ZB   0002
      DIMENSION Y(8),DY(8),Y1(8),Y2(8),Y3(8)                        ZB   0003
      IF(ABS (XOUT     - X)-ABS (HH)) 10,10,20                      ZB   0004
   10 HH=XCUT-X                                                     ZB   0005
      CALL STEP                                                     ZB   0006     4
      J          = J + 1                                            ZB   0007
   20 RETURN                                                        ZB   0008
      END                                                           ZB   0009
```

```
C     MAIN    OF NONSYMMETRIC STABILITY                                 MAIN0001
C     MAXIMUM NUMBER OF SEGMENTS IS 20, MAX COMPONENTS 9                MAIN0002
C     DEFINE FILE 2(3600,36,U,LINE2)                                    MAIN0003
C     DEFINE FILE 4(3600,72,U,LINE4)                                    MAIN0004
      DIMENSION DUMM(220)                                               MAIN0005
      DIMENSION Y(8,9),DY(8,9),MNM(9)                                   MAIN0006
      DIMENSION D(36,36),EI(36,36),CI(36,36),CB(36)                     MAIN0007
      DIMENSION DM(72,72),IA(8),IB(8)                                   MAIN0008
      DIMENSION PAR(3)                                                  MAIN0009
      DIMENSION PRE(3)                                                  MAIN0010
      DIMENSION SI(20),SF(20),NSEG(20),INT(20),ISS(20),NTP(20)          MAIN0011
      DIMENSION B11(20,4),B12(20,4),B22(20,4),B66(20,4)                 MAIN0012
      DIMENSION MLY(20),ZLY(20,5)                                       MAIN0013
      DIMENSION RHO(20,4)                                               MAIN0014
      COMMON NDE,X,Y,DY,HH,J9,JMAX,M9,XOUT,IFREQ,KT,DUMM,LINE1          MAIN0015
      COMMON XI,XF,ALFA,HT,R2,R1,R3,E,SP,CSP,INDEX,IBR,RHH,OMSQ,IVB,ISH MAIN0016
      COMMON MSEG,DM,D,DETC,NB,EI,CI,IA,IB,CB                           MAIN0017
      COMMON MNM,PAR,IPR,P,NOPR,P1,NST                                  MAIN0018
      COMMON SI,SF,NSEG,INT,ISS                                         MAIN0019
      COMMON IBRM,NFG,LCST                                              MAIN0020
      COMMCN LINE2,LINE4                                                MAIN0021
      COMMON C11,C12,C22,E11,E12,E22,D11,D12,D22,MLY,ZLY,C66,E66,D66    MAIN0022
      COMMON B11,B12,B22,B66,RHO                                        MAIN0023
  400 READ(5,10)                                                        MAIN0024
     1      IBRM,VAC,NBR,NXT,IVB,NOPR,NB                                MAIN0025    1
   10 FORMAT(7I5)                                                       MAIN0026
      IF(IBRM) 30,20,30                                                 MAIN0027
   20 CALL EXIT                                                         MAIN0028   10
   30 CONTINUE                                                          MAIN0029
      WRITE(6,31)                                                                  11
   31 FORMAT(1H1,10X,60HNONSYMMETRIC (PRESTRESS) EIGENVALUE PROGRAM JAN
     11969 VERSION)
      NDE=8                                                             MAIN0030
      DO 170 I=1,IBRM                                                   MAIN0031
      IBR=I                                                             MAIN0032
      WRITE(6,40) I                                                     MAIN0033   16
   40 FORMAT(1H0,10X,8HPART NO.,I3)                                     MAIN0034
      READ(5,50)                                                        MAIN0035
     1        SI(I),SF(I),NSEG(I),INT(I),ISS(I),NTP(I),MLY(I)           MAIN0036   17
   50 FORMAT(2F10.3,5I5)                                                MAIN0037
      WRITE(6,60)                                                       MAIN0038
     1       SI(I),SF(I),NSEG(I),INT(I),ISS(I),MLY(I)                   MAIN0039   25
   60 FORMAT(1H0,3HSI=,E12.5,4H SF=,E12.5,I5,9H SEGMENTS,I3,7H POINTS   MAIN0040
     1   , 11H  SHELL NO.,I3,I5,7H LAYERS)                              MAIN0041
      ISH=ISS(I)                                                        MAIN0042
      XI=SI(I)                                                          MAIN0043
      XF=SF(I)                                                          MAIN0044
      MSEG=NSEG(I)                                                      MAIN0045
      NST=INT(I)                                                        MAIN0046
      INDEX=1                                                           MAIN0047
      CALL INPUT                                                        MAIN0048   41
      CALL ORTHO                                                        MAIN0049   43
      READ(5,70) IBRT,NTR,LCST                                          MAIN0050   44
   70 FORMAT(3I5)                                                       MAIN0051
      WRITE (6,250)                                                     MAIN0052   48
```

```
MAIN        -  EFN   SOURCE STATEMENT  -  IFN(S)  -

      WRITE(6,71)                                                         49
   71 FORMAT(1H0,20X,23HPRESTRESS VARIABLES ARE)
      WRITE(6,80)                                             MAIN0053    50
   80 FORMAT(1H0,16H        X        ,16H       NPHI        ,     MAIN0054
     116H     NTHETA      ,16H   NTHETA PHI     )             MAIN0055
      IF(LCST-1) 110,90,90                                    MAIN0056
   90 CONTINUE                                                MAIN0057
  100 NFG=1                                                   MAIN0058
      CALL PGEN                                               MAIN0059    55
      NFG=2                                                   MAIN0060
  110 CONTINUE                                                MAIN0061
      DO 130 JS=1,NTR                                         MAIN0062
      READ(5,120)  X,(PAR(K),K=1,3)                           MAIN0063    59
  120 FORMAT(4F20.8)                                          MAIN0064
  130 WRITE(6,140) X,(PAR(K),K=1,3)                           MAIN0065    65
  140 FORMAT(1H , 4E16.8)                                     MAIN0066
      IF (LCST-1) 160,150,150                                 MAIN0067
  150 CONTINUE                                                MAIN0068
      CALL PGEN                                               MAIN0070    73
  160 CONTINUE                                                MAIN0071
  170 CONTINUE                                                MAIN0072
      INXC=0                                                  MAIN0073
  180 CONTINUE                                                MAIN0074
      WRITE(6,31)                                                         78
      WRITE(6,191) ALFA,DEL,ALFB,NFIN                                     79
  191 FORMAT(1H0,15HSTARTING OMEGA=,E12.5,12H  INCREMENT=,E12.5,14H  FIN
     1AL OMEGA=,E12.5,I5,12H EIGENVALUES)
      READ(5,190)                                             MAIN0075
     1      ALFA,DEL,ALFB,NFIN,NX,KT                          MAIN0076    80
  190 FORMAT(3F10.3,3I5)                                      MAIN0077
      DO 200 I=1,KT                                           MAIN0078
  200 MNM(I)=NX+(I-1)*NB                                      MAIN0079
      WRITE(6,210) IBRM,KT,(MNM(I),I=1,KT)                    MAIN0080    90
  210 FORMAT(1H0,10X,19HEIGENVALUE ANALYSIS,I5,7H PARTS,I5,
     1 14H  WAVE NUMBERS,9I5)                                 MAIN0082
      READ (5,220)  (IA(I),I=1,8)                             MAIN0083    97
      READ (5,220)  (IB(I),I=1,8)                             MAIN0084   104
  220 FORMAT (8I5)                                            MAIN0085
      WRITE(6,250)                                            MAIN008    111
      WRITE(6,240)  (IA(I),I=1.8)                             MAIN008    112
      WRITE(6,241)  (IB(I),I=1,8)                                        119
  240 FORMAT(1H0,42HAT STARTING EDGE, PRESCRIBED VARIABLES ARE,4I5,5X,26
     1HUNPRESCRIBED VARIABLES ARE,4I5)
  241 FORMAT(1H0,42HAT FINAL EDGE, UNPRESCRIBED VARIABLES ARE ,4I5,7X,24
     1HPRESCRIBED VARIABLES ARE,4I5)
  250 FORMAT (1H0)                                            MAIN0092
      IM=0                                                    MAIN0093
  260 REWIND 4                                                MAIN0094   126
      WRITE(6,31)                                                        127
      IF(IVB)  290,270,290                                    MAIN0095
  270 OMSQ=ALFA*ALFA                                          MAIN0096
      WRITE(6,280) ALFA                                                  130
  280 FORMAT(1H1,35HFREE VIBRATION ANALYSIS WITH OMEGA=,E12.5)
      GO TO 310                                               MAIN0099
  290 CONTINUE                                                MAIN0100
      WRITE (6,300)  ALFA                                     MAIN0101   132
```

```
 300 FORMAT(1H1,30HSTABILITY ANALYSIS WITH OMEGA=,E12.5)
 310 CONTINUE                                            MAIN0103
     DO 350 J=1,IBRM                                     MAIN0104
     IBR=J                                               MAIN0105
     XI=SI(J)                                            MAIN0106
     XF=SF(J)                                            MAIN0107
     MSEG=NSEG(J)                                        MAIN0108
     NST=INT(J)                                          MAIN0109
     ISH=ISS(J)                                          MAIN0110
     INDEX=3                                             MAIN0111
     CALL INPUT                                          MAIN0112     147
     CALL ORTHO                                          MAIN0113     149
     INDEX=4                                             MAIN0114
     NPL=NST+1                                           MAIN0115
     IF(ISH-2) 330,320,330                               MAIN0116
 320 CALL SYLSG                                          MAIN0117     155
     GO TO 350                                           MAIN0118
 330 CONTINUE                                            MAIN0119
     DO 340 M=1,MSEG                                     MAIN0120
 340 CALL SEGM(M)                                        MAIN0121     161
 350 CONTINUE                                            MAIN0122
     CALL TRIA                                           MAIN0123     167
     IF(DETC) 360,390,360                                MAIN0124
 360 CONTINUE                                            MAIN0125
 370 CONTINUE                                            MAIN0126
     IF (ALFA-ALFB) 380,390,390                          MAIN0127
 380 ALFA=ALFA+DEL                                       MAIN0128
     GO TO 260                                           MAIN0129
 390 INXC=INXC+1                                         MAIN0130
     IF(INXC-NXT) 180,400,400                            MAIN0131
     END                                                 MAIN0133
```

```
      SUBROUTINE SEGM(M)                                                  A  0001
C     SEGM    OF NONSYMMETRIC STABILITY                                   A  0002
      DIMENSION DUMM(220)                                                 A  0003
      DIMENSION SI(20),SF(20),NSEG(20),INT(20),ISS(20),NTP(20)            A  0004
      DIMENSION Y(8,9),DY(8,9),MNM(9)                                     A  0005
      DIMENSION D(36,36),EI(36,36),CI(36,36),CB(36)                       A  0006
      DIMENSION DM(72,72),IA(8),IB(8)                                     A  0007
      DIMENSION PAR(3)                                                    A  0008
      COMMON NDE,X,Y,DY,HH,J9,JMAX,M9,XOUT,IFREQ,KT,DUMM,LINE1            A  0009
      COMMON XI,XF,ALFA,HT,  R1,R2,R3,E, SP,CSP,INDEX,IBR,PN,PL,PC,ISH    A  0010
      COMMON MSEG,DM,D,DET,NB,EI,CI,IA,IB,CB                              A  0011
      COMMON MNM,PAR,IPR,P,NOPR,P1,NST                                    A  0012
      COMMON SI,SF,NSEG,INT,ISS                                           A  0013
      COMMON IBRM,NFG,LCST                                                A  0014
      COMMON LINE2,LINE4                                                  A  0015
      NPL=NST+1                                                           A  0016
      XST=NST                                                             A  0017
      SMXX = (XF - XI)/FLOAT(MSEG)                                        A  0018
      HH=0.01*SMXX                                                        A  0019
      WRITE (6,90)                                                        A  0020   3
      NH=NDE*KT                                                           A  0021
   10 JJ=0                                                                A  0022
      IF(NORR-1) 30, 30, 20
   20 CONTINUE                                                            A  0024
      WRITE (6,90)                                                        A  0025   7
   30 CONTINUE                                                            A  0026
      DO 240 N1=1,NDE                                                     A  0027
      IF(IBR-1) 60,40,60                                                  A  0028
   40 CONTINUE                                                            A  0029
      IF (M-1) 60,50,60                                                   A  0030
   50 L=IA(N1)                                                            A  0031
      GO TO 70                                                            A  0032
   60 L=N1                                                                A  0033
   70 DO 240 K1=1,KT                                                      A  0034
      DO 80 I=1,NDE                                                       A  0035
      DO 80 K=1,KT                                                        A  0036
   80 Y(I,K)=0.0                                                          A  0037
      Y(L,K1)=1.0                                                         A  0038
   90 FORMAT (1H0)                                                        A  0039
      X=XI                                                                A  0040
      IF(NOPR-1) 130,130,100
  100 CONTINUE                                                            A  0042
      DO 110 K=1,KT                                                       A  0043
  110 WRITE(6,120)      X,(Y(I,K),I=1,NDE)                                A  0044  35
  120 FORMAT(1H ,    F9.3,2X,8E13.6)                                      A  0045
      WRITE (6,90)                                                        A  0046  40
  130 CONTINUE                                                            A  0047
      IF(M-1) 140,140,150                                                 A  0048
  140 X=XI                                                                A  0049
  150 SMAX=X+SMXX                                                         A  0050
      XOUT=SMAX                                                           A  0051
      IPR=2                                                               A  0052
      CALL RUNGE                                                          A  0053  46
      IF(NOPR-1) 180,180,160
  160 CONTINUE                                                            A  0055
```

```
      DO 170 K=1,KT                                        A  0056
  170 WRITE(6,120)      X,(Y(I,K),I=1,NDE)                 A  0057   52
      WRITE (6,90)                                         A  0058   57
  180 CONTINUE                                             A  0059
      J=0                                                  A  0060
      JJ=JJ+1                                              A  0061
      DO 230 I=1,NDE                                       A  0062
      IF(IBR-IBRM) 210,190,190                             A  0063
  190 CONTINUE                                             A  0064
      IF (M-MSEG) 210,200,200                              A  0065
  200 L1=IB(I)                                             A  0066
      GO TO 220                                            A  0067
  210 L1=I                                                 A  0068
  220 DO 230 K=1,KT                                        A  0069
      J=J+1                                                A  0070
  230 DM(J,JJ)=Y(L1,K)                                     A  0071
  240 CONTINUE                                             A  0072
      DO 250 I=1,NH                                        A  0073
  250 WRITE (4)                                            A  0074
C  28 WRITE (4'LINE4)                                      A  0075
     1                  (DM(I,J),J=1,NH)                   A  0076   85
      IF(NOPR-1) 280,280,260
  260 CONTINUE                                             A  0078
      DO 270 I=1,NH                                        A  0079
  270 WRITE(6,290)   (DM(I,J),J=1,NH)                      A  0080   97
  280 CONTINUE                                             A  0081
  290 FORMAT (1H0, 8E12.5)                                 A  0082
      RETURN                                               A  0083
      END                                                  A  0064
```

```
      SUBROUTINE SYLSG
C     SEGM OF CYLINDRICAL SHELL
      DIMENSION DUMM(220)
      DIMENSION SI(20),SF(20),NSEG(20),INT(20),ISS(20)
      DIMENSION Y(8,9),DY(8,9),MMM(9)
      DIMENSION D(36,36),C1(36,36),CI(36,36),CR(36)
      DIMENSION DM(72,72),IA(8),IB(8)
      DIMENSION PAR(3)
      COMMON NDE,X,Y,DY,MM,JV,JMAX,M9,XOUT,IFREQ,KT,DUMM,LINE1
      COMMON XI,XF,ALFA,HT,  K1,K2,K3,E, SP,CSP,INDEX,IBN,PN,PL,PC,ISM
      COMMON MSEG,DM,D,DET,NM,E1,C1,IA,IB,CP
      COMMON MNM,PAR,IPR,P,NOPM,P1,NST
      COMMON SI,SF,NSEG,INT,ISS
      COMMON IBMM,NFG,LCST
      COMMON LINE2,LINE4
      SMXX = (XF - XI)/FLOAT(MSEG)
      NM=NDE*KT
      DO 60 M=1,MSEG
      IF (M-2) 10,10,20
   10 CALL SEGM(M)
      GO TO 60
   20 CONTINUE
      IF (M-MSEG) 40,30,30
   30 X=XF-SMXX
      CALL SEGM(M)
      GO TO 60
   40 CONTINUE
      DO 50 I=1,NM
   50 WRITE (4)
     1        (DM(I,J),J=1,NM)
   60 CONTINUE
      RETURN
      END
```

```
                  SUBROUTINE TRIA                                C   0001
C      TRIANG  OF NONSYMMETRIC STABILITY                         C   0002
      DIMENSION DUMM(220)                                        C   0003
      DIMENSION SI(20),SF(20),NSEG(20),INT(20),ISS(20),NTP(20)   C   0004
      DIMENSION Y(8,9),DY(8,9),MMP(9)                            C   0005
      DIMENSION D(36,36),E(36,36),C(36,36),CB(36)                C   0006
      DIMENSION DP(72,72),IA(6),IP(8)                            C   0007
      DIMENSION PAA(3)                                           C   0008
      DIMENSION Z(36),GA(36),GB(36) ,KE(36)                      C   0009
      COMMON NDE,X,Y,DY,MM,J9,JMAX,M9,NOUT,IPNEU,KT,DUMM,L6      C   0010
      COMMON XI,XF,ALFA,HT,  R1,R2,R3,E, SP,CSP,INDEX,IBA,PN,PL,PC,ISN C   0011
      COMMON NSEG,DP,D,DLTC,NR,EI,CI,IA,IB,C0                    C   0012
      COMMON NNM,PAA,IPA,P,NUPR,P1,NST                           C   0013
      COMMON SI,SF,NSEG,INT,ISS                                  C   0014
      COMMON IPRN,NFG,LCST                                       C   0015
      COMMON LINE2,LINE4                                         C   0016
      NRC=NDF*KT                                                 C   0019
      NPL=NRC+1                                                  C   0020
      NM=NRC/2                                                   C   0021
      DO 20 I=1,NM                                               C   0022
      GA(I)=C.0                                                  C   0023
   20 GB(I)=0.0                                                  C   0024
      LINE2=1                                                    C   0025
      LINE4=1                                                    C   0026
      REWIND 2                                                   C   0027    16
      REWIND  4                                                  C   0028    19
      DO 30 I=1,NM                                               C   0029
      DO 30 J=1,NM                                               C   0030
   30 C(I,J)=0.0                                                 C   0031
      DO 190 NP=1,IBRP                                           C   0032
      NSEG1=NSEG(NP)                                             C   0033
      IF(NOPR-1)  32, 31, 31
   31 CONTINUE
      WRITE(6,40) NP,NSEG1                                       C   0034    32
   40 FORMAT(1H0,10X,4HPART,I3,I5,10H   SEGMENTS)                C   0035
   32 CONTINUE
      DO 190 N=1,NSEG1                                           C   0036
      DO 50 I=1,NM                                               C   0037
C  36 READ(4'LINE4)                                              C   0038
   50 READ (4)                                                   C   0039
     1          (D(I,J),J=1,NM),(E(I,J),J=1,NM)                  C   0040    38
      L436=LINE4                                                 C   0041
   60 FORMAT (1H0, 8E12.5)                                       C   0042
      WRITE(6,70)                                                C   0043    40
   70 FORMAT (1H0)                                               C   0044
      DO 80 I=1,NM                                               C   0045
      DO 80 J=1,NM                                               C   0046
      DO 80 K=1,NM                                               C   0047
   80 EI(I,J)=EI(I,J)+C(I,K)*C(K,J)                              C   0048
      CALL INVERT (EI,NM,DET,ISC)                                C   0049    41
      DETE=DET                                                   C   0050
      ISCE=ISC                                                   C   0051
      DO 90 I=1,NM                                               C   0052
C  37 READ(4'LINE4)                                              C   0053
   90 READ (4)                                                   C   0054
```

```
      1           (D(I,J),J=1,NH),(DP(I,J),J=1,NH)                 C  0055   66
        L437=LINE4                                                 C  0056
        DO 100 I=1,NH                                              C  0057
        DO 100 J=1,NH                                              C  0058
        DO 100 K=1,NH                                              C  0059
    100 DM(I,J)=DP(I,J)+D( I,K)*C(K,J)                             C  0060
        DO 110 I=1,NH                                              C  0061
        DO 110 J=1,NH                                              C  0062
        C(I,J)=0.0                                                 C  0063
        DO 110 K=1,NH                                              C  0064
    110 C(I,J)=C(I,J)+DM(I,K)*E(K,J)                               C  0065
        IF(NP-IPRM)  150,120,120                                   C  0066
    120 CONTINUE                                                   C  0067
        IF(N-NSEG1) 150,130,150                                    C  0068
    130 DO 140 I3=1,NH                                             C  0069
        DO 140 J3=1,NH                                             C  0070
    140 DM(I3,J3)=C(I3,J3)                                         C  0071
    150 CONTINUE                                                   C  0072
        CALL INVERT (C,NH,DET,ISC)                                 C  0073   122
        DETC=DET                                                   C  0074
        ISCC=ISC                                                   C  0075
        DO 160 J=1,NH                                              C  0076
C    15 WRITE(2'LINE2)                                             C  0077
    160 WRITE (2)                                                  C  0078
      1            (E(I,J),I=1,NH)                                 C  0079   127
        L215=LINE2                                                 C  0080
        DO 170 J=1,NH                                              C  0081
C    16 WRITE(2'LINE2)                                             C  0082
    170 WRITE (2)                                                  C  0083
      1            (C(I,J),I=1,NH)                                 C  0084   136
        L216=LINE2                                                 C  0085
        IF(NOPR-1) 190,171,171
    171 CONTINUE
        WRITE(6,180) N,DETE,ISCE,DETC,ISCC,L436,L437,L215,L216     C  0086   144
    180 FORMAT(1H0,I5,7H  DETE=,E16.6,I5,10X,5HDETC=, E16.6,I5,5X,4I5)  C  0087
    190 CONTINUE                                                   C  0088
C       COFACTOR COMPUTATION                                       C  0089
        IF(NOPR-1) 241,211,211
    211 CONTINUE
        WRITE(6,200) DETC,ISC                                      C  0090   150
    200 FORMAT(1H0,6HDETCM=, E16.5, I5)                            C  0091
        WRITE(6,70)                                                C  0092   151
        WRITE(6,210) ALFA                                          C  0093   152
    210 FORMAT(1H0,6HALFA=   E16.5)                                C  0094
        WRITE(6,70)                                                C  0095   153
        WRITE(6,220)                                               C  0096   154
    220 FORMAT(1H0,9HCM MATRIX )                                   C  0097
        DO 230 I=1,NH                                              C  0098
    230 WRITE(6,240) (DP(I,J),J=1,NH)                              C  0099   158
    240 FORMAT (1H0, 6E12.4)                                       C  0100
        WRITE(6,70)                                                C  0101   164
    241 CONTINUE
        NINV=NH                                                    C  0102
        K12=NH-1                                                   C  0103
C       DELETE FIRST ROW                                           C  0104
C        DELETE KTH COLUMN                                         C  0105
```

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 C       - EFN   SOURCE STATEMENT  -  IFN(S)  -                  02/07/69                     PAGE
      DO 250 I=1,NH                                             C   0106
      DO 250 J=1,NH                                             C   0107
  250 D(I,J)=0.0                                                C   0108
      DO 310 K=1,NINV                                           C   0109
      DO 300 I=1,K12                                            C   0110
      IF (K-1) 280,280,260                                      C   0111
  260 KL=K-1                                                    C   0112
      DO 270 J=1,KL                                             C   0113
  270 D(I,J)=DM(I+1,J)                                          C   0114
      IF (K-NINV)  280,300,300                                  C   0115
  280 DO 290 L=K,K12                                            C   0116
  290 D(I,L)=DM(I+1,L+1)                                        C   0117
  300 CONTINUE                                                  C   0118
      CALL INVERT (D,K12,DET,ISC)                               C   0119    202
      KB(K)=ISC                                                 C   0120
  310 Z(K)=DET                                                  C   0121
      DO 320 K=2,NINV,2                                         C   0122
  320 Z(K)=-Z(K)                                                C   0123
      IF(NOPR-1) 351,321,321
  321 CONTINUE
      WRITE(6,330)                                              C   0124    210
  330 FORMAT(1H0,16H COFACTORS OF CM )                          C   0125
      DO 340 J=1,KT                                             C   0126
  340 WRITE(6,350) (Z(K),KB(K), K = J,NINV,KT)                  C   0127    222
  350 FORMAT(1H0,4(E16.5, I5,5X))                               C   0128
      WRITE (6,70)                                              C   0129    224
  351 CONTINUE
C     COFACTOR CHECK                                            C   0130
      DO 360 J=1,NH                                             C   0131
  360 D(1,J)=0.0                                                C   0132
      DO 370 J=1,NH                                             C   0133
      DO 370 K=1,NH                                             C   0134
  370 D(1,J)=D(1,J)+DM(J,K)*Z(K)*(10.0**KB(K))                  C   0135    245
      IF(NOPR-1) 391,371,371
  371 CONTINUE
      WRITE(6,380)                                              C   0136    254
  380 FORMAT(1H0,15H COFACTOR CHECK )                           C   0137
      WRITE (6,390)  (D(1,J),J=1,NH)                            C   0138    255
  390 FORMAT (1H0,4E16.5)                                       C   0139
      WRITE(6,70)                                               C   0140    262
  391 CONTINUE
      Z1=Z(1)                                                   C   0141
      DO 410 I=1,NH                                             C   0142
  400 KB(I)=KB(I)-KB(1)                                         C   0143
  410 Z(I)=Z(I)/Z1                                              C   0144
      DO 420 I=1,NH                                             C   0145
  420 Z(I)=Z(I)*(10.0**KB(I))                                   C   0146    280
      IF(NOPR-1) 441,421,421
  421 CONTINUE
      WRITE(6,430)                                              C   0147    286
  430 FORMAT(1H0,26H NORMALIZED END CONDITIONS)                 C   0148
      DO 440 J=1,KT                                             C   0149
  440 WRITE(6,390)  (Z(I),I=J,NH,KT)                            C   0150    290
      WRITE (6,70)                                              C   0151    296
  441 CONTINUE
      DO 450 I=1,NH                                             C   0152
```

```
  450 GA(I)=0.0                                            C   0153
      NHD=NDE/2                                            C   0154
      TWNH=2*NH                                            C   0155
      DO 460 I=1,NH                                        C   0156
  460 DM(2,I)=0.0                                          C   0157
      DO 870 LP=1,IBRM                                     C   0158
      LC=IBRM-LP+1                                         C   0159
      XI=SI(LC)                                            C   0160
      XF=SF(LC)                                            C   0161
      NSEGI=NSEG(LC)                                       C   0162
      XST=INT(LC)                                          C   0163
      IBR=LC                                               C   0164
      ISH=ISS(LC)                                          C   0165
      IF(LP-1) 480,480,470                                 C   0166
  470 L=NSEGI                                              C   0167
      GO TO 490                                            C   0168
  480 L=NSEGI+1                                            C   0169
  490 CONTINUE                                             C   0170
      INDEX=3                                              C   0171
      CALL INPUT                                           C   0172   331
      CALL ORTHO                                           C   0173   333
      INDEX=4                                              C   0174
      DO 870 K=1,L                                         C   0175
      IF(LP-IBRM) 550,500,500                              C   0176
  500 CONTINUE                                             C   0177
      IF(K-L) 550,510,510                                  C   0178
  510 CONTINUE                                             C   0179
      M=0                                                  C   0180
      DO 520 I=1,NHD                                       C   0181
      KD=IA(I)                                             C   0182
      J=NHD+I                                              C   0183
      KE=IA(J)                                             C   0184
      DO 520 J=1,KT                                        C   0185
      M=M+1                                                C   0186
      Y(KD,J)=GA(M)                                        C   0187
  520 Y(KE,J)=DM(2,M)                                      C   0188
      X=XI                                                 C   0189
      IPP=2                                                C   0190
      SMXX = (XF - XI)/FLOAT(NSEGI)                        C   0191
      HH=0.01*SMXX                                         C   0192
      SMAX=XI+SMXX                                         C   0193
      XOUT=SMAX                                            C   0194
      CALL RUNGE                                           C   0195   361
      DO 530 J=1,KT                                        C   0196
  530 WRITE (6,840)  (Y(I,J),I=1,NDE)                      C   0197   365
      WRITE (6,70)                                         C   0198   370
      WRITE(6,670) (IA(IC),IC=1,NDE)                       C   0199   371
      WRITE(6,820) XI                                      C   0200   376
      DO 540 J=1,KT                                        C   0201
  540 WRITE(6,840) (GA(I),I=J,NM,KT),(DM(2,I),I=J,NM,KT)   C   0202   379
      GO TO 870                                            C   0203
  550 CONTINUE                                             C   0204
      REWIND 2                                             C   0205   390
      IF (LC-1) 590,590,560                                C   0206
  560 LS=LC-1                                              C   0207
      DO 580 I7=1,LS                                       C   0208
```



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      KKKK = NSEG(I2)                                          C  0209
      DO 580 I3 = 1, KKKK                                      C  0210
      DO 570 J=1,NM                                            C  0211
  570 READ (2)  (E(I,J),I=1,NM)                                C  0212   402
      DO 580 J=1,NM                                            C  0213
  580 READ (2)  (C(I,J),I=1,NM)                                C  0214   410
  590 CONTINUE                                                 C  0215
      NMC=NSEG1-K+1                                            C  0216
      DO 610 I1=1,NMC                                          C  0217
      LLLL = TWNM                                              C  0218
      LINE2 = LINE2 - LLLL                                     C  0219
      DO 600 J=1,NM                                            C  0220
C  18 READ(2'LINE2)                                            C  0221
  600 READ (2)                                                 C  0222
     1             (E(I,J),I=1,NM)                             C  0223   427
      DO 610 J=1,NM                                            C  0224
C  19 READ(2'LINE2)                                            C  0225
  610 READ (2)                                                 C  0226
     1             (C(I,J),I=1,NM)                             C  0227   435
      LLLL = TWNM                                              C  0228
      LINE2 = LINE2 - LLLL                                     C  0229
      DO 620 I=1,NM                                            C  0230
  620 DM(4,I)=DM(2,I)                                          C  0231
      DO 630 I=1,NM                                            C  0232
      DM(1,I)=0.0                                              C  0233
      DO 630 J=1,NM                                            C  0234
  630 DM(1,I)=DM(1,I)+C(I,J)*DM(4,J)                           C  0235
      IF(LP-1) 640,640,680                                     C  0236
  640 CONTINUE                                                 C  0237
      IF(K-1) 650,650,680                                      C  0238
  650 DO 660 I=1,NM                                            C  0239
      DM(1,I)=Z(I)                                             C  0240
  660 DM(2,I)=D(1,I)                                           C  0241
      WRITE (6,70)                                             C  0242   473
      WRITE(6,661) ALFA, OMEGA                                           474
  661 FORMAT(1H0,34HDETERMINANT OF CM MATRIX AT OMEGA=,E13.5,3H IS,E13.5
     1)
      WRITE(6,662) KT                                                    475
  662 FORMAT(1H0,30X,12HSOLUTION FOR,I3,45H FOURIER COMPONENTS FOR EACH
     1VARIABLE FOLLOWS)
      WRITE(6,670) (IB(IC),IC=1,NDE)                           C  0243   476
  670 FORMAT(1H0,117,7113)                                     C  0244
      GO TO 780                                                C  0245
  680 CONTINUE                                                 C  0246
      M=0                                                      C  0247
      DO 690 I=1,NMD                                           C  0248
      I1=NMD+I                                                 C  0249
      DO 690 J=1,KT                                            C  0250
      M=M+1                                                    C  0251
      Y( I,J)=DM(1,M)                                          C  0252
  690 Y(I1,J)=DM(2,M)                                          C  0253
      IPR=2                                                    C  0254
      SMXX = (XF - XI)/FLOAT(NSEG1)                            C  0255
      HH=0.01*SMXX                                             C  0256
      IF(K-1) 700,700,710                                      C  0257
  700 X=XF-SMXX                                                C  0258
```

```
      GO TO 720
  710 X=X-2.0*SMAX
  720 SMAX=X+SMXA
      XOUT=SMAX
      CALL RUNGE
      DO 730 J=1,KT
  730 WRITE (6,840)  (Y(I,J),I=1,NDE)
      IF(K-2) 750,740,750
  740 WRITE(6,670) (I,I=1,NDE)
  750 CONTINUE
  760 CALL MODES
  770 CONTINUE
  780 CONTINUE
      IF(LP-1) 730,790,810
  790 CONTINUE
      IF(K-1) 800,800,810
  800 X=XF
  810 WRITE(6,820) X
  820 FORMAT(1H0,2HX=, E12.5)
      WRITE(6,70)
      DO 830 J=1,KT
  830 WRITE(6,840) (DM(1,I),I=J,NH,KT),(DM(2,I),I=J,NH,KT)
  840 FORMAT (1H ,8E13.6)
      WRITE(6,70)
      DO 850 I=1,NH
  850 DM(4,I)=DM(1,I)
      DO 860 I=1,NH
      DM(2,I)=0.0
      DO 860 J=1,NH
  860 DM(2,I)=DM(2,I)+F(I,J)*DM(4,J)
  870 CONTINUE
      RETURN
      END
```

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D            -  EFN   SOURCE STATEMENT  -  IFN(S)  -

      SUBROUTINE MODES                                                   D    0001
      DIMENSION Q(19)                                                    D    0002
      DIMENSION DUMM(220)                                                D    0003
      DIMENSION Y(8,9),DY(8,9),MNM(9)                                    D    0004
      DIMENSION D(36,36),EI(36,36),CI(36,36),CB(36)                      D    0005
      DIMENSION DM(72,72),IA(8),IB(8)                                    D    0006
      DIMENSION PAR(3)                                                   D    0007
      COMMON NDE,X,Y,DY,HH,J9,JMAX,M9,XOUT,IFREQ,KT,DUMM,LINE1           D    0008
      COMMON XT,XF,ALFA,HT,R2,R1,R3,E,SP,CSP,INDEX,IBR,RHH,OMSQ,IVB,ISH  D    0009
      COMMON MSEG,DM,D,DETC,NB,EI,CI,IA,IB,CB                            D    0010
      COMMON MNM,PAR,IPR,P,NOPR,P1,NS1                                   D    0011
      DO 10 I=1,19                                                       D    0012
      Q(I)=0.0                                                           D    0013
      XI=(I-1)                                                           D    0014
      ANG=XI*3.14159/18.0                                                D    0015
      DO 10 J=1,KT                                                       D    0016
      XJ=MNM(J)                                                          D    0017
      ARG=XJ*ANG                                                         D    0018
   10 Q(I)=Q(I)+Y(1,J)*COS(ARG)                                          D    0019    10
      WRITE(6,20)  Q(1)                                                  D    0020    16
   20 FORMAT(1H0,20X,30HMODE SHAPE  W AT TOP OF SHELL=, E12.5)           D    0021
      WRITE(6,30)  (Q(I),I=2,10)                                         D    0022    17
      WRITE(6,30)  (Q(I),I=11,19)                                        D    0023    24
   30 FORMAT(1H0, 9E12.5)                                                D    0024
      WRITE(6,40)                                                        D    0025    31
   40 FORMAT(1H0)                                                        D    0026
      RETURN                                                             D    0027
      END                                                                D    0028
```

```
      SUBROUTINE DIFFEQ                                                 E    0001
C     DIFFEQ  OF NONSYMMETRIC STABILITY                                 E    0002
      DIMENSION DUMM(220)                                               E    0003
      DIMENSION SI(20),SF(20),NSEG(20),INT(20),ISS(20),NIP(20)          E    0004
      DIMENSION Y(8,9),DY(8,9),MNM(9)                                   E    0005
      DIMENSION D(36,36),EI(36,36),CI(36,36),CB(36)                     E    0006
      DIMENSION DM(72,72),IA(8),IB(8)                                   E    0007
      DIMENSION PAR(3)                                                  E    0008
      DIMENSION THK(11),PHK(11),TAU(11)                                 E    0009
      DIMENSION B11(20,4),B12(20,4),B22(20,4),B66(20,4)                 E    0010
      DIMENSION MLY(20),ZLY(20,5)                                       E    0011
      DIMENSION RHO(20,4)                                               E    0012
      COMMON NDE,X,Y,DY,HH,J9,JMAX,M9,XOUT,IFREQ,KT,DUMM,LINE1          E    0013
      COMMON XI,XF,ALFA,HT,R2,R1,R3,E,SP,CSP,INDEX,IBR,RHH,OMSQ,IVB,ISH E    0014
      COMMON MSEG,DM,D,DET,N ,EI,CI,IA,IB,CB                            E    0015
      COMMON MNM,PAR,IPR,P,NOPR,P1,NST                                  E    0016
      COMMON SI,SF,NSEG,INT,ISS                                         E    0017
      COMMON IBRM,NFG,LCST                                              E    0018
      COMMON LINE2,LINE4                                                E    0019
      COMMON C11,C12,C22,E11,E12,E22,D11,D12,D22,MLY,ZLY,C66,E66,D66    E    0020
      COMMON B11,B12,B22,B66,RHO                                        E    0021
      IF(LCST-1) 20,10,10                                               E    0022
   10 CONTINUE                                                          E    0023
      NFG=3                                                             E    0024
      CALL PGEN                                                         E    0025    5
   20 CONTINUE                                                          E    0026
      INDEX=4                                                           E    0027
      CALL INPUT                                                        E    0028    8
      CALL ORTHO                                                        E    0029    10
      XN=N                                                              E    0030
      SYM=1                                                             E    0031
      DO 30 I=1,NDE                                                     E    0032
      DO 30 J=1,KT                                                      E    0033
   30 DY(I,J)=0.0                                                       E    0034
      DO 70  K=1,KT                                                     E    0035
   40 M1=MNM(K)                                                         E    0036
      XM=M1                                                             E    0037
      IF (SYM) 50,60,60                                                 E    0038
   50 XM=-XM                                                            E    0039
   60 CONTINUE                                                          E    0040
      SXR=SP*R1                                                         E    0041
      CXR=CSP*R1                                                        E    0042
      ELH=R2-SXR                                                        E    0043
      ELJ=R2+SXR                                                        E    0044
      ETH=R1*(XM*Y(7,K)+CSP*Y(3,K)+SP*Y(1,K))                           E    0045
      BTH   =R1*(SP*Y(7,K)+XM*Y(1,K))                                   E    0046
      THK(K+1)=R1*XM*BTH+CXR*Y(5,K)                                     E    0047
      DTH=-R1*(XM*Y(5,K)+CSP*BTH)                                       E    0048
      GTH=-R1*(XM*Y(3,K)+CSP*Y(7,K))                                    E    0049
      EN=Y(4,K)-C12*ETH-E12*THK(K+1)                                    E    0050
      EM=Y(6,K)-E12*ETH-D12*THK(K+1)                                    E    0051
      DEL=C11*D11-E11*E11                                               E    0052
      EPHI=(EN*D11-EM*E11)/DEL                                          E    0053
      PHK(K+1)=(EM*C11-EN*E11)/DEL                                      E    0054
      DY(1,K)=R2*Y(3,K)-Y(5,K)                                          E    0055
```

```
      DY(3,K)=EPHI-Y(1,K)*R2
      DY(5,K)=PHK(K+1)
      HKA=R1*(-2.0*XM*(Y(5,K)+CXR*Y(1,K))+XM*R2*Y(3,K)+CSP*(ELH-SXR)*
     1 Y(7,K))
      DY(7,K)=(Y(8,K)-(C66+SXR*E66)*GTH-(E66+SXR*D66)*HKA)/
     1 (C66+2.0*E66*SXR+D66*SXR*SXR)
      GPH=DY(7,K)
      TPM=E66*(GTH+DY(7,K))+D66*(HKA+SXR*DY(7,K))
      THN=C12*EPHI+C22*ETH+E12*PHK(K+1)+E22*THK(K+1)
      THM=E12*EPHI+E22*ETH+D12*PHK(K+1)+D22*THK(K+1)
      TPN=Y(8,K)-SXR*TPM
      TAU(K+1)=GTH+SXR*GPH
      DY(2,K)=R1*(SP*THN-CSP*Y(2,K)+XM*R1*(XM*THM-2.0*CSP*TPM))+
     1 R2*Y(4,K)
      DY(4,K)=R1*(CSP*(THN-Y(4,K))+XM*((R2+R1*SP)*TPM-Y(8,K)))-R2*Y(2,K)
      DY(6,K)=Y(2,K)+R1*(CSP*(THM-Y(6,K))-2.0*XM*TPM)
      DY(8,K)=R1*(CSP*(R2-R1*SP)*TPM-2.0*CSP*Y(8,K)+XM*(THN+R1*SP*THM))
   70 CONTINUE
      PHK(1)=0.0
      THK(1)=0.0
      TAU(1)=0.0
      DO 80 K=1,2
      PHK(K+9)=0.0
      THK(K+9)=0.0
   80 TAU(K+9)=0.0
      IF(IVB)  110,90,110
   90 CONTINUE
      DO 100 K=1,KT
      DY(2,K)=DY(2,K)+RHH*OMSQ*Y(1,K)
      DY(4,K)=DY(4,K)+RHH*OMSQ*Y(3,K)
  100 DY(4,K)=DY(4,K)+RHH*OMSQ*Y(7,K)
      GO TO 130
  110 CONTINUE
      DO 120 K=1,KT
  120 DY(2,K)=DY(2,K)+ALFA*(0.5*(PAR(1)*(PHK(K)+PHK(K+2))
     1 +PAR(2)    *(THK(K)+THK(K+2)))+PAR(3)    *(TAU(K)-TAU(K+2)))
  130 CONTINUE
      IF (R3-1.0)  140,160,140
  140 DO 150 I=1,NDE
      DO 150 J=1,KT
  150 DY(I,J)=DY(I,J)/R3
  160 CONTINUE
      RETURN
      END
```

```
            SUBROUTINE INPUT                                            F  0001
C     INPUT   OF NONSYMMETRIC STABILITY                                 F  0002
      DIMENSION DUMM(220)                                               F  0003
      DIMENSION SI(20),SF(20),NSEG(20),INT(20),ISS(20),NIP(20)          F  0004
      DIMENSION Y(8,9),DY(8,9),MNM(9)                                   F  0005
      DIMENSION D(36,36),EI(36,36),CI(36,36),CB(36)                     F  0006
      DIMENSION DM(72,72),IA(8),IB(8)                                   F  0007
      DIMENSION PAR(3)                                                  F  0008
      DIMENSION VN(20,4)
      COMMON NDE,X,Y,DY,HH,J9,JMAX,M9,XOUT,IFREQ,KT,DUMM,LINE1          F  0010
      COMMON XI,XF,ALFA,HT,  R1,R2,R3,E,SXN,CXS,INDEX,IBR,PN,PL,PC,ISH  F  0011
      COMMON NSEG,DM,D,DET,NF,EI,CI,IA,IB,CB                            F  0012
      COMMON MNM,PAR,IPR,P,NOPR,P1,NST                                  F  0013
      COMMON SI,SF,NSEG,INT,ISS                                         F  0014
      COMMON IBRR,NFG,LCST                                              F  0015
      COMMON LINE2,LINE4                                                F  0016
      GO TO (10,240,250,360),INDEX                                      F  0017
   10 READ (5,20)  ( VN(IBR,I),I=1,4)                                   F  0018   3
   20 FORMAT (4F10.5)                                                   F  0019
      HT=VN(IBR,1)                                                      F  0020
   60 GO TO (70,80,100,120,140,180,200,220,160), ISH                    F  0025
   70 RETURN                                                            F  0026
   80 WRITE (6,90)  ISH,VN(IBR,1),VN(IBR,2),VN(IBR,3)                   F  0027  13
   90 FORMAT (1H0,      20HCYLINDRICAL SHELL NO,I3,2X,                  F  0028
     1 4HH/L=,F10.6,2X,4HR/L=,F10.6, 6H  PHI=, F5.1)                    F  0029
      RETURN                                                            F  0030
  100 WRITE (6,110)  ISH,VN(IBR,1),VN(IBR,2), VN(IBR,4)                 F  0031  17
  110 FORMAT (1H0,        18HSPHERICAL SHELL NO,I3,4X,                  F  0032
     1 4HH/L=,F10.6,2X,4HR/L=,F10.6, 12H  DIRECTION=, F3.0 )            F  0033
      RETURN                                                            F  0034
  120 WRITE (6,130)  ISH,VN(IBR,1),VN(IBR,2), VN(IBR,4)                 F  0035  21
  130 FORMAT (1H0,      21HPARABOLOIDAL SHELL NO,I3,3X,                 F  0036
     1 4HH/L=,F10.6,2X,5H2P/L=,F10.6,  12H  DIRECTION=, F3.0)           F  0037
      RETURN                                                            F  0038
  140 WRITE (6,150)  ISH,VN(IBR,1),VN(IBR,2),VN(IBR,3),VN(IBR,4)        F  0039  25
  150 FORMAT (1H0,        20HELLIPSOIDAL SHELL NO,I3,3X,                F  0040
     1 4HH/L=,F10.6,2X,4HA/L=,F10.6, 2X,4HB/L=,F10.6,10H  DIRECTN=,F3.0)F  0041
      RETURN                                                            F  0042
  160 WRITE (6,170) ISH,VN(IBR,1),VN(IBR,2),VN(IBR,3),VN(IBR,4)         F  0043  30
  170 FORMAT (1H0,        20HHYPERBOLIC  SHELL NO,I3,3X,                F  0044
     1 4HH/L=,F10.6,2X,4HA/L=,F10.6, 2X,4HB/L=,F10.6,10H  DIRECTN=,F3.0)F  0045
      RETURN                                                            F  0046
  180 WRITE (6,190)  ISH,VN(IBR,1),VN(IBR,2),VN(IBR,3)                  F  0047  35
  190 FORMAT (1H0,        16HCONICAL SHELL NO,I3,2X,   4HH/L=,F10.6, 2X, F  0048
     14HPHI=,F10.6, 8H DEGREES, 2X,                   4HA/L=,F10.6)     F  0049
      RETURN                                                            F  0050
  200 WRITE (6,210)  ISH,VN(IBR,1),VN(IBR,2),VN(IBR,3),VN(IBR,4)        F  0051  39
  210 FORMAT (1H0,      17HTOROIDAL SHELL NO,I3,3X,                     F  0052
     1 4HH/L=,F10.6,2X,4HA/L=,F10.6, 2X,4HB/L=,F10.6,10H  DIRECTN=,F3.0)F  0053
      RETURN                                                            F  0054
  220 WRITE (6,230)  ISH, (VN(IBR,I),I=1,4)                             F  0055  44
  230 FORMAT (1H0,  16HGENERAL SHELL NO, I3, 5H H/L=,F10.6, 7H L/RFI=,  F  0056
     1F10.6, 5H R/L=,    F10.6,   4H FI=,  F6.2, 4H DEG)                F  0057
      RETURN                                                            F  0058
  240 CONTINUE                                                          F  0059
```

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F             - EFN    SOURCE STATEMENT  -  IFN(S)  -
      RETURN                                                         F   0060
  250 CONTINUE                                                       F   0061
      GO TO (260,280,290,300,310,330,340,350,320), ISH               F   0062
  260 CONTINUE                                                       F   0063
  270 RETURN                                                         F   0064
  280 CONTINUE                                                       F   0065
      R2=ABS (1.0/VN(IBR,2))                                         F   0066
      R1=0.0                                                         F   0067
      R3=1.0                                                         F   0068
      CXS=0.0                                                        F   0069
      ARG =VN(IBR,3)*1.745329E-02                                    F   0070
      SXN=SIN (ARG)                                                  F   0071    59
      GO TO 360                                                      F   0072
  290 CONTINUE                                                       F   0073
      R1=1.0/VN(IBR,2)                                               F   0074
      R3=ABS (R1)                                                    F   0075
      GO TO 360                                                      F   0076
  300 GO TO 330                                                      F   0077
  310 CONTINUE                                                       F   0078
      BSQ=(VN(IBR,3)/VN(IBR,2))*(VN(IBR,3)/ VN(IBR,2))               F   0079
      GO TO 360                                                      F   0080
  320 CONTINUE                                                       F   0081
      BSQ=(VN(IBR,3)/VN(IBR,2))*(VN(IBR,3)/ VN(IBR,2))               F   0082
      GO TO 360                                                      F   0083
  330 CONTINUE                                                       F   0084
      R1=0.0                                                         F   0085
      R3=1.0                                                         F   0086
      ALFA1= VN(IBR,2)*0.1745329E-01                                 F   0087
      SXN=SIN (ALFA1)                                                F   0088    75
      CXS=COS (ALFA1)                                                F   0089    76
      GO TO 360                                                      F   0090
  340 CONTINUE                                                       F   0091
      R1=1.0/VN(IBR,3)                                               F   0092
      R3=ABS (R1)                                                    F   0093
      GO TO 360                                                      F   0094
  350 CONTINUE                                                       F   0095
      R3=1.0                                                         F   0096
  360 CONTINUE                                                       F   0097
      GO TO(450,370,380,390,400,420,430,440,410), ISH                F   0098
  370 GO TO 450                                                      F   0099
  380 ARG=S*VN(IBR,4)                                                F   0100
      SXN=SIN (ARG)                                                  F   0101    87
      CXS=COS (ARG)                                                  F   0102    88
      R2=ABS (R1/SXN)                                                F   0103
      GO TO 450                                                      F   0104
  390 ARG=S*VN(IBR,4)                                                F   0105
      SXN=SIN (ARG)                                                  F   0106    92
      CXS=COS (ARG)                                                  F   0107    93
      R2=ABS (CXS/(SXN*VN(IBR,2)))                                   F   0108
      R1=ABS (CXS*CXS*CXS)/VN(IBR,2)                                 F   0109
      R3=ABS (R1)                                                    F   0110
      GO TO 450                                                      F   0111
  400 ARG=S*VN(IBR,4)                                                F   0112
      SXN=SIN (ARG)                                                  F   0113    99
      CXS=COS (ARG)                                                  F   0114    100
      R=SQRT (BSQ+(1.0-BSQ)*SXN*SXN)                                 F   0115    101
```

```
      R1=R*R*R/(BSC*VN(IBR,2))                          F   0116
      R2=ABS (R/(SXN*VN(IBR,2)))                        F   0117
      R3=ABS (R1)                                       F   0118
      GO TO 450                                         F   0119
  410 ARG=S*VN(IBR,4)                                   F   0120
      SXN=SIN (ARG)                                     F   0121   107
      CXS=COS (ARG)                                     F   0122   108
      R=SQRT (SXN*SXN-BSC*CXS*CXS)                      F   0123   109
      R1=-VN(IBR,2)*R*R*R/(VN(IBR,3)*VN(IBR,3))         F   0124
      R2=ABS (R/(SXN*VN(IBR,2)))                        F   0125
      R3=ABS (R1)                                       F   0126
      GO TO 450                                         F   0127
  420 CONTINUE                                          F   0128
      R2=ABS (1.0/((VN(IBR,3)+S)*CXS))                  F   0129
      GO TO 450                                         F   0130
  430 ARG=S*VN(IBR,4)                                   F   0131
      SXN=SIN (ARG)                                     F   0132   120
      CXS=COS (ARG)                                     F   0133   121
      R2=ABS (1.0/(VN(IBR,2)+VN(IBR,3)*SXN))            F   0134
      GO TO 450                                         F   0135
  440 CONTINUE                                          F   0136
      R1=VN(IBR,2)                                      F   0137
      R2=ABS (1.0/VN(IBR,3))                            F   0138
      ARG=  VN(IBR,4)*0.1745329E-01                     F   0139
      SXN=SIN (ARG)                                     F   0140   129
      CXS=COS (ARG)                                     F   0141   130
  450 RETURN                                            F   0142
      END                                               F   0143
```

```
      SUBROUTINE CRTFC
      DIMENSION DUMM(220)
      DIMENSION SI(20),SF(20),NSEG(20),INT(20),ISS(20),NTP(20)
      DIMENSION Y(8,9),YD(8,9),MNM(9)
      DIMENSION D(36,36),EI(36,36),CI(36,36),CB(36)
      DIMENSION DM(72,72),IA(6),IB(6)
      DIMENSION B11(20,4),B12(20,4),B22(20,4),B66(20,4)
      DIMENSION RHO(20,4)
      DIMENSION MLY(20),ZLY(20,5)
      DIMENSION PSR(20,4),IL1(20,4),IL2(20)
      DIMENSION PAR(3)
      COMMON NDE,S,Y,YD,HH,JY,JMAX,M9,XOUT,IFREQ,KT,DUMM,LINE1
      COMMON XX,XLD,ALFA,H,R1,R2,R3,E,SXN,CXS,INDEX,IBR,RHH,OMSQ,IVB,ISHG
      COMMON MSEG,DM,D,DET,NB,EI,CI,IA,IB,CB
      COMMON MNM,PAR,IPK,P,NOPK,P1,NST
      COMMON SI,SF,NSEG,INT,ISS
      COMMON IPRM,NFG,LCST
      COMMON LINE2,LINE4
      COMMON C11,C12,C22,E11,E12,E22,D11,D12,D22,MLY,ZLY,C66,E66,D66
      COMMON B11,B12,B22,B66,RHO
      GO TO (10,10,190,160),INDEX
   10 MK=MLY(IBR)
      DO 60 I=1,MK
      READ (5,20) B11(IBR,I),B12(IBR,I),B22(IBR,I),B66(IBR,I),
     1 AL1,AL2,RHO(IBR,I),IL1(IBR,I)                                   6
   20 FORMAT(7F10.6,I5)
      PSR(IBR,I)=B12(IBR,I)
      IF(IL1(IBR,I)) 30,60,40
   30 L3=-IL1(IBR,I)
      DY=FGEN (L3,I)                                                   22
      GO TO 60
   40 L1=IL1(IBR,I)
      L2=IL1(IBR,I)+5
      DO 50 J=L1,L2
   50 DY=FGEN (J,I)                                                    32
   60 CONTINUE
      READ (5,70)  (ZLY(IBR,I),I=1, 5),IL2(IBR)                        36
   70 FORMAT (5F10.6,I5)
      IF(IL2(IBR)) 100,100,80
   80 L2=IL2(IBR)+MK
      L1=IL2(IBR)
      DO 90 I=L1,L2
   90 DY=FGEN (I,1)                                                    55
  100 CONTINUE
      MK=MLY(IBR)
      DO 150 I=1,MK
      J=I+1
      IF(B22(IBR,I)-1.0E-6) 110,110,130
  110 E=B11(IBR,I)
      P=B12(IBR,I)
      P1=1.0-P*P
      B11(IBR,I)=E/P1
      B12(IBR,I)=P*E/P1
      B22(IBR,I)=B11(IBR,I)
      B66(IBR,I)=E*0.5/(1.0+P)
```

```
      WRITE (6,120) E,P                                                G 0056   75
 120 FORMAT (1H0, 21HYOUNG S MODULUS/EREF=,E16.6,2X, 16HPOISSONS RATIOG 0057
     1=,F10.6)                                                         G 0058
 130 WRITE (6,140) I, ZLY(IBR,I),ZLY(IBR,J), B11(IBR,I),E12(IBR,I),    G 0059
     1 B22(IBR,I),B66(IBR,I)                                           G 0060   76
 140 FORMAT( 1H0, 5HLAYER,    I3,      3H Z=,F10.6, 6H TO Z=, F10.6/   G 0061
     1 6H  B11=,  E10.3,5H B12=, E10.3,5H B22=,E10.3,5H B66=,F10.3 )   G 0062
 150 CONTINUE                                                          G 0063
      RETURN                                                           G 0064
 160 IF(IL2(IBR)) 170,170,200                                          G 0065
 170 DO 180 I=1,MK                                                     G 0066
      IF(IL1(IBR,I)) 220,180,220                                       G 0067
 180 CONTINUE                                                          G 0068
      RETURN                                                           G 0069
 190 L1=IL2(IBR)                                                       G 0070
      MK=MLY(IBR)                                                      G 0071
      L2=IL2(IBR)+MK                                                   G 0072
      IF(IL2(IBR)) 260,220,200                                         G 0073
 200 J=0                                                               G 0074
      DO 210 I=L1,L2                                                   G 0075
      J=J+1                                                            G 0076
 210 ZLY(IBR,J)=FGEN(I,2)                                              G 0077  113
 220 DO 250 I=1,MK                                                     G 0078
      IF(IL1(IBR,I)) 230,250,240                                       G 0079
 230 L3=-IL1(IBR,I)                                                    G 0080
      E=FGEN (L3,2)                                                    G 0081  126
      P=PSR(IBR,I)                                                     G 0082
      P1=1.0-P*P                                                       G 0083
      B11(IBR,I)=E/P1                                                  G 0084
      B22(IBR,I)=B11(IBR,I)                                            G 0085
      E12(IBR,I)=P*B11(IBR,I)                                          G 0086
      B66(IBR,I)=E*0.5/(1.0+P)                                         G 0087
      GO TO 250                                                        G 0088
 240 J=IL1(IBR,I)                                                      G 0089
      B11(IBR,I)= FGEN (J  ,2)                                         G 0090  138
      B12(IBR,I)= FGEN (J+1,2)                                         G 0091  140
      B22(IBR,I)= FGEN (J+2,2)                                         G 0092  142
      B66(IBR,I)= FGEN (J+3,2)                                         G 0093  144
 250 CONTINUE                                                          G 0094
 260 C11=0.0                                                           G 0095
      RHI=0.0                                                          G 0096
      C12=0.0                                                          G 0097
      C22=0.0                                                          G 0098
      C66=0.0                                                          G 0099
      E11=0.0                                                          G 0100
      E12=0.0                                                          G 0101
      E22=0.0                                                          G 0102
      E66=0.0                                                          G 0103
      D11=0.0                                                          G 0104
      D12=0.0                                                          G 0105
      D22=0.0                                                          G 0106
      D66=0.0                                                          G 0107
      H= ZLY(IBR,MK+1)-ZLY(IBR,1)                                      G 0108
      DO 270 I=1,MK                                                    G 0109
      J=I+1                                                            G 0110
      Z1=ZLY(IBR,J) -ZLY(IBR,I)                                        G 0111
```

```
      Z2=ZLY(IBR,J)*ZLY(IBR,J)-ZLY(IBR,I)*ZLY(IBR,I)                 G  0112
      Z3=ZLY(IBR,J)*ZLY(IBR,J)*ZLY(IBR,J)-ZLY(IBR,I)*ZLY(IBR,I)*ZLY(IBR,G  0113
     1I)                                                              G  0114
      C11=C11+B11(IBR,I)*Z1                                           G  0115
      C12=C12+B12(IBR,I)*Z1                                           G  0116
      C22=C22+B22(IBR,I)*Z1                                           G  0117
      C66=C66+B66(IBR,I)*Z1                                           G  0118
      E11=E11+B11(IBR,I)*Z2                                           G  0119
      E12=E12+B12(IBR,I)*Z2                                           G  0120
      E22=E22+B22(IBR,I)*Z2                                           G  0121
      E66=E66+B66(IBR,I)*Z2                                           G  0122
      D11=D11+B11(IBR,I)*Z3                                           G  0123
      D12=D12+B12(IBR,I)*Z3                                           G  0124
      D22=D22+B22(IBR,I)*Z3                                           G  0125
      D66=D66+B66(IBR,I)*Z3                                           G  0126
      RHH=RHH+RHO(IBR,I)*Z1                                           G  0127
  270 CONTINUE                                                        G  0128
      E11=0.5*E11                                                     G  0129
      E12=0.5*E12                                                     G  0130
      E22=0.5*E22                                                     G  0131
      E66=0.5*E66                                                     G  0132
      D11=0.33333333*D11                                              G  0133
      D12=0.33333333*D12                                              G  0134
      D22=0.33333333*D22                                              G  0135
      D66=0.33333333*D66                                              G  0136
      RETURN                                                          G  0137
      END                                                             G  0138
```

```
      SUBROUTINE INVERT (DPS,MAX,DETERM,ISC)
      DIMENSION DPS(36,36)
      DIMENSION DP(36,36),M(36), C(36)
      DOUBLE PRECISION DP,C,TEMP,D
      DO 10 I=1,MAX
      DO 10 J=1,MAX
   10 DP(I,J)=DPS(I,J)
      ISC=0
      DETERM = 1.
C
C     INITIALIZE BOOK-KEEPING ARRAY
      DO 20 I = 1, MAX
      M(I) = - I
   20 CONTINUE
      DO 240 II = 1, MAX
C     LOCATE LARGEST ELEMENT
      D = 0.0
      DO 70 K = 1, MAX
      IF (M(K)) 30,30,70
   30 DO 60 L = 1, MAX
      IF (M(L)) 40,40,60
   40 IF(DABS(D)-DABS(DP(K,L))) 50,50,60
   50 LD = L
      KD = K
      D = DP(K,L)
   60 CONTINUE
   70 CONTINUE
C     CALCULATE DETERMINANT
      IF(KD-LD) 80,90,80
   80 DETERM =-DETERM
   90 DETERM = D*DETERM
      XXM=1.0E-10
      XXP=1.0E 10
      DET= ABS(DETERM)
      IF(DET-1.0) 130,130,100
  100 XXP1=1.
      NXP=0
      DO 120 I=1,10
      XXP1=XXP1*XXP
      IF(DET-XXP1) 160,160,110
  110 NXP=NXP+10
  120 CONTINUE
  130 XXP1=1.0
      NXP=0
      DO 150 I=1,10
      XXP1=XXP1*XXM
      IF(DET-XXP1) 140,160,160
  140 NXP=NXP-10
  150 CONTINUE
  160 IF(NXP) 170,180,170
  170 DETERM=DETERM/10.0**NXP
  180 ISC=ISC+NXP
C     INTERCHANGE COLUMNS AND SUBSTITUTE IDENTITY ELEMENTS
C
      NEMP = -M(LD)
```

```
      M(LD)=M(KD)
      M(KD)= NEMP
      DO 190 I = 1, MAX
      C(I) = DP(I,LD)
      DP(I,LD) = DP(I,KD)
      DP(I,KD) = 0.0
  190 CONTINUE
      DP(KD,KD) = 1.
C     DIVIDE ROW BY LARGEST ELEMENT
      DO 200 J = 1, MAX
      DP(KD,J) = DP(KD,J)/D
  200 CONTINUE
C     REDUCE REMAINING ROWS AND COLUMNS
      DO 230 I = 1, MAX
      IF (I-KD) 210,230,210
  210 DO 220 J = 1, MAX
      DP(I,J) = DP(I,J) - C(I)*DP(KD,J)
  220 CONTINUE
  230 CONTINUE
  240 CONTINUE
C     INTERCHANGE ROWS
      DO 270 I = 1, MAX
      L=0
  250 L=L+1
      IF (M(L)-I) 250,260,250
  260 M(L)=M(I)
      M(I)=I
      DO 270 J = 1, MAX
      TEMP = DP(L,J)
      DP(L,J) = DP(I,J)
      DP(I,J) = TEMP
  270 CONTINUE
      DO 280 I=1,MAX
      DO 280 J=1,MAX
  280 DPS(I,J)=DP(I,J)
      RETURN
      END
```

```
      FUNCTION FGEN (NM,N1)
      DIMENSION Y(8,9),YD(8,9)
      DIMENSION XP(20,20),YP(20,20),SL(20,20),M(20)
      DIMENSION DUMM(220)
      COMMON NDF,S,Y,YD,MH,J9,JMAX,M9,XOUT,IFREQ,KT,DUMP,LINE1
      GO TO (10,90),N1
   10 READ  (5,20) NAMEG,  M(NM)
   20 FORMAT (A5,I5)
      WRITE  (6,30) NAMEG, NM,M(NM)
   30 FORMAT (1H0,  10X, A5,  32H  LINEAR FUNCTION GENERATOR NO. ,
     1I3, 5H FROM, I4, 7H POINTS)
      MK=M(NM)
      READ  (5,40) (XP(NM,I),  YP(NM,I), I=1,MK)
   40 FORMAT (8F10.5)
      MU=0
      MZ=10
      MX= (M(NM)-1)/10+1
      DO 70 J=1,MX
      L=AMINO (MK,   MZ)
      L1=MU+1
      WRITE  (6,50) (YP(NM,I), I=L1,L)
      WRITE  (6,60)  (XP(NM,I), I=L1,L)
   50 FORMAT (1H0, 13HY COORDINATES,3X,10F10.5)
   60 FORMAT (1H0, 13HX COORDINATES,3X,10F10.5)
      MZ=MZ+10
   70 MU=MU+10
      MM=M(NM)-1
      DO 80 I=1,MM
      SL(NM,I)= (YP(NM,I+1)-YP(NM,I))/ (XP(NM,I+1)-XP(NM,I))
   80 YP(NM,I)=YP(NM,I) -SL(NM,I)*XP(NM,I)
      RETURN
   90 MK=M(NM)
      DO 100 I=1,MK
      J=I
      IF(XP(NM,I)-S) 100,100,110
  100 CONTINUE
  110 FGEN= SL(NM,J-1)*S +YP(NM,J-1)
      RETURN
      END
```

J - EFN SOURCE STATEMENT - IFN(S) -

```
      SUBROUTINE RUNKUT
      DIMENSION Y(8,9),DY(8,9)
      DIMENSION Y1(8,9),Y2(8,9),Y3(8,9)
      COMMON N  ,X,Y,DY,HH,J ,JMAX,M ,XOUT,IFREQ,KT
      COMMON X1,X2,X3,Y1,Y2,Y3
      INDE9     = 0
      CALL       ADJSTP                                      3
      IF(J-JMAX) 10,10,50
   10 INDE9     = INDE9 + 1
      CALL INTPOL                                            8
      IF(J-JMAX) 20,20,50
   20 CALL STEP                                              12
      X1        = X2
      X2        = X3
      X3        = X
      DO 30 I = 1, N
      DO 30 K=1,KT
      Y1(I,K)=Y2(I,K)
      Y2(I,K)=Y3(I,K)
   30 Y3(I,K)=Y(I,K)
      IF (INDE9 - IFREQ) 10,40,40
   40 INDE9     = 0
      CALL       ADJSTP                                      32
      IF(J-JMAX) 10,10,50
   50 RETURN
      END
```

```
SUBROUTINE RUNGE                                      K   0001
DIMENSION Y(8,9),DY(8,9)                              K   0002
DIMENSION Y1(8),Y2(8),Y3(8)                           K   0003
COMMON N  ,X,Y,DY,HH,J ,JMAX,M ,XOUT,IFREQ,KT         K   0004
COMMON X1,X2,X3,Y1,Y2,Y3                              K   0005
J=1                                                   K   0006
JMAX=1                                                K   0007
IFREQ=3                                               K   0008
M=1                                                   K   0009
CALL RUNKUT                                           K   0010    6
RETURN                                                K   0011
END                                                   K   0012
```

```
      SUBROUTINE INTPCL                                             L   0001
      DIMENSION Y(8,9),DY(8,9)                                      L   0002
      COMMON NDE,X,Y,DY,HH,J ,JMAX,M ,XOUT,IFREQ,KT                 L   0003
   10 IF(ABS (XOUT    - X)-ABS (HH)) 20,20,30                       L   0004
   20 HH         = XOUT-X                                           L   0005
      CALL STEP                                                     L   0006    4
      J          = J + 1                                            L   0007
   30 RETURN                                                        L   0008
      END                                                           L   0009
```

```
      SUBROUTINE STEP                                        M  0001
      DIMENSION Y(8,9),Y1(8,9),P1(8,9),DY(8,9)               M  0002
      COMMON NDE,X,Y,DY,HH,J ,JMAX,M ,XOUT,IFREQ,KT          M  0003
      DO 10 I=1,NDE                                          M  0004
      DO 10 K=1,KT                                           M  0005
   10 Y1(I,K)=Y(I,K)                                         M  0006
      X1=X                                                   M  0007
      CALL DIFFEQ                                            M  0008   13
      DO 20 I=1,NDE                                          M  0009
      DO 20 K=1,KT                                           M  0010
      P1(I,K)=DY(I,K)*HH                                     M  0011
   20 Y(I,K)=Y1(I,K)+P1(I,K)*0.5                             M  0012
      X=X1+0.5*HH                                            M  0013
      CALL DIFFEQ                                            M  0014   29
      DO 30 I=1,NDE                                          M  0015
      DO 30 K=1,KT                                           M  0016
      P1(I,K)=P1(I,K)+2.0*HH*DY(I,K)                         M  0017
   30 Y(I,K)=Y1(I,K)+0.5*HH*DY(I,K)                          M  0018
      CALL DIFFEQ                                            M  0019   46
      DO 40 I=1,NDE
      DO 40 K=1,KT                                           M  0021
      P1(I,K)=P1(I,K)+2.0*HH*DY(I,K)                         M  0022
   40 Y(I,K)=Y1(I,K)+HH*DY(I,K)                              M  0023
      X=X1+HH                                                M  0024
      CALL DIFFEQ                                            M  0025   63
      DO 50 I=1,NDE                                          M  0026
      DO 50 K=1,KT                                           M  0027
   50 Y(I,K)=Y1(I,K)+(P1(I,K)+DY(I,K)*HH)/6.0                M  0028
      RETURN                                                 M  0029
      END                                                    M  0030
```

```
      SUBROUTINE ADJSTP                                     N  0001
      DIMENSION Y(8,9),DY(8,9)                              N  0002
      DIMENSION Y1(8,9),Y2(8,9),Y3(8,9)                     N  0003
      COMMON N  ,X,Y,DY,HH,J ,JMAX,M ,XOUT,IFREG,KT         N  0004
      COMMON X1,X2,X3,Y1,Y2,Y3                              N  0005
      KSL=0                                                 N  0006
      HFACT     = 1.0 E+31                                  N  0007
      HFACT1    = 1.0E+30                                   N  0008
      GO TO (30,10), M                                      N  0009
   10 H1        = HH                                        N  0010
      HH        = 2.0 * HH                                  N  0011
      X         = X1                                        N  0012
      DO 20 I = 1, N                                        N  0013
      DO 20 K=1,KT                                          N  0014
   20 Y(I,K)=Y1(I,K)                                        N  0015
      GO TO 100                                             N  0016
   30 KSL=1                                                 N  0017
   40 H1        = HH                                        N  0018
      XXX       = X                                         N  0019
      DO 50 I = 1, N                                        N  0020
      DO 50 K=1,KT                                          N  0021
   50 Y1(I,K)=Y(I,K)                                        N  0022
      X1        = X                                         N  0023
      CALL INTPOL                                           N  0024   30
      IF(J-JMAX) 60,60,250                                  N  0025
   60 CALL STEP                                             N  0026   34
      DO 70 I = 1, N                                        N  0027
      DO 70 K=1,KT                                          N  0028
   70 Y2(I,K)=Y(I,K)                                        N  0029
      X2        = X                                         N  0030
      CALL INTPOL                                           N  0031   47
      IF(J-JMAX) 80,80,250                                  N  0032
   80 CALL STEP                                             N  0033   51
      DO 90 I = 1, N                                        N  0034
      DO 90 K=1,KT                                          N  0035
      Y3(I,K)=Y(I,K)                                        N  0036
   90 Y(I,K)=Y1(I,K)                                        N  0037
      X3        = X                                         N  0038
      X         = XXX                                       N  0039
      HH        = 2.0 * HH                                  N  0040
  100 CALL STEP                                             N  0041   67
      DO 150 I = 1, N                                       N  0042
      DO 150 K=1,KT                                         N  0043
      DELY=ABS (Y (I,K)-Y3(I,K))/30.0                       N  0044
      IF(DELY-ABS (Y2(I,K))*1.0E-05)120,110,110             N  0045
  110 IF(ABS (Y2(I,K)) - 1.0E-04)  120,130,130              N  0046
  120 HFIRST    = 1.0E+30                                   N  0047
      GO TO 140                                             N  0048
  130 HFIRST=(ABS (Y2(I,K))* 1.0E-05/DELY   ) **0.2         N  0049   85
  140 CONTINUE                                              N  0050
  150 HFACT=AMIN1 (HFACT, HFIRST   )                        N  0051
      IF (HFACT1 - HFACT) 160,160,170                       N  0052
  160 HH        = 2.0 * H1                                  N  0053
      GO TO (40,230), M                                     N  0054
  170 HH        = H1 * HFACT                                N  0055
```

```
      N           - EFN    SOURCE STATEMENT  -  IFN(S)  -

      GO TO (180,230), M                                  N    0056
  180 IF(KSL) 220,220,190                                 N    0057
  190 KSL=0                                               N    0058
      IF(ABS (MM)-ABS (M1)) 200,220,220                   N    0059
  200 DO 210 I = 1, N                                     N    0060
      DO 210 K=1,KT                                       N    0061
  210 Y(I,K)=Y1(I,K)                                      N    0062
      K          = KAK                                    N    0063
      GO TO 40                                            N    0064
  220 KSL=0                                               N    0065
      M          = 2                                      N    0066
  230 DO 240 I = 1, N                                     N    0067
      DO 240 K=1,KT                                       N    0068
  240 Y(I,K)=Y3(I,K)                                      N    0069
  250 RETURN                                              N    0070
      END                                                 N    0071
```

```
      SUBROUTINE PGEN
      DIMENSION Y(8,9),YD(8,9)
      DIMENSION XP(21,20),YP(3,20,20),SL(3,20,20),NFT(20)
      DIMENSION SI(20),SF(20),NSEG(20),INT(20),ISS(20),NTP(20)
      DIMENSION MNM(9),DUMM(220)
      DIMENSION D(36,36),EI(36,36),CI(36,36),CB(36)
      DIMENSION DM(72,72),IA(8),IB(8)
      DIMENSION PAR(3)
      COMMON NDE,S,Y,YD,HH,JV,JMAX,M3,XOUT,IFREQ,KT,DUMM,LINE1
      COMMON XN,ALD,OMSQ,  H,H1,H2,H3,E,SAN,CXS,INDEX,IBR,PN,PL,PC,ISH
      COMMON MSEG,DM,D,DETC,NF,EI,CI,IA,IB,CB
      COMMON MNM,PAR,IPR,P,NOPR,P1,NST
      COMMON SI,SF,NSEG,INT,ISS
      COMMON IPRM,NFG,LCST
      WRITE (6,10) NFG
   10 FORMAT (1H0,I15)
      GO TO (20,30,80),NFG
   20 NCT=0
      RETURN
   30 NCT=NCT+1
      NFT(IBR)=NCT
      XP(NCT,IBR)= S
      DO 40 I=1,3
   40 YP(I,NCT,IBR)=PAR(I)
      IF(NCT-1) 70,70,50
   50 L=NCT-1
      DO 60 I=1,3
      SL(I,L,IBR)=  (YP(I,NCT,IBR)-YP(I,L,IBR))/(S-XP(L,IBR))
   60 YP(I,L,IBR)=  YP(I,L,IBR)-SL(I,L,IBR)*XP(L,IBR)
   70 RETURN
   80 J=NFT(IBR)
      DO 90 I=1,J
      L=I
      IF(XP(I,IBR)-S) 90,90,100
   90 CONTINUE
  100 IF(L-1) 110,110,120
  110 L=2
  120 DO 130 I=1,3
  130 PAR(I)=  SL(I,L-1 ,IBR)*S+YP(I,L-1,IBR)
      RETURN
      END
```

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13. ABSTRACT

This project was undertaken to present workable methods of analyses for thin, elastic shells of revolution, and to provide computer programs for performing such analyses. By means of these methods, the following problems for a thin, elastic shell of revolution can be solved: (1) stresses and deflections can be determined when the shell is subjected to arbitrary mechanical and/or thermal loads; (2) natural frequencies and mode shapes can be found for free vibration when the shell is subjected to or is free of prestress; (3) buckling loads, according to the classical stability theory, can be found when the shell is subjected to axisymmetric or sinusoidal nonsymmetric prestress. The results of the static and free-vibration analyses have been verified and compared to experiments on many occasions and should be regarded as acceptable. The buckling load, however, may or may not correspond to the actual collapse load of the shell.

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